I.ALLAKOV

SONLAR NAZARIYASIDAN MISOL VA MASALALAR

(yechimlari bilan)

O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA MAXSUS
TA'LIM VAZIRLIGI

I. ALLAKOV

SONLAR NAZARIYASIDAN MISOL VA MASALALAR

(yechimlari bilan)

O'quv qo'llanma O'zbekiston Respublikasi Oliy va o'rta maxsus ta'lim vazirligi tomonidan 5130100–matematika yo'nalishi talabalari uchun o'quv qo'llanma sifatida tavsiya etilgan.

ТОШКЕНТ – 2020

UO‘K: 511(075.8)
KBK: 22.13ya73
A 51

A51 **I. Allakov. Sonlar nazariyasidan misol va masalalar. O‘quv qo‘llanma. –T.: «Инновацион ривожланиш нашриёт-матбаа уйи», 2020. 348 bet.**

ISBN 978-9943-6726-4-2

O‘quv qo‘llanma uch qismdan iborat bo‘lib, uning birinchi qismida sonlar nazariyasida muhim bo‘lgan asosiy mavzular bo‘yicha qisqacha nazariy ma’lumotlar hamda ularning tatbiqlariga doir misol va masalalar berilgan. Ikkinchi qismida misol va masalalarning javoblari keltirilgan. Uchinchi qismida esa barcha misol va masalalar ishlab ko‘rsatilgan.

O‘quv qo‘llanma universitetdagi matematika ta’lim yo‘nalishi talabalariga mo‘ljallangan bo‘lib, undan algebra va sonlar nazariyasi fanining sonlar nazariyasi qismini o‘tishda, ayniqsa, amaliy mashg‘ulotlar va mustaqil ishlarni tashkil qilishda foydalanish mumkin. Shuningdek, o‘rta umumta’lim maktablari va akademik litseylar o‘quvchilarining sinfdan tashqari mashg‘ulotlarini tashkil qilishda hamda umuman matematikani mustaqil o‘rganuvchilar foydalanishlari mumkin.

UO‘K: 511(075.8)
KBK: 22.13ya73

Taqrizchilar:

Sh.A.Ayupov – O‘zbekiston Fanlar akademiyasi V.I.Romanovskiy nomli Matematika instituti direktori, akademik;

M.Mirsaburov – Termiz davlat universiteti matematik analiz kafedrasi mudiri, fizika-matematika fanlari doktori, professor;

X.X.Ro‘zimurodov – Samarqand davlat universiteti algebra va geometriya kafedrasi mudiri, fizika-matematika fanlari nomzodi, dotsent.

ISBN 978-9943-6726-4-2

KIRISH

O'quv qo'llanma uch qismdan iborat bo'lib, uning birinchi qismida sonlar nazariyasida muhim bo'lgan asosiy mavzular bo'yicha qisqacha nazariy ma'lumotlar hamda ularning tatbiqlariga doir misol va masalalar berilgan. Ikkinchi qismida misol va masalalarning javoblari keltirilgan. Uchunchi qismida barcha misol va masalalar ishlab ko'rsatilgan.

O'quv qo'llanma universitetdagi matematika ta'lim yo'nalishi talabalariga mo'ljallangan bo'lib, sonlar nazariyasi, algebra va sonlar nazariyasi fanlaridan darslarni o'tishda foydalanish mumkin. Shuningdek o'rta umumta'lim maktablari va akademik litseylar o'quvchilarining sinfdan tashqari mashg'ulotlarini tashkil qilishda hamda umuman matematikani mustaqil o'rganishuvchilar foydalanishlari mumkin.

Qo'llanmani yozishda Respublikamizda mavjud bo'lgan adabiyotlar:

1.Xojiyev J.X., Faynleyb A.S. Algebra va sonlar nazariyasi kursi. –T., «O'zbekiston», 2001. 304b.;

2.Isroilov M.I., Soleyev A.S. Sonlar nazariyasiga kirish. – T., "Fan". 2003. 190b.

singari darsliklaridan tashqari rus tilidagi:

1. Нестеренко Ю.В. Теория чисел. –М., Издателски центр "Академия". 2008. 272с

2. Виноградов И.М. Основи теории чисел. –.: Наука, 1981, 176с

kitoblardan va ingliz tilidagi:

1.Hardy G.H., Wright E. M. An introduction to the Theory of Numbers. 6th.ed., Oxford University Press. -2008, 480p.

2.ManinYu.I., Panchishkin A.A. Introduction to modern number theory Germany, 2007, English. 372p.

kitoblaridan ham foydalandik.

Qo'llanmani qo'lyozma holatida o'qib chiqib, uning mazmunini yaxshilash yuzasidan o'z fikrlarini bildirganlari uchun O'zbekiston Fanlar Akademiyasi V.I.Romanovskiy nomli Matematika instituti direktori, akademik Sh.A.Ayupovga, Termiz davlat universiteti

matematik analiz kafedrasi professori, fizika-matematika fanlari doktori M. Mirsaburovga, Samarqand davlat universiteti algebra va geometriya kafedrasi mudiri, fizika-matematika fanlari nomzodi, dotsent X.X.Ro'zimurodovga hamda Termiz davlat universiteti algebra va geometriya kafedrasi a'zolariga o'z minnatdorchiligimni bildiraman.

Qollanma to'g'risidagi fikr va mulohazalaringizni mamnuniyat bilan qabul qilamiz.

(Muallif iallakov@mail.ru).

I BOB. BUTUN SONLARNING BO‘LINISHI

I.1-§. Qoldiqli bo‘lish haqidagi teorema

Natural sonlar $1,2,3,\dots,n,\dots$ va ularga qarama-qarshi sonlar $-1,-2,-3,\dots,-n,\dots$ hamda 0 soni birgalikda butun sonlar deyiladi. Butun sonlar nazariyasida qoldiqli bo‘lish haqidagi teorema muhim ahamiyatga ega: ixtiyoriy butun a va $m>0$ sonlari uchun $a=mq+r$, $0\le r<m$ tenglikni qanoatlantiruvchi yagona butun q va r sonlari jufti mavjud. Bu yerda a-bo‘linuvchi, m-bo‘luvchi yoki modul, q to‘liqsiz (chala) bo‘linma va r qoldiq.

Agar $r=0$ bo‘lsa, a soni m ga bo‘linadi deyiladi va $a \vdots b$ ko‘rinishida yoziladi.

$a=mq+r, 0\le r<m$ munosabatni $\frac{a}{m}=q+\frac{r}{m}$ $(0\le\frac{r}{m}<1)$ ko‘rinishda yozish mumkin.

Bunday holda, q soni $\frac{a}{m}$ sonning butun qismi, $\frac{r}{m}$ esa uning kasr qismi hisoblanadi.

Shuning bilan birga yig‘indining bo‘linish alomati muhim tatbiqlarga ega: agar, $a \vdots m$ va b$\vdots$ m bo‘lsa, u holda, $(a+b) \vdots m$ bo‘ladi.

Quyidagi teskari teorema o‘rinli ekanligini qayd qilib o‘tish muhim: agar $(a+b) \vdots m$ va $a \vdots m$ bo‘lsa, u holda $b\vdots$ m bo‘ladi.

Sonlarning bo‘linishi refleksivlik $a \vdots a$ va tranzitivlik xossalariga ham ega, ya’ni $a \vdots b$ va $b \vdots c$ lardan $a \vdots c$ kelib chiqadi.

1. 13 ga bo‘lganda, to‘liqsiz bo‘linma 17 teng bo‘ladigan eng katta butun sonni toping.

2. Agar bo‘linuvchi va to‘liqsiz bo‘linma mos holda 1) 25 va 3 2) -30 va -4 bo‘lsa, bo‘luvchi va qoldiqni toping.

3. Isbotlang:

a) toq natural sonning kvadratini 8 ga bo‘lganda qoldiq 1ga teng bo‘ladi.

b) ketma-ket ikkita natural son kvadratlari yig‘indisini 4 ga bo‘lganda qoldiq 1ga teng.

4. $p\ge 5$ tub sonni 6 ga bo‘lganda qoldiq 1 yoki 5 bo‘lishini isbotlang.

5. $p\ge 5$ tub sonning kvadratini 24 ga bo‘lganda 1 qoldiq hosil bo‘lishini isbotlang.

6. Agar ikki butun sondan har birini m natural soniga bo'lganda 1 qoldiq qolsa, u holda ularning ko'paytmasini m ga bo'lgandagi qoldiq ham 1 ga teng bo'lishini isbotlang.

7. $3m + 2$ $(m = 1,2, ...)$ ko'rinishdagi sonlar butun sonning kvadratidan iborat emas ekanligini isbotlang.

8. Matematik induksiya metodidan foydalanib 15 ning ixtiyoriy natural darajasi 15^n ni 7 ga bo'lsak qoldiq 1 ga teng bo'lishini ko'rsating.

9. Barcha $2^{2^n} + 1$ $(n = 2,3, ...)$ ko'rinishdagi sonlar 7 raqami bilan $2^{4^n} - 5$ $(n = 1,2, ...)$ ko'rinishdagi sonlar 1 raqami bilan tugashi – ni isbotlang.

10. Ikkita toq sonning kvadratlari yig'indisi butun sonning kvadratiga teng emasligini isbotlang.

11. Pifagor uchburchagining (tomonlari natural sonlarda ifodalanadigan to'g'ri burchakli uchburchakda) hech bo'lmaganda bitta kateti 3 ga bo'linishini isbotlang.

12. Pifagor uchburchagi tomonlaridan hech bo'lmaganda bittasi 5 ga bo'linishini isbotlang.

13. $S_n = 1 + 2 + 3 + \cdots + n$ yig'indini 5 ga bo'lgandagi qoldiq 1 bo'ladigan barcha n natural sonlarni toping.

14. Agar $(ax - by) \vdots m$, $(a - b) \vdots m$ hamda b va m lar 1 dan farqli umumiy natural bo'luvchiga ega bo'lmasa, u holda $(x - y) \vdots m$ ekanligini isbotlang.

15. $4^n + 15n - 1$ $(n = 1,2, ...)$ ko'rinishdagi sonlar 9 ga karrali ekanligini isbotlang.

16. Natural argumentli $f(n) = 10^n + 18n - 1$ va $F(n) = 3^{2n+3} + 40n - 27$ funksiyalar qiymatlari mos ravishda 27 va 64 ga karrali ekanligini isbotlang.

17. $\frac{n}{2n^2+1}$ va $\frac{n}{n^2+n+1}$ ko'rinishdagi kasrlar sof davriy o'nli kasrlarga aylanishini isbotlang.

18. Agar ikkita uch xonali sonlarning yig'indisi 37 ga bo'linsa, u holda ulardan birini ikkinchisining davomidan yozish natijasida hosil bo'lgan olti xonali sonning 37 ga bo'linishini isbotlang.

19. Quyidagilarni isbotlang:

1) $(m^5 - m) \vdots 5$, 2) $m(m^2 + 5) \vdots 6$ 3) $m(m + 1)(2m + 1) \vdots 6$

20. $2n + 1$ ta ketma-ket natural sonlar yig'indisi $2n + 1$ ga karrali ekanligini isbotlang.

21.$7 \cdot 11 \cdot 13 = 1001$ ekanligini bilgan holda 7, 11 va 13 ga bo'linishning umumiy belgisini keltirib chiqaring va uni 368312 soniga qo'llang.

22. Raqamlari yig'indisi bir xil bo'lgan sonlar ayirmasining 9 ga karrali ekanligini isbotlang.

23.$S_n = 7 + 77 + 777 + \cdots + \underbrace{77\ldots7}_{n\ ta}$ – yig'indini hisoblang.

24.$48, 4488, 444888, \ldots$ sonlarni ikkita ketma-ket juft sonlarning ko'paytmasi shaklida ifodalash mumkinligini ko'rsating.

25.16, 1156, 111556, 11115556, sonlarning to'liq kvadrat bo'lishini ko'rsating.

26. Ixtiyoriy n natural soni uchun $(n+1)(n+2)\ldots(n+n)$ ning 2^n ga bo'linishini isbotlang.

I. 2-§. Eng katta umumiy bo'luvchi (EKUB) va eng kichik umumiy karrali (EKUK)

Berilgan $a_1, a_2, \ldots, a_n$ sonlarning barchasini bo'luvchi sonlarga ularning umumiy bo'luvchilari deyiladi. Umumiy bo'luvchilarining eng kattasiga berilgan sonlarning eng katta umumiy bo'luvchi (EKUB) deyiladi va uni $(a_1, a_2, \ldots, a_n)$ ko'rinishda belgilaymiz.

Berilgan $a_1, a_2, \ldots, a_n$ sonlarning barchasiga bo'linadigan sonlarga ularning umumiy karralilari (bo'linuvchilari) deyiladi. Umumiy karralilarining eng kichigiga berilgan sonlarning eng kichik umumiy karralisi (EKUK) deyiladi va uni $[a_1, a_2, \ldots, a_n]$ ko'rinishda belgilaymiz. Ta'rifdan $(a_1, a_2, \ldots, a_n) \geq 1$ va $[a_1, a_2, \ldots, a_n] \geq 1$ ekanligi kelib chiqadi.

Bu paragrafda masalalar yechimini topishda EKUB va EKUK ning quyidagi ikki asosiy xossasidan foydalanamiz:

1. Berilgan sonlar EKUBi ularning ixtiyoriy umumiy bo'luvchisiga bo'linadi.
2. Berilgan sonlarning ixtiyoriy umumiy karralisi ularning EKUKiga bo'linadi.

Bir nechta sonlarning EKUB va EKUKini topishda

$$(a_1, a_2, \ldots, a_{n-1}, a_n) = ((a_1, a_2, \ldots, a_{n-1}), a_n); \; [a_1, a_2, \ldots, a_{n-1}, a_n] = [[a_1, a_2, \ldots, a_{n-1}], a_n]$$

rekurrent formulalardan foydalanib, ikkita sonning EKUB va EKUKlarini topishga keltiramiz.

Ikkita sonning EKUBini ularning kanonik yoyilmasi (tub ko'paytuvchilar ko'paytmasiga yoyilmasi) yoki Evklid algoritmidan foydalanib topish mumkin.

a va b lar natural sonlar bo'lib $a > b$ bo'lsin. U holda qoldiqli bo'lish haqidagi teoremaga asoslangan quyidagi jarayonga Evklid algoritmi deyiladi:

$$\left.\begin{array}{ll} a = bq_1 + r_1, & 0 \le r_1 < b \\ b = r_1 \cdot q_2 + r_2, & 0 \le r_2 < r_1 \\ r_1 = r_2 \cdot q_3 + r_3, & 0 \le r_3 < r_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots & \\ r_{n-2} = r_{n-1}q_n + r_n, & 0 \le r_n < r_{n-1} \\ r_{n-1} = r_n \cdot q_n & \end{array}\right\} \quad (1)$$

Bu yerda, $b > r_1 > r_2 > \cdots > r_{n-1} > r_n$ bajarilgani uchun jarayon albatta chekli bo'ladi. Evklid algoritmidagi noldan farqli oxirgi qoldiq r_n berilgan a va b sonlarning EKUBi bo'ladi, ya'ni $r_n = (a,b)$. Agar $a_1, a_2, \ldots, a_n$ sonlari uchun $(a_1, a_2, \ldots, a_n) = 1$ bo'lsa, ular o'zaro tub, $i_2 \neq i_1$ bo'lganda $(a_{i_1}, a_{i_2}) = 1$ bo'lsa, juft-jufti bilan o'zaro tub sonlar deb ataladi. Ikkita a va b sonlarining EKUB va EKUK lari $(a,b) \cdot [a,b] = a \cdot b$ tenglik orqali bog'langan, ammo bu ko'p hollarda bir nechta sonlar uchun o'rinli emas.

Agar berilgan sonlar juft-jufti bilan o'zaro tub bo'lsa, ularning EKUKi berilgan sonlarning ko'paytmasiga teng bo'ladi.

27. Evklid algoritmidan foydalanib berilgan sonlarning EKUBini toping:

1) 546 va 231; 2) 1001 va 6253; 3) 1517 va 2257.

28. a). $(420, 126, 525)$ va $[420, 126, 525]$;

b). $(529, 1541, 1817)$ va $[529, 1541, 1817]$ ni toping.

29. a). $[6, 35, 143] = 6 \cdot 35 \cdot 143$; b). $[n, n+1] = n(n+1)$ ekanligini isbotlang.

30. Ikkita ketma-ket juft sonlarning EKUBi 2ga, ikkita ketma-ket toq sonlarning EKUBi esa 1ga teng ekanligini isbotlang.

31. $(cb, bc, ca) \vdots (a,b,c)^2$ ekanligini isbotlang.

32. Agar$(a,b)=1$ bo'lsa, u holda $(a+b,a-b)$ 1 ga yoki 2 ga teng ekanligini isbotlang.

33. Agar $\frac{a}{b}$ qisqarmaydigan kasr bo'lsa, $\frac{a}{a+b}$ kasr qisqarmaydigan kasr bo'la oladimi?

34. Ikkita toq sonlar ayirmasi 2^n ga teng. Bu sonlar o'zaro tub ekanligini isbotlang.

35. Quyidagi sonlarning EKUBini toping:

$a)$ $d=(a,b)$ va $m=[a,b]$ $b)$ $a{\cdot}b$ va $[a,b]$

$c)$ $a+b$ va ab, bunda $(a,b)=1$ $d)$ $a+b$ va $m=[a,b]$.

36. Quyidagilarni toping:

$a)$ $(n,2n+1)$, $b)$ $(10n+9,n+1)$, $c)$ $(3n+1,10n+3)$

37.$x=[a,b]$ bo'lganda va faqat shunday bo'lgandagina $\left(\frac{x}{a},\frac{x}{b}\right)=1$ bo'lishini isbotlang.

38.a,b,c toq sonlar uchun $(a,b,c)=\left(\frac{a+b}{2},\frac{a+c}{2},\frac{b+c}{2}\right)$ tenglik o'rinli ekanligini isbotlang.

39.1). Agar $a=cq+r$ va $b=cq_1+r_1$ bo'lsa, u holda $(a,b,c)=(c,r,r_1)$ ekanligini isbotlang. Bu yerda a,b,q,q_1,r,r_1- manfiy bo'lmagan butun sonlar; $c-$ musbat butun son.

2). Birinchi qismda isbotlangan qoida bo'yicha : a) (299, 391, 667), b) (588, 2058, 2849) larni toping.

40. $(a,b)=(5a+3b,13a+8b)$ tenglik o'rinli ekanligini isbotlang.

41. Uchta ketma-ket natural sonlarning EKUB va EKUKlari nimaga teng ekanligini toping.

42. n, a, b – natural sonlar va $(a,b)=1$ bo'lsa, nab sonni $ax+by$ ko'rinishda tasvirlang, bu yerda x,y lar ham natural sonlar.

43. $a=899, b=493$ uchun $d=(a,b)$ ni toping va uni $d=ax+by$ ko'rinishda ifodalab x,y larning qiymatlarini aniqlang.

44. Evklid teoremasini isbotlang: Agar $(a,c)=(b,c)=1$ bo'lsa, u holda $(ab,c)=1$bo'ladi.

45. Ikkita natural sonning EKUBi ular ayirmasidan katta bo'lishi mumkinmi?

46. Agar $(a,c)=1$ bo'lsa, u holda $b \vdots (ab,c)$ o'rinli ekanligini isbotlang.

47. Agar $(a,b)=1$ bo'lsa, u holda $(ac,b)=(c,b)$ekanligini isbotlang.

48. m, n va k natural sonlar uchun $m\cdot n\cdot k=[m,n,k]\cdot(mn,mk,nk)$ munosabat o'rinli ekanligini isbotlang.

49. Quyidagi tenglamalar sistemasini natural sonlarda yeching:

$a)\begin{cases}x+y=150\\(x,y)=30\end{cases}$; $b)\begin{cases}(x,y)=45\\\frac{x}{y}=\frac{11}{7}\end{cases}$; $c)\begin{cases}xy=8400\\(x,y)=20\end{cases}$;

$d)\begin{cases}\frac{x}{y}=\frac{5}{9}\\(x,y)=28\end{cases}$; $e)\begin{cases}xy=20\\[x,y]=10\end{cases}$.

50. $(a-bq)\,\vdots\, m$ $(0\le b\le 9)$ bo'lganda va faqat shu holdagina $N=10a+b$ natural son $m=10q+1$ ga bo'linishini isbotlang.

51. $a+b(q+1)\,\vdots\, m$ bo'lganda, va faqat shu holdagina $N=10a+b(0\le b\le 9)$ natural son $m=10q+9$ ga bo'linishini isbotlang.

52. $N=\overline{a_n\dots a_1a_0}$ soni 19 ga bo'linishi uchun, $N_1=\overline{a_n\dots a_2a_1}+2a_0$ sonning 19 ga bo'linishi zarur va yetarli ekanligini isbotlang va misollarda ko'rib chiqing.

53. 52-misoldagi qoida bo'yicha $N=3086379$ sonining 19 ga bo'linish bo'lin masligini aniqlang.

I.3-§.Tub va murakkab sonlar

Agar natural son faqat ikkita bo'linuvchi (bir va o'zi) ga ega bo'lsa, bunday natural sonlar tub sonlar deb ataladi. Agar natural son ikkitadan ortiq bo'luvchilarga ega bo'lsa, bunday sonlar murakkab sonlar deyiladi.

Tub sonlar (va ularning natural darajalari) juft-juft o'zaro tub. Birdan farqli berilgan a sonining eng kichik bo'luvchisi p tub son bo'ladi va $p\le\sqrt{a}$ bajariladi. Har bir murakkab sonni tub sonlar ko'paytmasi ko'rinishida yagona usulda tasvirlash mumkin (bu tasdiqga arifmetikaning asosiy teoremasi deyiladi), ya'ni a ni $a=p_1p_2\cdots p_n$ ko'rinishda yozish mumkin.

Agar bu yoyilmada p_1 soni α_1 marta, p_2 soni α_2 marta va hokazo p_k soni α_k marta qatnashsa $(k\le n)$, uni $a=p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k}$ ko'rinishda yozish mumkin. Bu yerdan a ning ixtiyoriy bo'luvchisi d ni

$$a=p_1^{\beta_1}p_2^{\beta_2}\cdots p_k^{\beta_k}\ (0\le\beta_i\le\alpha_i, i=\overline{1,n})\qquad(*)$$

ko'rinishda ifodalash mumkin ekanligi kelib chiqadi.

Berilgan (N_0, N_1), $(N_0 < N_1)$ oraliqdagi tub sonlarni ajratish uchun Eratosfen g'alviri deb ataluvchi usuldan foydalaniladi. Unga ko'ra berilgan oraliqdagi 2 ga bo'linadigan sonlarni o'chirib chiqamiz. Qolgan sonlar orasidan 3 ga karralilarini o'chirib chiqamiz, keyin esa 5 ga karrali sonlarni o'chirib chiqamiz va hokazo davom etib p ga (p bu $\sqrt{N_1}$ dan katta bo'lmagan va unga eng yaqin turgan tub son) bo'linadigan barcha sonlarni o'chirib chiqamiz. Bunda p ga karrali sonlarni o'chirishni p^2 dan boshlash kifoya. O'chmay qolgan sonlar izlanayotgan tub sonlar bo'ladi.

Berilgan sonning tub yoki murakkab ekanligini aniqlashda ham shunga o'xshash usuldan foydalanish mumkin. Berilgan a sonining tub yoki murakkab ekanligini aniqlash uchun uni $p \leq \sqrt{a}$ shartni qanoatlantiruvchi barcha tub sonlarga bo'lib ko'ramiz. Agar ularning birortasiga ham bo'linmasa a tub son, aks holda murakkab son bo'ladi.

54. $6n + 1$ $(n = 1,2, \dots)$ ko'rinishidagi toq sonni, tub sonlar ayirmasi ko'rinishida ifodalab bo'lmasligini isbotlang.

55. Tub sonlar ayirmasi ko'rinishida tasvirlanadigan barcha toq sonlarni toping.

56. $N = 3m + 2$ $(m = 1,2, \dots)$ sonning kvadratini natural son kvadrati va tub sonning yig'indisi ko'rinishida ifodalash mumkin emasligini isbotlang.

57. a murakkab sonning eng kichik tub bo'luvchisi $\sqrt{a}$ dan katta emasligini isbotlang. Bu teorema $a = p$ tub son o'rinli bo'ladimi?

58. Oldingi masaladagi teoremadan foydalanib
1) 127 2) 919
3) 7429 sonlarining tub yoki murakkab ekanligini aniqlang.

59. 1) 100 va 110, 2) 190 va 200, 3) 200 va 220,
4) 2640 va 2680 sonlari orasidagi barcha tub sonlarni toping.

60. n va $n!$ $(n > 2)$ natural sonlari orasida hech bo'lmaganda bitta tub son joylashganini isbotlang.

61. 20 ta ketma-ket murakkab sonni yozing.

62. n ning shunday natural qiymatlarini topingki n, $n + 10$ va $n + 14$ sonlarning barchasi tub sonlardan iborat bo'lsin.

63. Shunday p tub sonni topingki $2p^2 + 1$ ham tub son bo'lsin.

64. $4p^2 + 1$ va $6p^2 + 1$ sonlarning har ikkalasi ham tub son bo'ladigan p tub sonni toping.

65. Quyidagi sonlarning bir vaqtda tub son bo'lmasligini isbotlang:

1). $P+5$ va $p+10$; 2) $p, p+2$ va $p+5$; 3) 2^n-1 va 2^n+1, bunda $n>2$.

66. Agar p va $8p^2+1$ tub sonlar bo'lsa, u holda $8p^2+2p+1$ ham tub son ekanligini isbotlang.

67. $2^{18}+3^{18}$ ni tub ko'paytuvchilarga ajrating.

68. $a>3$ butun son va m, n lar natural sonlar 3 ga bo'lganda mos ravishda 1 va 2 qoldiqli bo'lsa uchta $a, a+m, a+n$ sonlarining bir vaqtda tub son bo'lmasligini isbotlang.

69. $n>1$ natural son bo'lsa, n^4+4 va n^4+n^2+1 larning murakkab son bo'lishini isbotlang.

70. $3, 5$ va 7 sonlari yagona egizak tub sonlar uchligi ekanligini isbotlang: (ya'ni ayirmasi 2 ga teng arifmetik progresiya tub sonlar uchligini tashkil etishini isbotlang).

71. $3n+2$ $(n=1,2,\dots)$ ko'rinishdagi tub sonlarning eng kattasi mavjud emasligini isbotlang.

72. $p_{n+1}<p_1\cdot p_2\cdot\dots\cdot p_n$ ekanligini isbotlang, bunda p_i $(i=1,2,\dots,n)$ –birinchi n ta tub son va p_{n+1} soni p_n dan keyingi tub son.

73. $p_n>2n$ ekanligini isbotlang, bunda $n=5,6,\dots$.

74. Matematik induksiya metodidan foydalanib $p_n\le 2^{2^n}$ ekanligini isbotlang. Bunda p_n bilan n –tub son belgilangan va tenglik faqatgina $n=1$ bo'lgandagina bajariladi.

75. Agar 2^n-1 tub son bo'lsa, u holda n ning ham tub son bo'lishini isbotlang.

II BOB. SONLI FUNKSIYALAR

II.1-§. $\pi(x)$ – funksiyasi

$\pi(x)$ funksiyasi x ning musbat qiymatlarida aniqlangan bo'lib, x dan katta bo'lmagan tub sonlarning sonini ifodalaydi. $\pi(x)$ ning qiymati tub sonlar jadvalidan foydalanib bevosita hisoblash yo'li bilan aniqlanadi. x ning kamma qiymatlarida esa

$$\pi(x) \approx \frac{x}{lnx} \quad \text{va} \quad \pi(x) \approx \int_2^x \frac{du}{lnu}$$

formulalardan foydalanib taqribiy topiladi .

76. Hisoblang: 1) $\pi(5)$; 2) $\pi(10)$; 3) π (25); 4) $\pi(37)$; 5) $\pi(200)$; 6) $\pi(1000)$.

77. $\pi(x) \approx \frac{x}{lnx}$ formuladan foydalanib $\pi(x)$ ning taqribiy qiymatini toping va nisbiy xatosini hisoblang. 1)$\pi(100)$, 2) $\pi(500)$, 3) $\pi(1000)$, 4) $\pi(3000)$.

78. $y = \pi(x)$ funksiyaning grafigini chizing va undan foydalanib $\pi(x) = \frac{x}{2}$ tenglamani yeching.

79. Chebishev tengsizligi $a < \pi(x) : \frac{x}{lnx} < b$, (bunda a va b lar $a < b$, $0 < a < 1$, $b > 1$ shartlarni qanoatlantiruvchi o'zgarmas sonlardir) dan foydalanib $x \to \infty$ da $\frac{\pi(x)}{x} \to 0$ ning bajarilishini ko'rsating.

80. p-tub sonlar uchun $\frac{\pi(p-1)}{p-1} < \frac{\pi(p)}{p}$ tengsizlikning, m-murakkab son uchun $\frac{\pi(m)}{m} < \frac{\pi(m-1)}{m-1}$ tengsizlikning bajarilishini isbotlang.

II. 2-§. Butun qism va kasr qism funksiyalari

$y = [\,x\,]$ – funksiyasi x ning barcha haqiqiy qiymatlarida aniqlangan bo'lib, x dan katta bo'lmagan va unga eng yaqin turgan butun sonni ifodalaydi. Bu funksiyaga x ning butun qismi deyiladi.

Tushunarliki, $[x] \le x < [x] + 1$ qo'sh tengsizlik o'rinli. x ni hamma vaqt $x = [x] + \alpha$, (bunda $0 \le \alpha < 1$) ko'rinishda yozish mumkin. Bundan $\alpha = \{x\} = x - [x]$. Bu tenglik yordamida aniqlanuvchi

$y = \{x\}$ – funksiyaga kasr qism funksiyasi yoki x ning kasr qismi deyiladi.

Agar x_1 va x_2 sonlardan hech bo'lmaganda bittasi butun son bo'lsa, u holda

$$[x_1 + x_2] = [x_1] + [x_2]$$

tenglik o'rinli bo'ladi.

Sonning butun qismi uchun $\left[\frac{x}{m}\right] = \left[\frac{[x]}{m}\right]$ ayniyat o'rinli. $n!$ sonning kanonik yoyilmasida p tub son

$$\left[\frac{m}{p}\right]+\left[\frac{m}{p^2}\right]+\cdots+\left[\frac{m}{p^s}\right]$$

daraja ko'rsatgich bilan qatnashadi, bu yerda s, $p^s \le m < p^{s+1}$ tengsizlikdan aniqlanadi.

81. Sonlarning butun qismini toping: a) -2,7; b) $2+\sqrt[3]{987}$; c) $\frac{7-\sqrt{21}}{2}$; d) $\frac{10}{3+\sqrt{3}}$; e) $1,(3)+2tg\frac{\pi}{4}$; i) $3+\sin\frac{13\pi}{7}$; j) $3-2\cos\frac{90\pi}{181}$; f) $2-\lg 2512$; l=$2-\lg\overline{abcd}$; k) $\sqrt{30}+\sqrt[3]{10}$.

82. $[\pi]^{[e]}+[e]=[e]^{[\pi]}+[\pi]$ tenglikni isbotlang. Bu yerda $\pi = 3,14...$ – aylana uzunligining diametriga nisbati va

$$e=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=2,7....$$

83. $\left[\frac{p}{4}\right]$ ning $\frac{p-1}{4}$ yoki $\frac{p-3}{4}$ ga tengligini isbotlang. Bu yerda $p > 2$ tub son.

84. $\left[\frac{a}{m}\right]=\frac{a-r}{m}$ tenglikni isbotlang , bu yerda r soni $a$ni m bo'lgandagi qoldiq.

85. $\frac{[nx]}{n}\le x<\frac{[nx]}{n}+\frac{1}{n}$, $n=1,2,...$ tengsizlikni isbotlang.

86. $\left[\frac{x+y}{n}\right]$ning $\left[\frac{x}{n}\right]+\left[\frac{y}{n}\right]$ ga yoki $\left[\frac{x}{n}\right]+\left[\frac{y}{n}\right]+1$ ga teng ekanligini isbotlang.

87. Arap m –toq son bo'lsa, u holda $\left[\frac{m}{2}\right]=\frac{m-1}{2}$ ekanligini isbotlang.

88. Funksiya grafigini chizing: $a)\ y = [x]$; $b)\ y = \{x\}$;

$c)\ y = \left[-\frac{x}{2}\right];$ $d)\ y = \left[\frac{x^2}{2} - 1\right];$ $e)\ y = [sinx].$

89. Tenglamani yeching.

a) $\left[x^2\right] = 2;$ b) $\left[3x^2 - x\right] = x + 1;$ c) $[x] = \frac{3}{4}x$ d) $\left[x^2\right] = x.$

90. $[12,4m] = 87$ tenglamani qanoatlantiruvchi m natural sonning mavjud emasligini isbotlang.

91. $[-x]$ va $[x]$ funksiyalar orasidagi bog'lanishni aniqlang.

92. $[x_1 + x_2 + \cdots + x_n] \geq [x_1] + [x_2] + \cdots + [x_n]$ tengsizlikni isbotlang.

93. $[nx] \geq n[x]$ tengsizlikni isbotlang, bunda $n = 1, 2, 3, \ldots$

94. 10^6 va 10^7 sonlarning orasida 786 ga karrali nechta natural son bor.

95. 1000 dan kichik nechta natural son 5 ga ham 7 ga ham bo'linmaydi.

96. 36 soni bilan o'zaro tub, 100 dan katta bo'lmagan natural sonlar sonini toping.

97. 2017! soni nechta nol bilan tugaydi.

98. $p^n! = 1 \cdot 2 \cdot 3 \cdots p^n$ning kanonik yoyilmasida p tub soni qanday daraja ko'rsatkich bilan ishtirok etadi.

99. 100! ko'paytmada 6 soni qanday daraja ko'rsatkich bilan ishtirok etadi.

100. 11! sonining kanonik yoyilmasini toping.

101. $N = \frac{101 \cdot 102 \cdots 1000}{7^{\alpha}}$ son butun son bo'ladigan eng katta natural sonni toping.

102. $(2m)!!$ sonining kanonik yoyilmasida p tup soni qanday daraja ko'rsatkich bilan qatnashishini toping.

103. x ning $[x] - 2\left[\frac{x}{2}\right] = 1$ tenglama to'g'ri tenglikka aylanadigan qiymatlarini toping.

104. $\left[ax^2 + bx + c\right] = d$ (bu yerda $a \neq 0, d$ –butun son) ko'rinishdagi tenglama yechimining mavjudlik shartini toping .

105. a va b lar natural sonlar, $f(x)$ berilgan kesmada manfiy bo'lmagan uzluksiz funksiya bo'lsa, $a \leq x \leq b,\ 0 \leq y \leq f(x)$ egri chiziqli trapetsiyada nechta butun koordinatali nuqtalar bo'ladi.

106. $x^2 + y^2 = 6{,}5^2$ doirada nechta butun koordinatali nuqta bor.

107. 12317 dan katta bo'lmagan va 1575 bilan o'zaro tub bo'lgan butun musbat sonlarning sonini aniqlang.

II.3-§. Berilgan sonning bo'luvchilari soni va bo'luvchilari yig'indisini ifodalovchi funksiyalar

$\tau(n)$ va $\sigma(n)$ funksiyalari n ning barcha natural qiymatlarida aniqlangan bo'lib, mos ravishda n ning barcha natural bo'luvchilari sonini va barcha natural bo'luvchilari yig'indisini ifodalaydi. Ta'rifdan $\tau(1) = \sigma(1) = 1$ ekanligi kelib chiqadi. Agar n ning kanonik yoyilmasi $n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_k^{\alpha_k}$ bo'lsa, $\tau(n)$ va $\sigma(n)$ lar mos ravishda quydagi formulalar yordamida topiladi:

$$\tau(n) = (\alpha_1 + 1)(\alpha_2 + 1) \ldots (\alpha_k + 1), (1)$$

$$\sigma(n) = \frac{p_1^{\alpha_1+1}-1}{p_1-1} \cdot \frac{p_2^{\alpha_2+1}-1}{p_2-1} \cdot \ldots \cdot \frac{p_k^{\alpha_k+1}-1}{p_k-1}. \qquad (1)$$

Ikkala funksiya ham multiplikativ funksiya ya'ni $(m, n) = 1$ lar uchun

$$\tau(m \cdot n) = \tau(m) \cdot \tau(n), \sigma(m \cdot n) = \sigma(m) \cdot \sigma(n)$$

tengliklar o'rinli.

108. Quydagi sonlarning barcha natural bo'luvchilari soni va bo'luvchilari yig`indisini toping; 1) 375; 2) 720; 3) 957; 4) 988; 5) 990; 6) 1200;

7) 1440; 8) 1500; 9) 1890; 10) 4320.

109. Berilgan sonlarning barcha natural bo'luvchilarini toping: 1) 360; 2) 720; 3) 954; 4) 988; 5) 600.

110. Noma'lum natural son x faqat ikkita tub bo'luvchiga ega ekanligi va uning bo'luvchilari soni 6 ga, bo'luvchilarining yig'indisi 28 ga teng bo'lsa, shu sonni toping.

111. $N = p^{\alpha} \cdot p^{\beta}(p, q$ lar turli tub sonlar) bo'lsin. Agar N^2 soni 15 ta har xil bo'luvchilarga ega bo'lsa, N^3nechta natural bo'luvchilarga ega bo'ladi.

112. $\tau(x)$ va $\sigma(x)$ larning grafigni sxematik tasvirlang.

113. Har bir egizak tub sonlar juftligi $p_1 < p_2$ uchun $\sigma(p_1) = \varphi(p_2)$ ekanligini isbotlang. Bunda $\varphi(a)$ –Eyler funksiyasi.

114. $\sigma(m) = 2m - 1$ tenglamaning m natural sonlarda cheksiz ko'p yechimga ega ekanligini isbotlang.

115.1). Agar $(m,n) = d > 1$ bo'lsa, $\tau(m\,n)$ va $\tau(m)\tau(n)$ larda qaysi katta?

2). Agar $(m,n) = d > 1$ bo'lsa, $\sigma(m\,n)$ va $\sigma(m)\sigma(n)$ lardan qaysi katta?

116. m natural sonining barcha natural bo'luvchilarining ko'paytmasi $\delta(m)$uchun formula chiqaring va $\delta(10)$ ni toping.

117. O'zining natural bo'luvchilarining ko'paytmasiga teng bo'lgan barcha natural sonlar to'plami barcha tub sonlar to'plami bilan ustma-ust tushishini isbotlang.

118. $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^k$ sonining bo'luvchilarining k- darajalarining yig'indisi $\sigma_k(n)$ uchun formula chiqaring.

119. $\sigma_k(n)$uchun (118-misoldagi) formuladan foydalanib hisoblang: 1) $\sigma_2(12)$; 2) $\sigma_2(18)$, 3) $\sigma_3(36)$, 4) $\sigma_2(16)$, 5) $\sigma_3(8)$.

120. $\sigma(n) = 2n$ tenglikni qanoatlantiruvchi n natural sonlarga mukammal sonlar deyiladi. 28, 496, 8128 sonlarining mukammal sonlar ekanligini tekshiring.

121. $\sigma(n) < 2n$ shartni qanoatlantiruvchi n soniga yetarli sondagi bo'luvchilarga ega emas, $\sigma(n) > 2n$ shartni qanoatlantiruvchilarga esa ortiqcha bo'luvchilarga ega bo'lgan son deyiladi. $N = p^n$ sonining yetarli bo'luvchilarga ega emasligini isbotlang. Bunda p tup son, n –natural son.

122. $N = p^\alpha \cdot q^\beta$ ko'rinishdagi toq natural sonning yetarli bo'luvchilariga ega emasligini isbotlang. Bunda p, q lar turli tub sonlar, α, β lar natural sonlar.

123. 1). Barcha bo'luvchilarining ko'paytmasi 5832 ga teng bo'lgan n natural sonini toping.

2). Barcha bo'luvchilarining ko'paytmasi $3^{30} \cdot 5^{40}$ ga teng bo'lgan n natural sonini toping.

124. $N = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}$ –ko'rinishdagi kanonik yoyilmaga ega bo'lgan sonni necha xilda 2 ta har xil ko'paytuvchiga ajratish mumkun.

125. Agar $5N$ soni N soniga qaraganda 8 ta ko'p, $7N$ soni N soniga qaraganda 12 ta ko'p, $8N$ soni ega N soniga qaraganda 18 ta ko'p bo'luvchiga ega bo'lsa, $N = 2^\alpha \cdot 5^\beta \cdot 7^\gamma$ sonini toping.

126. N soni $N = 2^x \cdot 3^y \cdot 5^z$ ko'rinishiga ega. Agar N ni 2 ga bo'lsak, hosil bo'lgan sonning bo'luvchilari soni N ning bo'luvchilari sonidan 30 taga kam. Agar N ni 3 ga bo'lsak, hosil bo'lgan sonning bo'lnuvchilari soni N ning bo'luvchilari sonidan 35 taga kam. Agarda N sonini 5 ga bo'lsak, hosil bo'lgan sonning bo'linuvchilari soni N ning bo'linuvchilari sonidan 42 ta kam bo'ladi. Shu N sonini toping.

127. Agar $2^{\alpha+1}-1$ soni tub son bo'lsa, u holda $2^{\alpha}(2^{\alpha+1}-1)$ sonining mukammal son ekanligini isbotlang (Evklid teoremasi).

128. Agar $2^{\alpha+1}-1$ tub son bo'lsa, $2^{\alpha}(2^{\alpha+1}-1)$ ning yagona juft mukammal son ekanligini isbotlang (Eyler teoremasi).

129. Bo'luvchilar yig'indisi o'zidan 3 marta katta bo'lgan $2^{\alpha}\cdot p_1\cdot p_2$, ($p_1, p_2$ lar toq tub sonlar) ko'rinishidagi eng kichik sonni toping. (Ferma masalasi).

130. Berilgan natural sonning aniq kvadrat bo'lishi uchun, uning har xil natural bo'luvchilari sonining toq bo'lishi zarur va yetarli ekanligini isbotlang.

II.4-§. Eyler funksiyasi

Eyler funksiyasi – m dan katta bo'lmagan va m bilan o'zaro tub sonlar sonini bildiradi va $\varphi(m)$ orqali belgilanadi. Agar $m=p$– tub son bo'lsa, u holda ta'rifdan $\varphi(p)=p-1$ ekanligi va agar $m=p^{\alpha}$ bo'lsa $\varphi(p^{\alpha})=p^{\alpha}-p^{\alpha-1}=p^{\alpha}\left(1-\frac{1}{p}\right)$; umuman agar $m=p_1^{\alpha_1}p_2^{\alpha_2}\dots p_n^{\alpha_n}$ bo'lsa, u holda

$$\varphi(m)=p_1^{\alpha_1}p_2^{\alpha_2}\dots p_n^{\alpha_n}\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\dots\left(1-\frac{1}{p_n}\right)$$
$$=m\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\dots\left(1-\frac{1}{p_n}\right)$$

ekanligi kelib chiqadi. Eyler funksiyasi multiplikativ funksiyadir, ya'ni u aynan nolga teng emas hamda $(m,n)=1$ shartni qanoatlantiruvchi m,n lar uchun $\varphi(mn)=\varphi(m)\varphi(n)$ bajariladi.

131. $y=\varphi(x)$ funksiyaning o'zgarishini grafik shaklda tasvirlang. Bu yerda x- natural son, $\varphi(x)$ – Eyler funksiyasi.

132. Hisoblang: 1) $\varphi(125)$, 2) $\varphi(1000)$, 3) $\varphi(180)$, 4) $\varphi(360)$, 5) $\varphi(1440)$,
6) $\varphi(1890)$, 7) $\varphi(11^3)$, 8) $\varphi(23^2)$, 8) $\varphi(12\cdot19)$, 10) $\varphi(24\cdot28\cdot45)$.

133. Maxraji m ga teng qisqarmas musbat to'g'ri kasrlarning soni nechta.

134. 1 dan 120 gacha natural sonlar orasida 30 bilan o'zaro tub bo'lmagan sonlar soni nechta.

135. Quyidagi formulalarning o'rinli ekanligini ko'rsating: *a)* $\varphi(2^{\alpha}) = 2^{\alpha-1}$; *b)* $\varphi(p^{\alpha}) = p^{\alpha-1}\varphi(p)$; *c)* $\varphi(m^{\alpha}) = m^{\alpha-1}\varphi(m)$ (m, α lar natural sonlar, p esa tub son).

136. $\varphi(2m)$ ning qiymati $\varphi(m)$ yoki $2\varphi(m)$ bo'lishi mumkinligini isbotlang. Bu hollarning har biri uchun o'rinli kriteriyani toping.

137. Quyidagi tengliklarni o'rinli ekanligini isbotlang:

$$a)\ \varphi(4n+2) = \varphi(2n+1);\ b)\ \varphi(4n) = \begin{cases} 2\varphi(n), \text{agar}(n,2) = 1 \text{ bo'lsa;} \\ 2\varphi(2n), \text{agar}(n,2) = 2 \text{ bo'lsa.} \end{cases}$$

138. Tenglamani yeching: *a)* $\varphi(5^{x}) = 100$;
b) $\varphi(7^{x}) = 294$; *c)* $\varphi(p^{x}) = p^{x-1}$; *d)* $\varphi(3^{x} \cdot 5^{x}) = 600$, bunda x va y natural sonlar.

139. Agar $m \geq 3$ bo'lsa $\varphi(m)$ning qiymati juft son ekanligini isbotlang.

140. Agar $\varphi(x) = a$ tenglamaning $x = m$ ildizi bo'lsa, u holda $x = 2m$ ham ildiz bo'lishini isbotlang. Bu yerda $(m, 2) = 1$.

141. Agar $(m, n) > 1$ bo'lsa, $\varphi(m \cdot n)$ va $\varphi(m) \cdot \varphi(n)$ sonlarini taqqoslang.

142. $\varphi(m \cdot n) = \varphi(m) \cdot \varphi(n) \cdot \frac{d}{\varphi(d)}$ ekanligini isbotlang. Bu yerda $(m, n) = d$.

143. Agar $\delta = (m, n)$ va $\mu = [m, n]$ bo'lsa, $\varphi(m \cdot n) = \varphi(\delta) \cdot \varphi(\mu)$ ekanligini isbotlang.

144. $\varphi(1) + \varphi(p) + \varphi(p^{2}) + \ldots + \varphi(p^{\alpha})$ yig'indini toping. Bunda α-natural son.

145. Gauss ayniyatini isbotlang: $\varphi(d_1) + \varphi(d_2) + \ldots + \varphi(d_k) = m$, $\left(\sum_{d \backslash m} \varphi(d) = m\right)$, bunda $d_i -$ m ning natural bo'luvchilari.

146. m bilan o'zaro tub va $m$dan kichik natural sonlar yig'indisi $\left(S = \sum_{\substack{x \leq m, \\ (x,m)=1}} 1\right)$ uchun formula chiqaring.

147. p bilan o'zaro tub va p dan katta bo'lmagan natural sonlar yig'indisi p^2 bilan o'zaro tub va p^2dan katta bo'lmagan natural sonlar sonidan ikki marta kam bo'lishini isbotlang.

148. Tenglamani yeching:
1) $\varphi(x)=p-1$, 2) $\varphi(x)=14$, 3) $\varphi(x)=8$, 4) $\varphi(x)=12$.

149. Tenglamani yeching: $a)\ \varphi(x)=2^{\alpha}$; $b)\ \varphi(p^{x})=6\cdot p^{x-2}$.

150. Tenglamani yeching: $\varphi\ (m)=3600$, bu yerda $m=3^{\alpha}\cdot 5^{\beta}\cdot 7^{\gamma}$.

151. Tenglamani yeching: $\varphi\ (x)=120$, bu yerda $x=p_1\cdot p_2$ va $p_1-p_2=2$.

152. Tenglamani yeching: $\varphi\ (m)=11424$, bu yerda $m=p_1^2\cdot p_2^2$.

153. Tenglamani tekshiring: $a)\ \ \varphi(x)=\varphi(px)$; $b)\ \ \varphi(px)=p\varphi(x)$;

$c)\ \ \varphi(p_1x)=\varphi(p_2x)$; $p_1\ ,p_2$ turli tub sonlar.

154. Tenglamani yeching: $a)\ \varphi(x)=\frac{x}{2}$; $b)\ \ \varphi(x)=\frac{x}{3}$; $c)\ \varphi(x)=\frac{x}{4}$.

155. Tenglamani tekshiring: $\varphi(p^{x})=a$.

156. Eyler funksiyasi xossalaridan foydalanib barcha tub sonlar to'plami cheksiz ekanligini isbotlang.

157. Maxraji 2 dan n gacha bo'lgan barcha musbat to'g'ri, qisqarmas kasrlar sonini aniqlang.

158. 300 dan kichik va u bilan EKUBi 20 ga teng bo'lgan natural sonlarning sonini aniqlang.

III BOB. TAQQOSLAMALAR NAZARIYASI ELEMENTLARI

III. 1-§.Taqqoslamalar va ularning asosiy xossalari

Agar ikkita butun a va b sonni $m \in N$ ga bo'lganda hosil bo'lgan qoldiqlar o'zaro teng bo'lsa, a va b sonlar m moduli bo'yicha teng qoldiqli yoki taqqoslanuvchi sonlar deyiladi va $a \equiv b(mod\ m)$ ko'rinishda belgilanadi. m modul bo'yicha taqqoslanuvchi sonlarning ayirmasi shu modulga qoldiqsiz bo'linadi.

Agar $a=b+mt$ bo'lib, b ni m ga bo'lgandagi qoldiq r bo'lsa, a ni ham m ga bo'lgandagi qoldiq r ga teng bo'ladi. Agar $a=mq+r$ bo'lsa, $a \equiv r(mod\ m)$ deb yozish mumkin. Agar $a \vdots m$ bo'lsa, $a \equiv 0(mod\ m)$ bo'ladi.

Taqqoslamalar quyidagi asosiy xossalarga ega:

1. Har bir butun son ixtiyoriy modul bo'yicha o'z-o'zi bilan taqqoslanadi.
2. Taqqoslamaning ikkala tomonini o'zaro almashtirish mumkin(simmetriklik).
3. Taqqoslamalar tranzitivlik xossasiga ega.
4. Bir xil modulli taqqoslamalarni hadlab qo'shish (ayirish), hadlab ko'paytirish mumkin.
5. Taqqoslamaning ikkala tomonini modul bilan o'zaro tub bo'lgan ularning umumiy bo'luvchisiga bo'lish mumkin.
6. Taqqoslamaning ikkala qismi va modulini bir xil songa bo'lish (ko'paytirish) mumkin.
7. Agar taqqoslama biror m modul bo'yicha o'rinli bo'lsa, u shu modulning ixtiyoriy bo'luvchisi m_1 moduli bo'yicha ham o'rinli bo'ladi.
8. Agar taqqoslama bir necha modul bo'yicha o'rinli bo'lsa, u shu modullarning eng kichik umumiy karralisi bo'yicha ham o'rinli bo'ladi.

159. Qanday modul bo'yicha barcha butun sonlar o'zi bilan taqqoslanadi.

160. 8 modul bo'yicha taqqoslanuvchi butun sonlarga misollar keltiring.

161. Quyidagi taqqoslamalardan qaysilari o'rinli:a) $1 \equiv -5(mod\ 6)$, b) $546 \equiv 0(mod\ 13)$, c) $2^3 \equiv 1(mod\ 4)$, d) $3m \equiv -1(mod\ m)$.

162. Quyidagi taqqoslamalarning o'rinli ekanligini isbotlang:

$a) 121 \equiv 13145(mod\ 2)$, $b)\ 121347 \equiv 92817(mod\ 10)$,
c) $31 \equiv -9(mod\ 10)$, $d)\ (m-1)^2 \equiv 1(mod\ m)$,
$e)\ 2m+1 \equiv (m+1)^2(mod\ m)$.

163. Quyidagi taqqoslamalarning o'rinli emasligini isbotlang.
$a)\ 5^{1812} \equiv 1964(mod\ 25)$, $b)\ 7^{103} \equiv 3(mod\ 87)$,
$c)\ 4^{1965} \equiv 25(mod\ 10)$, $d)\ 30 \cdot 17 \equiv 81 \cdot 19(mod\ 6)$,
$e)\ (2n+1)(2m+1) \equiv 2k(mod\ 6)$, bu yerda n, m va k –butun sonlar.

164. Har bir butun son berilgan modul bo'yicha o'zining qoldig'i bilan taqqoslanishini isbotlang.

165. x soni $x \equiv 2(mod 10)$ shartni qanoatlantiradi. Bu shartni parametrik tenglama ko'rinishida yozing va x ning bir nechta qiymatini toping.

166. Quyidagi taqqoslamalarni qanoatlantiruvchi x ning barcha qiymatlarini toping: a) $x \equiv 0(mod\ 3)$, $b)\ x \equiv 1(mod\ 2)$.

167. $a)$ $20 \equiv 8(mod\ m)$ $b)$ $3p+1 \equiv p+1(mod\ m)$ shartni qanoatlantiruvchi m ning qiymatini toping.

168. Agar $x = 13$ soni $x \equiv 5(modm)$ taqqoslamani qanoatlantirishi ma'lum bo'lsa, bu taqqoslamada modulning mumkin bo'lgan qiymatlarini toping.

169. 10 modul bo'yicha taqqoslanuvchi butun sonlarga misollar keltiring.

170. Quyidagi taqqoslamalardan qaysilari o'rinli:
$a)\ 1 \equiv -11(mod\ 6)$, $b)\ 3n \equiv n^2(mod\ n)$, $c)\ 2^6 \equiv 1(mod\ 7)$,
$d)\ 3m \equiv 1(mod\ m)$.

171. $x \equiv 7(mod5)$ taqqoslamani qanoatlantiruvchi x ning barcha qiymatlarini toping.

172. Butun koeffitsiyentli $F(x,y,z) = ax^3 + bx^2y + cxyz + dz$ ko'phad argumentlarining qiymatlari berilgan modul bo'yicha taqqoslanuvchi bo'lsa, u holda ko'phad qiymatlari ham shu modul bo'yicha taqqoslanuvchi bo'lishini isbotlang.

173. Agar $3^n \equiv -1(mod\ 10)$ bo'lsa, unda $3^{n+4} \equiv -1(mod\ 10)$ bo'lishini isbotlang, bu yerda n – natural son.

174. $2^{5n} - 1$ soni 31 ga bo'linishini isbotlang, bu yerda n –natural son.

175. Agar $x = 3n + 1, n = 0,1,2, ...$ bo‘lsa, $1 + 3^x + 9^x$ soni 13 ga bo‘linishini isbotlang.

176. $(a + b)^p \equiv a^p + b^p (mod\ p)$ o‘rinli bo‘lishini isbotlang.

177. Agar $a \equiv b(mod p^n)$ bo‘lsa, $a^p \equiv b^p(mod\ p^{n+1})$ ekanligini isbotlang.

178. Agar $ax \equiv bx(modm)$ bo‘lsa, u holda $a \equiv b\left(mod\frac{m}{(x,m)}\right)$ ekanligini isbotlang.

179. $a_{i+1} = 0$ bo‘lganda $\overline{a_{i+1}a_i} = a_i$ deb hisoblab, agar $\overline{a_4a_3a_2a_1} \equiv 0(mod\ 33)$ bo‘lsa, u holda $a_4 + \overline{a_3a_2} + \overline{a_1a_0} \equiv 0(mod\ 33)$ ekanligini isbotlang.

180. $p - i \equiv -i(modp)$, (bu yerda $i = 1,2, ..., n$ ekanligidan foydalanib

1) $C_{p-1}^n \equiv (-1)^n (mod\ p)$; 2) $C_{p-2}^n \equiv (-1)^n (n + 1)(mod\ p)$

o‘rinli ekanligini isbotlang.

181. 1) 9^{9^9} 2) $7^{9^{9^9}}$ sonlarning oxirgi ikki raqamini toping.

182. $p^{p+2} + (p + 2)^p \equiv 0(mod\ 2p + 2)$ taqqoslama o‘rinli ekanligini isbotlang, bu yerda $p > 2$.

183. $-\frac{p-1}{2}, -\frac{p-3}{2}, ..., -1, 0, 1, ..., \frac{p-3}{2}, \frac{p-1}{2}$ sonlarning $p>2$ modul bo‘yicha o‘zaro taqqoslanmasligini isbotlang.

184. $i \equiv i - m(mod\ m)$ ekanligidan foydalanib $\sum_{i=1}^m i^n \equiv 0(mod\ m)$ o‘rinli ekanligini isbotlang, bu yerda n va m lar toq sonlar.

185. $2^{3^n} \equiv -1(mod\ 3^{n+1})$ taqqoslama o‘rinli ekanligini isbotlang, bu yerda $n = 1,2,3, ...$

186. 185- masaladagi taqqoslamadan foydalanib $2^m + 1 \equiv 0(modm)$ shartni qanoatlantiruvchi cheksiz ko‘p $m>1$ natural sonlarning mavjudligini isbotlang.

187. $m > 1$-toq son va n – natural son uchun $(m - 1)^{m^n} \equiv -1(mod\ m^{n+1})$ ekanligini isbotlang.

188. 187-masaladagi taqqoslama yordamida $2^{2x} + 1 \equiv 0(modx)$ shartni qanoatlantiruvchi natural x sonlarning cheksiz to‘plami mavjudligini isbotlang.

189. $N = 3^{2^{4n+1}} + 2$ va $M = 2^{3^{4n+1}} + 3$, (bunda $n = 1,2,3, ...$) ko‘rinishdagi sonlarning murakkab son ekanligini isbotlang.

190. $2^x + 7^y = 19^z$ va $2^x + 5^y = 19^z$ tenglamalarning natural sonlarda yechimga ega emasligini isbotlang.

191. Agar $\frac{11a+2b}{19}$ ko'rinishdagi sonlarning butun ekanligi ma'lum bo'lsa, (a, b –butun sonlar) $\frac{18a+5b}{19}$ ko'rinishdagi son ham butun son ekanligini isbotlang.

192. Agar n toq son bo'lsa, $n^2 - 1 \equiv 0(mod 8)$ ning o'rinli ekanligini isbotlang.

193. $2^{11\cdot 31} \equiv 2(mod 11 \cdot 31)$ ning o'rinli ekanligini ko'rsating.

194. Agar $p>2$ tub son bo'lsa, $1^{2k+1} + 2^{2k+1} + 3^{2k+1} + \dots + (p-1)^{2k+1} \equiv 0(mod p)$ ning o'rinli ekanligini ko'rsating.

III.2-§.Berilgan modul bo'yicha chegirmalar sinflari

m modul bo'yicha Z-butun sonlar to'plamini quyidagicha m ta sinfga ajratamiz. m ga bo'lganda bir xil qoldiq qoladigan sonlar to'plamini bitta sinf deb qaraymiz. Ixtiyoriy $a \in Z$ sonini $a = mq + r, 0 \le r < m$ ko'rinishda tasvirlash mumkin bo'lgani uchun, $r = 0, 1, 2, \dots, m-1$ qoldiqlarga mos ravishda

$$C_0, C_1, C_2, \dots, C_{m-1} \qquad (1)$$

sinflarga ega bo'lamiz. C_i sinfning elementlari $a = mq + i$ shaklga ega bo'lib , q ga har xil qiymatlar berish natijasida bu sinfning barcha elementlarini hosil qilish mumkin. (1) ga m moduli bo'yicha chegirmalar sinflari deyiladi. m moduli bo'yicha chegirmalar sinflari to'plami $\frac{Z}{mZ} = \{C_0, C_1, C_2, \dots, C_{m-1}\}$ da qo'shish

$$C_i + C_j = \begin{cases} C_{i+j}, & \text{agar } i + \mathrm{j} < m \text{ bo'lsa;} \\ C_{i+j-m}, & \text{agar } i + \mathrm{j} \ge \mathrm{m} \text{ bo'lsa} \end{cases} \qquad (2)$$

munosabat bilan, ko'paytirish esa

$$C_i \cdot C_j = \begin{cases} C_{ij}, & \text{agar } i\mathrm{j} < m \text{ bo'lsa;} \\ C_r, & \text{agar } i\mathrm{j} \ge \mathrm{m} \text{ bo'lib } ij = mq + r \text{ bo'lsa} \end{cases} \qquad (3)$$

munosabat bilan aniqlanadi.

m moduli bo'yicha chegirmalar sinflarining har biridan bittadan element olib tuzilgan sonlar to'plami m modul bo'yicha *chegirmalarning to'la sistemasi* deyiladi.

Chegirmalarning m modul bo'yicha to'la sistemasi sifatida odatda qulaylik uchun $\{0, 1, 2, ..., m-1\}$ – manfiy bo'lmagan eng kichik chegirmalarning to'la sistemasi; $\{1, 2, ..., m-1, m\}$ – musbat eng kichik chegirmalarning to'la sistemasi; m juft bo'lsa $\{0; \pm1; \pm2; \ldots, \pm(m\text{-}2)/2; m/2\}$, m toq bo'lsa $\{0; \pm1; \pm2; \ldots, \pm(m\text{-}1)/2\}$ – absolyut qiymati jihatidan eng kichik chegirmalarning to'la sistemasi olib qaraladi.

Berilgan sonlar to'plami biror m modul bo'yicha chegirmalarning to'la sistemasini hosil qilishi uchun bu to'plam elementlari quyidagi ikki shartni qanoatlantirishi kerak:

1) ular m modul bo'yicha har xil sinflarning vakillari bo'lishi;

2) ularning soni m ga teng bo'lishi kerak .

Bu yerda quyidagi teorema keng qo'llaniladi

1-teorema. Agar x o'zgaruvchi m modul bo'yicha chegirmalarning to'la sistemasini qabul qilsa, u holda $(a,m)=1$ va b esa ixtiyoriy butun son bo'lganda $ax+b$ chiziqli forma ham m modul bo'yicha chegirmalarning to'la sistemasini qabul qiladi.

m moduli bo'yicha chegirmalarning to'la sistemasidan m bilan o'zaro tub bo'lganlarini ajratib olib sistema tuzsak hosil bo'lgan sistemaga m moduli bo'yicha *chegirmalarning keltirigan sistemasi* deyiladi.Ta'rifdan chegirmalarning keltirilgan sistemasida $\varphi(m)$ta chegirma mavjud ekanligi kelib chiqadi.

2-teorema. Agar $(a,m)=1$ bo'lib x o'zgaruvchi m modul bo'yicha chegirmalarning keltirilgan sistemasini qabul qilsa, u holda ax ham m modul bo'yicha chegirmalarning keltirilgan sistemasini qabul qiladi.

p –tub moduli bo'yicha eng kichik musbat chegirmalarning keltirilgan sistemasi $1,2,3,\ldots,p-1$, ularning to'la sistemasi $1,2,3,\ldots,p-1,p$ dan p ni tushurib qoldirib hosil qilinadi. Shuningdek, p –tub moduli bo'yicha eng katta manfiy chegirmalarning keltirilgan sistemasi $-(p-1), -(p-2), ..., -2, -1$; $p > 2$ –tub moduli bo'yicha absolyut qiymati jihatidan eng kichik chegirmalarning keltirilgan sistemasi $\pm1, \pm2, ..., \pm\frac{p-1}{2}$ lardan iborat bo'ladi.

195. 10 moduli bo'yicha barcha sinflarni taqqoslama ko'rinishda yozing.

196. Berilgan modullar bo'yicha chegirmalarning to'la va keltirilgan sistemalarini uch xil (musbat eng kichik chegirmalar, manfiy va absolyut qiymati jihatidan eng kichik chegirmalar sistemalari) ko'rinishlarida yozing:

1) $m = 9$, 2) $m = 8$, 3) $p = 13$, 4) $m = 12$, 5) $p = 7$, 6) $m = 10$.

197. 10 modul bo'yicha barcha sinflarni $x = 10q + r, 0 \leq r < 10$ formula yordamida yozing.

198. Chegirmalarning barcha sinflarini ko'rsating: *a*) 10 modul bilan o'zaro tub bo'lgan; b)10 modul bilan EKUBi 2 ga teng, d) 10 moduli bilan EKUBi 5ga teng;

e) 10 modul bilan EKUBi 10ga teng.

199. *m* modul bo'yicha har bir sinf, *md* modul bo'yicha *d* ta sinfdan tuzilganligini isbotlang.

200. 10 moduli bo'yicha bir nechta chegirmalarning to'la sistemasini toping.

201. *m* moduli bo'yicha chegirmalar sinflari to'plamining halqa bo'lishligini isbotlang. Bunda sinflar yig'indisi va ko'paytmasi mos ravishda (2) va (3) tengliklar yordamida aniqlanadi.

202. $20, -4, 22, 18, -1$ sonlari qanday modul bo'yicha chegirmalarning to'la temasini tashkil etadi.

203. $20, 31, -8, -5, 25, 14, 8, -1, 13$ va 6 sonlar sistemasining 10 moduli bo'yicha chegirmalarning keltirilgan sistemasini tashkil etmasligini isbotlang.

204. Istalgan *m* ta ketma-ket kelgan butun sonlar *m* modul bo'yicha chegirmalarning to'la sistemasini tashkil qilishini isbotlang.

205. $-\frac{m-1}{2}, -\frac{m-3}{2}, \dots, -1, 0, 1, \dots, \frac{m-3}{2}, \frac{m-1}{2}$ sonlar *m*- toq modul bo'yicha chegirmalarning to'la sistemasini tashkil qilishini isbotlang.

206. 10 moduli bo'yicha hech bo'lmaganda bitta $3x - 1$ ko'rinishdagi chegirmalarning to'la sistemasini toping.

207. 4 moduli bo'yicha $5x$ ko'rinishdagi hech bo'lmaganda bitta chegirmalarning to'la sistemasini toping.

208. Agar $ax_i + b$ $(i = 1, 2, 3, \dots, m)$ ko'rinishdagi son *m* modul bo'yicha chegirmalarning to'la sistemasini tashkil etsa, unga mos x_i sonlar ham *m* modul bo'yicha chegirmalarning to'la sistemasini tashkil qilishini isbotlang.

209. Агар $a_n x_i^n + a_{n-1} x_i^{n-1} + \dots + a_1 x_i + a_0, (i = 1,2, \dots, m)$ ko'rinishdagi sonlar *m* modul bo'yicha chegirmalarning to'la sistemasini hosil qilsa, u holda unga mos x_i sonlar ham m modul bo'yicha chegirmalarning to'la sistemasini hosil qiladi va aksincha ekanligini *isbotlang.*

210. 6 moduli bo'yicha bir nechta chegirmalarning keltirilgan sistemasini tuzing.

211. Nima uchun -5, 13, 11, -21, 5 sonlar sistemasi 12 moduli bo'yicha chegirmalarning keltirilgan sistemasini tashkil etmaydi.

212. p modul bo'yicha chegirmalarning keltirilgan sistemasi $p-1$ ta chegirmadan tuzilganligini isbotlang.

213. $-\frac{p-1}{2}, -\frac{p-3}{2}, \dots, -1, 1, \dots, \frac{p-3}{2}, \frac{p-1}{2}$ sonlar sistemasi $p>2$ modul bo'yicha chegirmalarning keltirilgan sistemasini tashkil etishini isbotlang.

214. $5, 5^2, 5^3, 5^4, 5^5, 5^6$ sonlar sistemasining 7 modul bo'yicha chegirmalarning keltirilgan sistemasi ekanligini isbotlang.

215. Agar $ax_i, (i = 1,2, \dots, \varphi(m))$ sonlari m modul bo'yicha chegirmalarning keltirilgan sistemasini tashkil qilsa, u holda ularga mos x_i sonlarining ham m modul bo'yicha chegirmalarning keltirilgan sistemasini tashkil etishini isbotlang (yuqoridagi ikkinchi teoremaga teskari teorema).

216. Agar $(a,m)=1$, $b\equiv 0(mod m)$ va x o'zgaruvchining qiymatlari m modul bo'yicha chegirmalarning keltirilgan sistemasini tashkil etsa, unda $ax+b$ funksiyaning qiymatlari ham m modul bo'yicha chegirmalarning keltirilgan sitsemasini tashkil qilishini isbotlang.

217. Agar $(a,m)=d$ va x o'zgaruvchining qiymatlari $\frac{m}{d}$ modul bo'yicha chegirmalarning to'la sistemasini tashkil etsa, u holda $\frac{a}{d}x+b$ funksiyaning mos qiymatlari ham $\frac{m}{d}$ modul bo'yicha chegirmalarning to'la sistemasini tashkil qilishini isbotlang.

218. Agar $(a,m)=d$ va x o'zgaruvchining qiymatlari $\frac{m}{d}$ modul bo'yicha chegirmalarning keltirilgan sistemasini tashkil etsa, u holda $\frac{a}{d}x$ funksiyaning mos qiymatlari ham $\frac{m}{d}$ modul bo'yicha chegirmalarning keltirilgan sistemasini tashkil qilishini isbotlang.

219. $m=9$ moduli bo'yicha chegirmalarning to'la va keltirilgan sistemalarini 3 xil (musbat, manfiy bo'lmagan, absolyut qyimati jihatidan eng kichik chegirmalar) ko'rinishda yozing.

III. 3-§. Eyler va Ferma teoremalari

Eyler teoremasi. Agar $m > 1$ va $(a, m)=1$ bo‘lsa, $a^{\varphi(m)} \equiv 1(mod m)$ bo‘ladi. Xususiy holda, agar $m= p$ tub songa teng bo‘lsa, Eyler teoremasidan quyidagi Ferma teoremasi kelib chiqadi.

Ferma teoremasi. Agar p tub son va $(a, p)=1$ bo‘lsa, u holda $a^{p-1} \equiv 1(mod\ m)$ bo‘ladi.

Ferma teoremasidan ixtiyoriy a butun musbat soni uchun $a^p - a \equiv 0(modp)$ ning bajarilishi kelib chiqadi.

220. a) agar $(a, 7) = 1$ bo‘lsa, $(a^{12} - 1) \vdots 7$; b) agar $(a, 65) = (b, 65) = 1$ bo‘lsa, $(a^{12} - b^{12}) \vdots 65$ ekanligini isbotlang.

221. Kanonik yoyilmasiga 2 va 5 kirmaydigan n natural sonining $12-$ darajasining birliklar xonasidagi raqami 1 ga teng ekanligini isbotlang.

222. $a^{p-1} + p - 1$ ko‘rinishdagi son murakkab ekanligini isbotlang, bu yerda $a \not\equiv 0(modp)$.

223. $2^{11 \cdot 31} \equiv 2(mod 11 \cdot 31)$ekanligini isbotlang .

224. 2^{30} sonni 13 ga bo‘lgandagi qoldiqni toping.

225. 3^{59} sonini 17 ga bo‘lgandagi qoldiqni toping.

226. $a^{n(p-1)+1} \equiv a(modp)$ ekanligini isbotlang.

227. 317^{259}sonini 15 ga bo‘lgandagi qoldiqni toping.

228. $3^{80} + 7^{80}$ sonini *11* ga bo‘lgandagi qoldiqni toping.

229. $3^{100} + 4^{100}$ sonini *7* ga bo‘lgandagi qoldiqni toping.

230. 197^{157} sonini 35 ga bo‘lgandagi qoldiqni toping.

231. $n = 73 \cdot 37$ uchun $2^{n-1} \equiv 1(mod\ n)$ ekanligini ko‘rsating.

232. $1^{30} + 2^{30} + \ \dots\ + 10^{30} \equiv -1(mod 11)$ekanligini isbotlang.

233. Ixtiyoriy x butun soni uchun 1) $x^7 \equiv x(mod 42)$; 2) $x^{13} \equiv x(mod 2730)$ ekanligini isbotlang.

234. Agar p va q lar har xil tub sonlar bo‘lsa, $p^{q-1} + q^{p-1} \equiv 1(modpq)$ ekanligini isbotlang.

235. 2^{100} sonining oxirgi ikkita raqamini toping.

236. 3^{100} sonining oxirgi raqamini toping.

237. 243^{402} sonining oxirgi uchta raqamini toping.

238. Agar $(n, 6) = 1$ bo‘lsa, $n^2 \equiv 1(mod 24)$ ekanligini isbotlang.

239. Agar p tub son bo'lsa, $\sum_{i=1}^{p-1} i^{k(p-1)} + 1 \equiv 0(mod p)$ taqqoslamaning o'rinli ekanligini ko'rsating.

240. Agar p tub son bo'lsa, $(\sum_{i=1}^{n} a_i)^p \equiv \sum_{i=1}^{n} a_i^p (mod p)$ taqqoslamaning o'rinli ekanligini ko'rsating.

241. Agar $(a, m) = 1$ bo'lsa $a^x \equiv 1(mod m)$ taqqoslamaning eng kichik natural yechimi φ(m) ning bo'luvchisi ekanligini isbotlang.

242. Agar $N = \sum_{i=1}^{n} a_i$ soni 30ga bo'linsa, u holda $M = \sum_{i=1}^{n} a_i^5$ sonining ham 30 ga bo'linishini isbotlang.

243. Ixtiyoriy butun sonning 100 –darajasi 125 ga bo'linadi yoki 125 ga bo'lganda 1 qoldiq qolishini isbotlang.

244. Agar $(a, 10) = 1$ bo'lsa, $a^{100n+1} \equiv a(mod 1000)$ ning bajarilishini ko'psating. Bunda n natural son.

245. m va n lar natural sonlar bo'lsalar, $a^{6m} + a^{6n} \equiv 0(mod 7)$ taqqoslamaning faqat a soni 7 ga karrali bo'lgandagina o'rinli ekanligini isbotlang.

246. $5^{p^2} + 1 \equiv 0\ (mod p^2)$ taqqoslamani qanoatlantiruvchi p tub sonini toping.

247. $p > 3$ tub son bo'lsa, p va $2p + 1$ lar tub sonlar bo'lsalar, u holda $4p + 1$ ning murakkab son ekanligini ko'rsating.

IV BOB. BIR NOMA'LUMLI TAQQOSLAMALAR

IV. 1-§. Bir noma'lumli taqqoslamalar (umumiy ma'lumotlar)

Ixtiyoriy darajali taqqoslamalar yechimlari sinflari. Faraz qilaylik $f(x)$ n-darajali butun koeffitsiyentli ko'phad bo'lsin, ya'ni $f(x)=a_0x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n$. U holda

$$f(x)\equiv 0(mod m) \qquad (1)$$

taqqoslamaga n-darajali bir noma'lumli taqqoslama deyiladi.

(1) da a_0 soni m ga bo'linmaydi, ya'ni $a_0\not\equiv 0(mod m)$. (1) ni yechish bu uni qanoatlantiruvchi barcha x larni topish yoki uning yechimining yo'q ko'rasatish demakdir. Lekinda agar x_1 (1) ning yechimlaridan biri bo'lsa, ya'ni $f(x_1)\equiv 0(mod m)$ bo'lsa, u holda $x\equiv x_1(\mathrm{mod}\, m)$ taqqoslamani qanoatlantiruvchi barcha sonlar ham (1) ning yechimi bo'ladi. Haqiqatan ham, $x\equiv x_1(\mathrm{mod}\, m)$ ni $x=x_1+mt,\ t\in Z$ deb yoza olamiz. Buni (1) ga olib borib qo'ysak:

$$f(x)=a_0(x_1+mt)^n+a_1(x_1+mt)^{n-1}+\ldots+a_{n-1}(x_1+mt)+a_n=$$
$$=a_0x_1^n+a_1^{n-1}x_1^{n-1}+\ldots+a_{n-1}x_1+a_n+mT(x_1)=f(x_1)+mT(x_1).$$

Bundan taqqoslamaga o'tsak, $f(x_1+mt)\equiv 0(mod m)$ ni hosil qilamiz. Shuning uchun ham (1) ning yechimi, deganda alohida olingan bitta x_1 son emas, balki x_1+mt sinf bitta yechim deb tushuniladi. m modul bo'yicha m ta chegirmalar sinflari mavjud bo'lganligi sababli (1) ning barcha yechimlarini m moduli bo'yicha chegirmalarning to'la sistemasidagi chegirmalarni qo'yib sinab ko'rish yo'li bilan topish mumkin. Bu usulga tanlash usuli deyiladi.

Agarda bir xil noma'lumli ikkita taqqoslamaning yechimlari to'plami bir xil bo'lsa, ular teng kuchli taqqoslamalar deyiladi. Quyidagi almashtirishlar natijasida hosil bo'lgan taqqoslamalar teng kuchlidir:

1)taqqoslamaning ikkala tomoniga yoki uning istalgan tomoniga modulga karrali bo'lgan sonni qo'shish;

2)taqqoslamaning ikkala tomonini modul bilan o'zaro tub songa ko'paytirish yoki bo'lish;

3) taqqoslamaning ikkala tomonini va modulini bir xil songa bo'lish;

Agarda berilgan taqqoslamani ixtiyoriy butun son qanoatlantirsa, u holda bu taqqoslamaga ayniy taqqoslama deyiladi. Ayniy taqqoslamaga misol sifatida Ferma teoremasidan kelib chiqadigan $x^p-x\equiv 0(\mathrm{mod}\, p)$

(p-tub son) taqqoslamani olish mumkin. Shuningdek, agar $f(x)$ ko'phadning barcha koeffitsiyentlar m ga bo'linsa, $f(x)\equiv 0(\mathrm{mod}\,m)$ taqqoslama ayniy taqqoslama bo'ladi.

248. Eng kichik manfiy bo'lmagan chegirmalarning to'la sistemasidagi chegirmalarni sinash yo'li bilan quyidagi taqqoslamalarning yechimini toping:

$a)\ 5x^2-15x+22\equiv 0(mod3),\ b)\ x^2+2x+2\equiv 0\ (mod5),$
$c)\ x^2-2x+2\equiv 0(mod3),\qquad d)\ x^3-2\equiv 0(mod5),$
$e)\ 2x^3-3x^2+2x-1\equiv 0(mod7),\qquad f)\ 2x\equiv 7(mod15),$
$i)\ 2x^3+3x-5\equiv 0(mod7).$

249. $x^3-x+1\equiv 0(mod3)$ taqqoslamani yeching.

250. Avvalo soddalashtirib, keyin absolyut qiymati jihatidan eng kichik chegirmalarni sinab ko'rish yo'li bilan quyidagi taqqoslamalarni yeching:

$a)\ 90x^{20}+46x^2-52x+46\equiv 0(mod15),$
$b)\ 25x^3-36x^2-18x+13\equiv 0(mod12),$
$c)\ 21x+4\equiv 7(mod6),\qquad d)\ x^5-2x^3+13x-1\equiv 0(mod4).$

251. $7x^3+12x^2-x+24\equiv 0(mod3)$ taqqoslamani noma'lum x ning barcha butun qiymatlarining qanoatlantirishini tekshiring.

252. Quyidagi taqqoslamalarni noma'lum x ning barcha butun qiymatlarining qanoatlantirishini tekshiring:

$a)x^3-x+6\equiv 0(mod3);\qquad b)\ x(x^2-1)\equiv 0(mod6);$
$c)\ 20x^5+x^4-10x^3-1\equiv 0(mod5);\ d)x^{13}-26x^{12}-x\equiv 0(mod13).$

253. Quyidagi taqqoslamalarni noma'lum x ning birorta ham butun qiymatlarining qanoatlantirmasligini tekshiring:

$a)5x\equiv 4(mod5);\quad b)\ x^2-2x+3\equiv 0(mod4);\quad c)\ 20x^5+5x^4-10x^3-1\equiv 0(mod5);\quad d)x^{13}-26x^{12}-x+5\equiv 0(mod13).$

254. $a)$ $(m,n)=1$ bo'lsa, n-darajali $x^n+a_1x^{n-1}+a_2x^{n-2}+\cdots+a_n\equiv 0(modm)$ taqqoslamani yangi o'zgaruvchi y kiritish yo'li bilan $(n-1)-$ darajali hadi qatnashmagan $y^n+b_2y^{n-2}+\cdots+b_n\equiv 0(modm)$ ko'rinishdagi taqqoslamaga keltirish mumkin ekanligini ko'rsating.

255. 254.*a)* dan foydalanib $x^3 + 5x^2 + 6x - 8 \equiv 0(mod13)$ taqqoslamani uch hadli $y^3 + px + q \equiv 0(mod13)$ taqqoslama ko'rinishiga keltiring.

256. $x^{\varphi(60)} \equiv 1(mod60)$ taqqoslamani yeching.

IV.2-§. Bir noma'lumli birinchi darajali taqqoslamalar.

Birinchi darajali $a_0x + a_1 \equiv 0(modm)$ taqqoslamani hamma vaqt

$$ax \equiv b(\text{mod}m) \qquad (1)$$

ko'rinishga keltirish mumkin. Shuning uchun ham biz (1) ni tekshiramiz. Avvalo, faraz etaylik $(a,m)=1$ bo'lsin. U holda x o'zgaruvchi m moduli bo'yicha chegirmalarning to'la sistemasini qabul qilsa ax ham shu sistemasini qabul qiladi. Shuning uchun ham x ning faqat bitta qiymatida ax soni b tegishli bo'lgan sinfga qarashli bo'ladi. Shu qiymatda $ax_1 \equiv b(modm)$ga ega bo'lamiz. Shunday qilib, agar $(a,m)=1$ bo'lsa, (1) taqqoslama bitta (yagona) $x \equiv x_1(modm)$ (yoki $x \equiv x_1 + mt,\ t = 0, \pm1, \pm2, \dots$) yechimga ega bo'lar ekan.

Endi, faraz etaylik $(a,m) = d > 1$ bo'lsin. Bu holda agar b soni d ga bo'linsa, $a = a_1 \cdot d,\ b = b_1 \cdot d,\ m = m_1 \cdot d$ deb olib (1) dan

$$a_1x \equiv b_1(\text{mod } m_1), \qquad (a_1, m_1) = 1 \qquad (2)$$

taqqoslamani hosil qilamiz. Bu (2) taqqoslama esa yuqorida qarab chiqilgan holga ko'ra yagona yechim $x \equiv x_1(\text{mod}m_1)$ ga ega bo'ladi. Biz m moduli bo'yicha $(m=m_1 \cdot d)$ (1) taqqoslamaning yechimlarini topishimiz kerak. Buning uchun (2) ning yechimlari

$$\dots, x_1 - m_1,\ x_1,\ x_1 + m_1, \dots, x_1 + (d-1)m_1,\ x_1 + dm_1, \dots \qquad (3)$$

$m_1d = m$ modul bo'yicha nechta har xil sinfga tegishli ekanligini aniqlashimiz kerak. Tushunarliki (3) dagi sonlar d ta sinfga tegishli bu sinflar sifatida

$$x_1,\quad x_1 + m_1,\quad \dots,\quad x_1 + (d-1)m_1 \qquad (4)$$

larni olish mumkin. Demak, (1) ning bu holda d ta yechimiga ega bo'lamiz.

Agarda $(a,m)=d>1$ bo'lib b soni d ga bo'linmasa, u holda (1)-taqqoslama birorta ham yechimga ega emas. Chunki bu holda (1) dan $a_1dx = b + m_1dt$ yoki $= b = d(a_1x - m_1t)$ tenglikka ega bo'lamiz.

b soni *d* ga bo'linmaganligi uchun bu tenglikning bajarilishi mumkin emas. Shunday qilib biz quyidagilarni isbotladik:

1). Agar *(a,m)=1* bo'lsa, (1) taqqoslama yagona yechimga ega;

2) Agarda *(a,m)=d>1* va *b* soni *d* bo'linsa, (1) taqqoslama *d* ta yechimga ega;

3) Agarda *(a,m)=d>1* va *b* soni *d* bo'linmasa, (1) taqqoslama birorta ham yechimga ega emas.

(1)-taqqoslamaning yechimini topish uchun quyidagi usullardan foydalanish mumkin:

1) tanlash usuli (bu usulda *m* moduli bo'yicha chegirmalarning to'la sistemasidagi chegermalar qo'yib sinab ko'riladi. Bu usul sodda, lekin *m* modul katta bo'lsa, chegirmalar sinflari soni ko'p bo'lgan uchun amaliy jihatdan noqulaydir);

2) taqqoslamalarning xossalaridan foydalanib koeffitsiyentlarini almashtirish usuli (bu usulda taqqoslamalarning xossalaridan foydalanib *x* noma'lumning oldidagi koeffitsiyent 1 bilan almashtiriladi. Bu usul ham koeffitsiyentlar katta bo'lgan holda aniq yo'llanma (algoritm) bo'lmagani uchun unchalik ham qulay emas. Bunday hollarda (1) ning yechimining topish uchun aniq formulaga ega bo'lish qulaydir);

3) Eyler teoremasidan foydalanib yechish usuli (bu usulda yechim $x \equiv a^{\varphi(m)-1} \cdot b(\mathrm{mod}\, m)$ formula yordamida topiladi) ;

4) uzluksiz (zanjirli) kasrlardan foydalanib yechish usuli mavjud. (Bu usulda yechim $x \equiv (-1)^{n-1} b P_{n-1} (\mathrm{mod}\, m)$ formula yordamida topiladi. Bu yerda P_{n-1} soni $\frac{a}{m}$ kasrning uzluksiz kasrlarga yoyilmasidagi $(n-1)-$ munosib kasrning surati (munosib kasrlar mavzusiga qarang)).

Taqqoslamalardan foydalanib $ax + by = c$ ko'rinishdagi birinchi darajali ikki noma'lumli, butun koeffitsiyentli aniqmas tenglamalarni butun sonlarda yechish mumkin. Berilgan tenglamani $ax = c + b(-y)$ ko'rinishda, buni esa $ax \equiv c(modb)$ ko'rinishda yozish mumkin. Bu taqqoslamaning yechimini yuqorida qarab chiqilgan usullardan biri bilan topamiz. $x = x_1 + bt, t \in Z$ bo'lsin. U holda x ning bu qiymatini berilgan tenglamaga qo'yib y ni aniqlaymiz: $a(x_1 + bt) + by = c \rightarrow y = \frac{1}{b}(c - ax_1 - abt) = \frac{c - ax_1}{b} - at$, ya'ni $y = y_1 - at, t \in Z$ ga ega bo'lamiz.

257. Quyidagi taqqoslamalarning yechimga ega yoki ega emasligini tekshiring, agar yechimga ega bo'lsa, uni tanlash usuli bilan toping:

$a)\ 5x \equiv 3(mod6)$, $b)\ 8x \equiv 3(mod10)$, $c)\ 2x \equiv 6(mod8)$,
$d)\ 3x \equiv -6(mod7)$, $e)\ 4x \equiv 3(mod12)$, $f)\ 6x \equiv 5(mod9)$,
$g)\ 5x \equiv 7(mod8)$.

258. Quyidagi taqqoslamalarning yechimga ega yoki ega emasligini tekshiring, agar yechimga ega bo'lsa, uni taqqoslamalarning xossalaridan foydalanib koeffitsiyentlarini almashtirish usuli bilan toping:

$a)\ 5x \equiv 3(mod7)$, $b)\ 8x \equiv 3(mod11)$, $c)\ 4x \equiv 6(mod8)$,
$d)\ 4x \equiv 25(mod13)$, $e)\ 11x \equiv 3(mod12)$, $f)\ 7x \equiv 5(mod9)$,
$g)\ 5x \equiv 7(mod8)$, $h)\ 7x \equiv 6(mod15)$.

259. Quyidagi taqqoslamalarning yechimga ega yoki ega emasligini tekshiring, agar yechimga ega bo'lsa uni Eyler teoremasidan foydalanib toping:

$a)\ 13x \equiv 3(mod19)$, $b)\ 27x \equiv 7(mod58)$, $c)\ 5x \equiv 7(mod10)$,
$d)\ 3x \equiv 8(mod13)$, $e)\ 25x \equiv 15(mod17)$, $f)\ 29x \equiv 35(mod12)$,
$g)\ 3x \equiv 7(mod11)$.

260. Quyidagi taqqoslamalarning yechimga ega yoki ega emasligini tekshiring, agar yechimga ega bo'lsa uni uzluksiz kasrlardan foydalanib toping:

$a)\ 13x \equiv 1(mod27)$, $b)\ 37x \equiv 25(mod117)$, $c)\ 113x \equiv 89(mod311)$, $d)\ 221x \equiv 111(mod360)$, $e)\ 23x \equiv 667(mod693)$, $f)\ 143x \equiv 41(mod221)$, $g)\ 20x \equiv 13(mod43)$.

261. Quyidagi taqqoslamalarning yeching: $a)\ 12x \equiv 9(mod15)$,
$b)\ 12x \equiv 9(mod18)$, $c)\ 20x \equiv 10(mod25)$,
$d)\ 10x \equiv 25(mod35)$,
$e)\ 39x \equiv 84(mod93)$, $f)\ 90x + 18 \equiv 0(mod138)$,
$g)\ 15x \equiv 35(mod55)$.

262. Quyidagi aniqmas tenglamalarni taqqoslamalardan foydalanib yeching:

$a)\ 5x + 4y = 3$, $b)\ 17x + 13y = 1$, $c)\ 91x - 28y = 35$,
$d)\ 2x + 3y = 4$, $e)\ 4x - 3y = 2$, $f)\ 3x - 7y = 1$,
$g)\ 7x + 6y = 11$.

263. $a)$. $x = -100$ va $x = 150$ to'g'ri chiziqlar orasida joylashgan va $8x - 13y + 6 = 0$ to'g'ri chiziqda yotuvchi butun koordinatali nuqtalar sonini aniqlang.

b). $x = 1$va $x = 200$ to'g'ri chiziqlar orasida joylashgan va $5x - 7y - 8 = 0$ to'g'ri chiziqda yotuvchi butun koordinatali nuqtalar sonini aniqlang.

264. x ning qanday butun qiymatlarida quyidagi funksiyalar butun qiymat qabul qiladi: $a)\ f(x) = \frac{9x-1}{7}$; $b)\ f(x) = \frac{7x-1}{15}$;

$$c)\ f(x) = \frac{2x-1}{11}.$$

265. *a*). G'allani tashish uchun 60 kg va 80 kg lik qoplar mavjud. 440 kg g'allani tashish uchun nechta 60 kg va 80 kg lik qoplar kerak bo'ladi.

b). 1490 so'mga 30 so'mlik va 50 so'mlik markalardan necha dona sotib olish mumkin.

d). 6000 so'mga 200 va 250 so'mlik daftarlardan necha dona sotib olish mumkin.

266.a). 523 sonining o'ng tomoniga shunday uchta raqam yozingki, hosil bo'lgan olti xonali son 7,8 va 9 ga bo'linsin.

b). 32 sonining o'ng tomoniga shunday ikkita raqam yozingki, hosil bo'lgan to'rt xonali son 3 va 7 ga bo'linsin.

IV. 3-§. Bir noma'lumli birinchi darajali taqqoslamalar sistemasini yechish

Ushbu birinchi darajali taqqoslamalar sistemasi

$$\grave{A}_1\tilde{o} \equiv \hat{A}_1(\mathrm{mod}\ m_1),\ A_2x \equiv B_2(\mathrm{mod}\ m_2),\ \cdots\ , A_kx \equiv B_k(\mathrm{mod}\ m_k) \qquad (1)$$

berilgan bo'lsin. Bu sistema yechimga ega bo'lishligi uchun avvalo (1) dagi har bir taqqoslama yechimga ega bo'lishi kerak. Bu taqqoslamalarning har birini yechib (1) ni quyidagicha yozib olish mumkin.

$$x \equiv b_1(\mathrm{mod}\ m_1,\ \ x \equiv b_2(\mathrm{mod}\ m_2), \cdots, x \equiv b_k(\mathrm{mod}\ m_k). \qquad (2)$$

(2) sistemani yechaylik. (2) ning birinchi taqqoslamasidan

$$x = b_1 + m_1t_1, \qquad t_1 \in Z. \qquad (3)$$

Bulardan (2) dagi ikkinchi taqqoslamani qanoatlantiruvchilarini ajratib olamiz:

$$b_1 + m_1 t_1 \equiv b_2 (\mathrm{mod}\, m_2). \quad (4)$$

Bundan $m_1 t_1 \equiv b_2 - b_1 (mod m_2)$. Faraz etaylik $(m_1, m_2) = d$ bo'lsin. U holda agarda b_2-b_1 ayirma d ga bo'linmasa, (4) taqqoslama yechimga ega emas. Agarda $d|b_2$-b_1 bo'lsa, (4) d ta yechimga ega va

$$\frac{m_1}{d} t_1 \equiv \frac{b_2 - b_1}{d}\left(\mathrm{mod}\frac{m_2}{d}\right), \qquad \left(\frac{m_1}{d}, \frac{m_2}{d}\right) = 1 \quad (5)$$

taqqoslama yagona $t_1 \equiv t' \left(mod \frac{m_2}{d}\right)$ yoki $t_1 = t' + \frac{m_2}{d} t_2$, $t_2 \in Z$ yechimga ega. $t_1$ning bu qiymatini (3) ga olib borib qo'yib (2) dagi birinchi 2 ta taqqoslamani qanoatlantiruvchi

$$x = b_1 + m_1\left(t' + \frac{m_2}{d} t_2\right) = b_1 + m_1 t' + \frac{m_1 m_2}{d} t_2 = b_1 + m_1 t' + [m_1, m_2] t_2$$

ni topamiz. Agarda $x_2 = b_1 + m_1 t'$ deb olsak, u holda

$$x = x_2 + [m_1, m_2] t_2 \quad \text{yoki} \quad x \equiv x_2 (\mathrm{mod}[m_1, m_2])$$

ni hosil qilamiz. Shu usulni davom ettirib $x \equiv x_k (mod\ [m_1, m_2, \ldots, m_k])$ ni, ya'ni (2) ning yechimini hosil qilamiz. (2)- sistemada $(m_i, m_j) = 1,\ i \neq j\ ,\ M = m_1 \cdot m_2 \ldots m_k, M_i = \frac{M}{m_i}$ bo'lsin.U holda (2) -sistemaning yechimi $x \equiv x_0 (\mathrm{mod} M)$ bo'ladi. Bu yerda

$$x_0 = M_1 \cdot M_1' b + M_2 \cdot M_2' b_2 + \ldots + M_k M_k' \cdot b_k \quad (6)$$

va $M_1', M_2', \ldots, M_k'$ lar ushbu taqqoslamalar sistemasidan aniqlanadi:

$$M_1 M_1' \equiv 1\ (\ \mathrm{mod}\ m_1), M_2 \cdot M_2' \equiv 1 (\mathrm{mod}\, m_2),\ \cdots,\ M_k M_k' \equiv 1 (\mathrm{mod}\, m_k). \quad (7)$$

(2)-sistemani yechish qadimgi xitoy masalasi deb ataluvchi $m_1$ga bo'lganda b_1, m_2 ga bo'lganda b_2, ...,m_k ga bo'lganda b_k qoldiq qoluvchi x sonini toping degan masalaning o'zginasidir.

267. Taqqoslamalar sistemasini yeching:

1) $\begin{cases} x \equiv 6 \pmod{15} \\ x \equiv 18 \pmod{21}, \\ x \equiv 3 \pmod{12} \end{cases}$ 2) $\begin{cases} x \equiv 13 \pmod{14} \\ x \equiv 6 \pmod{35} \\ x \equiv 26 \pmod{45} \end{cases}$ 3) $\begin{cases} x \equiv 19 \pmod{56} \\ x \equiv 3 \pmod{24} \\ x \equiv 7 \pmod{20} \end{cases}$ 4) $\begin{cases} x \equiv 4 \pmod{5} \\ x \equiv 1 \pmod{12} \\ x \equiv 7 \pmod{14} \end{cases}$

5) $\begin{cases} x \equiv 13 \pmod{16} \\ x \equiv 3 \pmod{10} \\ x \equiv 9 \pmod{14} \end{cases}$ 6) $\begin{cases} x \equiv 9 \pmod{10} \\ x \equiv 10 \pmod{15} \\ x \equiv 11 \pmod{12} \end{cases}$ 7) $\begin{cases} x \equiv 7 \pmod{9} \\ x \equiv 2 \pmod{7} \\ x \equiv 3 \pmod{12} \end{cases}$ 8) $\begin{cases} x \equiv 5 \pmod{12} \\ x \equiv 2 \pmod{8} \\ x \equiv 2 \pmod{11} \end{cases}$

9) $\begin{cases} x \equiv 7 \pmod{10} \\ x \equiv 2 \pmod{5} \\ x \equiv 8 \pmod{9} \end{cases}$ 10) $\begin{cases} x \equiv 8 \pmod{7} \\ x \equiv 3 \pmod{11} \\ x \equiv 9 \pmod{13} \end{cases}$ 11) $\begin{cases} x \equiv 2 \pmod{5} \\ x \equiv 8 \pmod{11} \\ x \equiv 12 \pmod{15}. \end{cases}$

268. Modullari juft-jufti bilan o'zaro tub bo'lgan taqqoslamalar sistemasini yeching.

1) $\begin{cases} x \equiv 1 \pmod{6} \\ x \equiv 2 \pmod{7} \\ x \equiv 3 \pmod{11}, \end{cases}$ 2) $\begin{cases} 2x \equiv 3 \pmod{5} \\ x \equiv 2 \pmod{7} \\ 3x \equiv 4 \pmod{11}, \end{cases}$ 3) $\begin{cases} 3x \equiv 1 \pmod{17} \\ 4x \equiv 3 \pmod{5} \\ 2x \equiv 5 \pmod{9}, \end{cases}$ 4) $\begin{cases} 5x \equiv 2 \pmod{9} \\ 3x \equiv -1 \pmod{13} \\ x \equiv 6 \pmod{11}, \end{cases}$

5) $\begin{cases} 6x \equiv 1 \pmod{35} \\ 3x \equiv 4 \pmod{17} \\ 10x \equiv 7 \pmod{13}, \end{cases}$ 6) $\begin{cases} 8x \equiv 7 \pmod{17} \\ 5x \equiv 11 \pmod{6} \\ x \equiv -1 \pmod{19}, \end{cases}$ 7) $\begin{cases} 11x \equiv -4 \pmod{18} \\ 7x \equiv 1 \pmod{11} \\ 3x \equiv 5 \pmod{7}, \end{cases}$ 8) $\begin{cases} 21x \equiv -2 \pmod{23} \\ 12x \equiv 3 \pmod{9} \\ x \equiv 6 \pmod{11}, \end{cases}$

9) $\begin{cases} x \equiv 3 \pmod{29} \\ x \equiv -5 \pmod{12} \\ 2x \equiv 7 \pmod{11}, \end{cases}$ 10) $\begin{cases} 6x \equiv 5 \pmod{31} \\ x \equiv -2 \pmod{29} \\ 5x \equiv 3 \pmod{27}, \end{cases}$ 11) $\begin{cases} x \equiv 1 \pmod{7} \\ x \equiv 3 \pmod{9} \\ x \equiv 5 \pmod{11}. \end{cases}$

269. m_1, m_2, m_3 sonlariga bo'lganda mos ravishda r_1, r_2, r_3 qoldiq qoluvchi eng kichik natural sonni toping.

№	m_1	m_2	m_3	r_1	r_2	r_3	№	m_1	m_2	m_3	r_1	r_2	r_3
1	7	8	9	1	2	3	7	13	21	23	9	1	13
2	3	4	5	1	2	3	**8**	3	5	8	2	4	1
3	9	10	13	3	5	6	**9**	3	5	8	2	4	1
4	4	5	7	2	3	4	**10**	5	7	9	4	6	1
5	3	7	8	2	4	5	**11**	7	13	17	6	12	16
6	7	13	17	4	9	1							

270. a ning qanday qiymatida berilgan taqqoslamalar sistemasi yechimga ega?

1) $\begin{cases} x \equiv 5 \pmod{18} \\ x \equiv 8 \pmod{21} \\ x \equiv a \pmod{35}, \end{cases}$ 2) $\begin{cases} x \equiv a \pmod{7} \\ x \equiv 2 \pmod{9} \\ x \equiv 7 \pmod{11}, \end{cases}$ 3) $\begin{cases} x \equiv 5 \pmod{12} \\ x \equiv a \pmod{11} \\ x \equiv 3 \pmod{15}, \end{cases}$ 4) $\begin{cases} x \equiv 11 \pmod{20} \\ x \equiv 1 \pmod{15} \\ x \equiv a \pmod{18}, \end{cases}$

5) $\begin{cases} x \equiv 19 \pmod{24} \\ x \equiv 10 \pmod{21} \\ x \equiv a \pmod{9}, \end{cases}$ 6) $\begin{cases} x \equiv 6 \pmod{15} \\ x \equiv 18 \pmod{21} \\ x \equiv a \pmod{11}, \end{cases}$ 7) $\begin{cases} x \equiv 19 \pmod{56} \\ x \equiv 3 \pmod{24} \\ x \equiv a \pmod{20}, \end{cases}$ 8) $\begin{cases} x \equiv 3 \pmod{5} \\ x \equiv 2 \pmod{7} \\ x \equiv a \pmod{9}, \end{cases}$

9) $\begin{cases} x \equiv 1 \pmod{3} \\ x \equiv 5 \pmod{7} \\ x \equiv a \pmod{11}, \end{cases}$ 10) $\begin{cases} x \equiv 14 \pmod{19} \\ x \equiv 5 \pmod{25} \\ x \equiv a \pmod{10} \end{cases}$ 11) $\begin{cases} x \equiv 5 \pmod{11} \\ x \equiv 4 \pmod{7} \\ x \equiv a \pmod{9}. \end{cases}$

271. Absissalar o‘qining qaysi butun nuqtalarida shu nuqtalardan o‘tkazilgan perpendikular berilgan to‘g‘ri chiziqlarning barchasini bir vaqtda butun koordinatali nuqtalarda kesadi.

1) $x = 2 + 5y, x = 1 + 8y, x = 3 + 11y$;
2) $4x - 7y = 9, 2x + 9y = 15, 5x - 13y = 12$;
3) $3x - 5y = 1, 2x + 3y = 3, 5x - 7y = 7$;
4) $x + 7y = 2, x - 5y = 3, 2x + 7y = 6$;
5) $2x - 3y = 1, x - 5y = 3, x - 11y = 2$;
6) $11x + 5y = 6, 10x + 11y = 9, 12x + 13y = -1$;
7) $3x - 7y = 5, 5x - 8y = 4, 11x + 13y = -2$;
8) $10x - 9y = 1, x - 7y = 3, x + 5y = 2$;
9) $11x + 17y = 5, 19x - 37y = 1, 11x - 7y = 4$;
10) $x - 19y = 2, 5x - 13y = 1, 10x + 13y = -3$;
11) $x - 7y = 5, 3x + 8y = 7, x = 11 + 3y$.

272. a). Agar o‘nlik sanoq sistemasidagi $N = 4x87y6$ sonining 56 ga bo‘linishi ma’lum bo‘lsa uni toping.

b). Agar o‘nlik sanoq sistemasidagi $N = xyz138$ sonining 7 ga bo‘linishi, $138xyz$ sonini 13 ga bo‘lganda qoldiq 6, $x1y3z8$ sonini 11 ga bo‘lganda 5 qoldiq qolishi ma’lum bo‘lsa, N ni toping.

d). Agar o‘nlik sanoq sistemasidagi $N = 13xy45z$ sonining 792 ga bo‘linishi ma’lum bo‘lsa, x, y, z larni toping.

273. Taqqoslamalar sistemasini yeching:

$a)\begin{cases} x+3y\equiv 5(mod7) \\ 4x\equiv 5(mod7), \end{cases}$ $b)\begin{cases} 9y\equiv 15(mod12) \\ 7x-3y\equiv 1(mod12), \end{cases}$ $c)\begin{cases} x\equiv 2(mod4) \\ x-2y\equiv 1(mod4), \end{cases}$

$d)\begin{cases} 9y\equiv 15(mod12) \\ 3x-7y\equiv 1(mod12), \end{cases}$ $e)\begin{cases} 3x-5y\equiv 1(mod12) \\ 9y\equiv 15(mod12). \end{cases}$

274. Taqqoslamalar sistemasini yeching:

$a)\begin{cases} x+2y\equiv 3(mod5) \\ 4x+y\equiv 2(mod5), \end{cases}$ $b)\begin{cases} x+2y\equiv 0(mod5) \\ 3x+2y\equiv 21(mod5), \end{cases}$

$c)\begin{cases} 3x+4y\equiv 29(mod143) \\ 2x-9y\equiv -84(mod143), \end{cases}$

$d)\begin{cases} x+2y\equiv 4(mod5) \\ 3x+y\equiv 2(mod5), \end{cases}$ $e)\begin{cases} x+5y\equiv 5(mod6) \\ 5x+3y\equiv 1(mod6), \end{cases}$

$f)\begin{cases} 5x-y\equiv 3(mod6) \\ 2x+2y\equiv -1(mod6). \end{cases}$

$g)\begin{cases} x-y\equiv 2(mod6) \\ 4x+2y\equiv 2(mod6), \end{cases}$ $h)\begin{cases} 4x-y\equiv 2(mod6) \\ 2x+2y\equiv 0(mod6). \end{cases}$

275. a)

$$\begin{cases} a_1x+b_1y\equiv c_1 \\ a_2x+b_2y\equiv c_2 \end{cases}(modm) \qquad (1)$$

taqqoslamalar sistemasida $D=\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ bilan m o'zaro tub bo'lsa, u yagona yechimga ega ekanligini isbotlang.

b) (1)-taqqoslamalar sistemasida $(D,m)=d>1$ bo'lsa, uning yechimga ega bo'lmaslik shartini toping.

c) (1)-taqqoslamalar sistemasida $D\equiv D_1\equiv D_2\equiv 0(modm)$ bo'lsa, uning yechimlari to'plami (1) dagi 1-taqqoslamaning yechimlari to'plami bilan bir xil bo'lishini isbotlang. Bunda

$$D_1=\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix},\quad D_2=\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}.$$

IV. 4-§. Tub modul bo'yicha n-darajali taqqoslamalar

$$f(x)=a_0x^n+a_1x^{n-1}+\cdots+a_n\equiv 0(modp) \qquad (1)$$

ko'rinishdagi taqqoslamaga tub modul bo'yicha *n*-darajali taqqolama deyiladi. Bunda p-tub son, $a_0 \not\equiv 0(mod p)$, n —butun musbat son, a_i – koeffitsiyentlar butun sonlar.

Eng avvalo $a_0, a_1, \dots, a_n$ sonlarini p moduli bo'yicha absolyut qiymati jihatidan eng kichik chegirmalar bilan almashtirsak (1) taqqoslama biroz soddaroq ko'rinishga keladi. Masalan:

$$25x^3 + 17x^2 - 13 \equiv 0(mod 11) \qquad (1')$$

ni

$$3x^3 - 5x^2 - 2 \equiv 0(mod 11) \qquad (2')$$

ko'rinishda yozish mumkin.

Ikkinchidan (1) ni hamma vaqt bosh hadining koeffitsiyenti 1 ga teng bo'lgan holga keltirish mumkin, chunki $aa_0 \equiv 0(mod p)$ taqqoslama $(a_0, p) = 1$ bo'lgani uchun yagona yechimga ega va (1) ning ikkala tomonini a ga ko'paytirsak x^n ning koeffitsiyentini 1 bilan almashtirish mumkin bo'ladi. Masalan: $3a \equiv 1(mod 11) \to a \equiv 4(mod 11)$. Shuning uchun ham (2') ning ikkala tomonini 4 ga ko'paytiramiz, u holda

$$12x^3 - 20x^2 - 8 \equiv 0(mod 11) \to x^3 + 2x^2 + 3 \equiv 0(mod 11).$$

Uchinchidan ushbu teorema yordamida berilgan taqqoslamani ancha soddalashtirish mumkin.

1-teorema. Agar (1) da $n \geq p$ bo'lsa, uni darajasi $p - 1$ dan katta bo'lmagan taqqoslama $R(x) \equiv 0(mod p)$ taqqoslama bilan almashtirish mumkin. Bunda $R(x)$ ko'phaq $f(x)$ ni $x^p - x$ ga bo'lishdan chiqqan qoldiq.

Amaliyotda $f(x)$ ni $x^p - x$ ga bo'lish shart emas. Buning uchun x^m ni darajasini *p-1* dan katta bo'lmagan had bilan almashtirish uchun m ni *p-1* ga bo'lamiz. $m = (p-1)k + r$. U holda $x \equiv x^p(mod p)$ taqqoslamaning ikki tomonini mos ravishda x^{r-1}, $x^{(p-1)1+r-1}, \dots$

$\dots, x^{(p-1)(k-1)+r-1}$ larga ko'paytirsak, $x^r \equiv x^{(p-1)+r}$,

$$x^{p-1+r} \equiv x^{2(p-1)+r}, \dots, x^{(p-1)(k-1)+r} \equiv x^{k(p-1)+r} \equiv x^m$$

hosil bo'ladi. Bulardan $x^m \equiv x^r(\text{modp})$. Bu esa yuqoridagi teoremaning yana bir isbotidir.

Misol. $x^8 + 2x^7 + x^5 - x^4 - x + 3 \equiv 0(mod 5)$ taqqoslamani darajasi 4dan katta bo'lmagan taqqoslama bilan almashtiring.

$$x^{4\cdot2+0} + 2x^{4\cdot1+3} + x^{4\cdot1+1} - x^{4\cdot1} - x + 3 \equiv 0(mod 5)$$
$$\to 1 + 2x^3 + x - x^0 - x + 3 \equiv 0(mod 5) \to 2x^3 + 3$$
$$\equiv 0(mod 5).$$

2-teorema(yechimlari soni haqida teorema).p-tub moduli bo'yicha n-darajali ($n \leq p-1$) taqqoslama n-tadan ortiq bo'lmagan ildizga ega.

Agarda $a_0 \not\equiv 0(mod p)$shartdan voz kechsak bu teoremadan quyidagi natija kelib chiqadi.

Natija. Agar p-tub modul bo'yicha n-darajali taqqoslama n tadan ortiq ildizga ega bo'lsa, uning barcha koeffitsiyentlari p ga bo'linadi.

3-teorema(Vilson teoremasi). Agar p tub son bo'lsa, u holda

$$(p-1)!+1 \equiv 0(mod p) \quad (3)$$

Bu taqqoslama agar p tub son bo'lmasa, bajarilmaydi. Haqiqatan ham agarda $p = p_1 \cdot d, 1 < d < p,$ bo'lsa $(p-1)!$ soni d ga bo'linadi, u holda $(p-1)!+1$ soni d ga bo'linmaydi, shuning uchun ham p ga bo'linmaydi. Demak, ushbu teskari teorema ham o'rinli ekan.

4-teorema. Agar butun musbat p soni uchun (3)taqqoslama o'rinli bo'lsa, p-tub son bo'ladi.

Shunday qilib Vilson teoremasini tub sonlarni aniqlash kriteriyasi deb qabul qilish mumkin, lekin $(p-1)!+1$ soni katta p lar uchun juda katta son bo'lgani uchun amaliyotda qo'llash noqulay.

276. Quyidagi taqqoslamalarning avval darajasini pasaytirib keyin yeching.

a). $6x^{10} - 12x + 1 \equiv 0(mod5),$

b). $x^5 - 2x^3 + x^2 - 2 \equiv 0(mod3),$

c).$x^5 - 7x^4 + 9x^2 - x + 13 \equiv 0(mod3),$

d). $x^7 - x^6 + 5x^2 - 3 \equiv 0(mod5),$

e). $x^5 + x^4 + x^3 - x^2 - 2 \equiv 0(mod5),$ f). $x^7 - 6 \equiv 0(mod5),$

g). $x^8 + 2x^7 + x^5 - x + 3 \equiv 0(mod5),$

h). $6x^4 + 17x^2 - 16 \equiv 0(mod3),$

i). $4x^7 - 2x^3 + 8 \equiv 0(mod5),$

j). $3x^7 - 2x^6 + 2x^2 + 13 \equiv 0(mod5).$

277. Quyidagi taqqoslamalarni berilgan modul bo'yicha ko'paytuvchilarga ajrating.

a). $x^3 + 4x^2 - 3 \equiv 0(mod5),$

b). $x^4 + x^3 - x^2 + x - 2 \equiv 0(mod5),$

c). $x^4 + x + 4 \equiv 0(mod11),$ d). $x^2 + 2x + 2 \equiv 0(mod5),$

e). $3x^3 - 1 \equiv 0(mod5),$

f). $2x^4 + x^3 - 3x^2 + 2x - 2 \equiv 0(mod11),$

g). $x^4 - 7x^3 + 13x^2 + 21x + 23 \equiv 0(mod7),$

h). $2x^4 + x^3 - 3x^2 + 2x - 2 \equiv 0(mod5)$,

i). $2x^3 + 5x^2 - 2x - 3 \equiv 0(mod7)$,

j). $x^4 - 2x^2 + x + 4 \equiv 0(mod7)$.

278. Quyidagi taqqoslamalarning 1-teoremadan foydalanib darajasini pasaytiring va yechimlarini toping:

a). $8x^{20} - 15x^{19} + 7x^{18} + 28x^{17} - 4x^{16} + 30x^{15} + 10x^6 - 4x^3 + 23x^2 - 21x - 11 \equiv 0(mod13)$,

b). $x^{10} + x^8 + x^7 - x^4 - x^2 + 4x - 3 \equiv 0(mod7)$,

c). $x^{101} + 3x^{15} + x^{11} - 3x^5 + 9x^2 + 10x - 5 \equiv 0(mod11)$,

d). $2x^{35} - 17x^{15} + 13x^8 - 3x^5 + 12x + 5 \equiv 0(mod11)$,

e). $x^{12} - 2x^7 + x^3 + 1 \equiv 0(mod5)$.

279. Quyidagi teoremani isbotlang: $f(x) = x^n + \sum_{i=1}^{n} a_i x^{n-i}$ va $n < p$ bo'lsin. $f(x) \equiv 0(modp)$ taqqoslamaning n ta yechimga ega bo'lishi uchun $x^p - x$ ni $f(x)$ ga bolishdan chiqqan qoldiqning barcha koeffitsiyentlarining p ga bo'linishi zarur va yetarli.

280. Agar $a \not\equiv 0(mod7)$ va $b \not\equiv 0(mod7)$ bo'lsa, $x^3 + ax + b \equiv 0(mod7)$ taqqoslamaning uchta yechimga ega bo'lmasligini isbotlang.

281. Tub modul bo'yicha taqqoslama $x^n \equiv a(modp)$ ning $(a,p) = 1$ va $n < p$ bo'lganda n ta yechimga ega bo'lishining zaruriy va yetarli shartini toping.

282.280-misolda topilgan shartdan foydalanib quyida berilgan $x^n \equiv a(modp)$ ko'rinishdagi taqqoslamalarning n ta yechimga ega yoki yechimga ega emas ekanligini aniqlang va yechimga ega bo'lsa, ularni toping.

a). $x^3 \equiv 1(mod7)$; b). $x^2 \equiv 2(mod5)$; c). $x^5 \equiv 10(mod11)$;
d). $x^4 \equiv 1(mod11)$; e). $x^6 \equiv 3(mod7)$; f). $x^4 \equiv 3(mod13)$.

283. Agar $p-$ tub son bo'lsa, $(p-2)! \equiv 1(modp)$ ekanligini ko'rsating.

284. p va $p+2$ sonlarining "egizak" tub sonlar bo'lishi uchun $4[(p-1)! + 1] + p \equiv 0\ (modp(p+2))$ shartning bajarilishi yetarli va zarur ekanligini (Klement teoremasini) isbotlang.

285. Vilson teoremasidan foydalanib p soni $p = 4n + 1$ ko'rinishdagi tub son bo'lganda $(2n)!$ sonining $x^2 \equiv -1(modp)$ taqqoslamani qanoatlantirishini isbotlang.

286. p tub son bo'lganda quyidagi taqqoslamalarning o'rinli ekanligini isbotlang. a). $a^p + a(p-1)! \equiv 0(modp)$;
b). $a^p(p-1)! + a \equiv 0(modp)$.

287. Leybnits teoremasi "$p > 2$ sonining tub son bo'lishi uchun $(p-2)! - 1 \equiv 0(modp)$ shartning bajarilishi zarur va yetarli" ni isbotlang.

288. 279-misoldagi teoremani quyidagi taqqoslamalarni yechishga qo'llang:
a). $x^2 + 2x + 2 \equiv 0(mod5)$, b). $3x^3 - 4x^2 - 2x - 4 \equiv 0(mod7)$.

IV.5-§. Murakkab modul bo'yicha yuqori darajali taqqoslamalar

Murakkab modulli taqqoslamani yechishni tub modul bo'yicha taqqoslamani yechishga keltirish mumkin. Bunda ushbu teoremadan foydalaniladi:

Teorema. Agar

$$f(x) \equiv 0(\text{mod}\, M) \qquad (1)$$

taqqoslamaning moduli M juft-jufti bilan o'zaro tub bo'lgan ko'paytuvchilarga ajratilgan $M = m_1 m_2 \ldots m_k$, $(m_i, m_j) = 1$ bo'lsa, u holda:

1). (1) taqqoslama ushbu taqqoslamalar sistemasi

$$f(x) \equiv 0(\text{mod}\, m_1), f(x) \equiv 0(\text{mod}\, m_2), \cdots, f(x) \equiv 0(\text{mod}\, m_k) \qquad (2)$$

ga teng kuchlidir.

2). Agarda (1) taqqoslama N ta yechimga ega bo'lib, (2) ning birinchisi n_1, ikkinchisi n_2 va x.k. oxirgisi n_K ta yechimga ega bo'lsa, u holda $N = n_1 \cdot n_2 \cdot \ldots \cdot n_k$ bo'ladi.

Yuqoridagi teoremaga asosan murakkab modul boyicha taqqoslamani hamma vaqt

$$f(x) \equiv 0(\text{mod}\, p^\alpha), \qquad p - \text{туб сон}, \ \alpha \ge 1 \qquad (1')$$

ko'rinishdagi taqqoslamani yechishga keltirish mumkin. Bu taqqoslamani tanlash usuli bilan yechish p^α katta son bo'lganda ancha noqulay (1) ni yechishni

$$f(x) \equiv 0(\text{mod}\, p) \qquad (3)$$

ni yechishga keltirish mumkin. Ma'lumki (1') ni qanoatlantiruvchi har bir x_1 soni (3) ni ham qanoatlantiradi. Shuning uchun ham (1') ning yechimlarini (3) ning yechimlari orasidan qidirish kerak. Buni ketma-ket

(3) dan p bo'yicha, keyin p^2 va h.k. taqqoslamalarga o'tib bajarish mumkin.

Faraz etaylik (3) ning birorta yechimi topilgan bo'lsin:

$$x \equiv x_1 (\bmod p) \quad ya'ni \qquad x = x_1 + pt_1, \qquad t \in Z \tag{4}$$

(4) dan

$$f(x) \equiv 0 (\bmod p^2) \tag{5}$$

taqqoslamani qanoatlantiruvchilarini ajratib olamiz.

$$f(x_1 + pt_1) \equiv 0 (\bmod p^2).$$

Bu taqqoslamaning chap tomonini hisoblash uchun $f(x_1 + pt_1)$ ning Teylor qator yoyilmasidan foydalanish qulay:

$$f(x_1) + pt_1 \cdot f'(x_1) + \frac{(pt_1)^2}{2!} f''(x_1) + \ldots + \frac{(pt_1)^k}{k!} f^{(k)}(x_1),$$

bu yerdagi har bir qo'shiluvchi butun son. Bundan foydalanib oxirgi taqqoslamani quyidagicha yozish mumkin:

$$f(x_1) + pt_1 \cdot f'(x_1) \equiv 0 (\bmod p^2) \tag{6}$$

Bu yerda $p \setminus f(x_1)$ bo'lgan uchun

$$\frac{f(x_1)}{p} + t_1 f'(x_1) \equiv 0 (\bmod p)$$

yoki

$$t_1 f'(x_1) \equiv -\frac{f(x_1)}{p} (\bmod p). \tag{7}$$

Bu yerda quyidagi uchta hol bo'lishi mumkin:

A. $p \nmid f'(x_1)$ bo'lsa, (7) dan $t_1 \equiv t' (\bmod p)$, ya'ni $t_1 = t' + pt_2$, $t_2 \in Z$.

Buni (4) ga qo'ysak, $x = x_1 + p(t' + pt_2) = x_1 + pt' + p^2 t_2$ hosil bo'ladi. $t_2 = 0$ ää $x = x_1 + pt'$. Bundan (5) ning bitta $x_2 = x_1 + pt'$ yechimi hosil bo'ladi. Demak $x = x_2 + p^2 t_2$. Buni

$$f(x) \equiv 0 (\bmod p^3) \tag{8}$$

taqqoslamaga olib borib qo'yib, yuqoridagi singari mulohaza yuritib t_2 *ni topamiz.*

$f(x_2 + p^2 t_2) \equiv 0 (\bmod p^3)$ yoki $f(x_2) + p^2 t_2 f'(x_2) \equiv 0 (\bmod p^3)$, bu yerda $p^2 \mid f(x_2)$

bo'lgani uchun

$$\frac{f(x_2)}{p^2} + t_2 f'(x_2) \equiv 0 (\bmod p). \tag{9}$$

$x_2 \equiv x_1\ (modp)$ bo'lgani uchun $f'(x_1) \equiv f'(x_2)(modp)$. Bunda shart bo'yicha $f'(x_1) \not\equiv 0(modp)$ va demak, $f'(x_2) \not\equiv 0(modp)$.

Demak, (9) yagona yechimga ega.

$t_2 \equiv t_2'(\mathrm{mod}\, p)$, ya'ni $t_2 = t_2' + pt_3$, $t_3 \in Z$. U holda

$x = x_2 + p^2(t_2' + pt_3) = x_2 + p^2t_2' + p^3t_3$ yoki $x = x_3 + p^3t_3$, ya'ni $x \equiv x_3(\mathrm{mod}\, p^3)$.

Shu jarayonni takrorlab $x \equiv x_\alpha(modp^\alpha)$ ni hosil qilamiz.

Shunday qilib, $p \nmid f'(x_1)$ holda (3) ning har bir yechimi (1') ning bitta yechimiga olib keladi.

Б. Agarda $p | f'(x_1)$ bo'lib, (7) ning o'ng tomoni esa p ga bo'linmasa (7) va demak (5) va (1') ham yechimga ega emas.

B.Agarda $p | f'(x_1)$ bo'lib, (7) ning o'ng tomoni ham p ga bo'linsa, (7) ayniy taqqoslamaga aylanadi, uni (4) dagi ixtiyoriy butun son t_1 qanoatlantiradi. Lekin bu yechimlar p^2 moduli bo'yicha p ta sinfga tegishli bo'ladi, ya'ni (5) taqqoslama p ta yechimga ega bo'ladi. Keyin bu yechimlardan umumiy usul bilan p^3 moduli bo'yicha taqqoslamani qanoatlantiruvchilarini ajratib olamiz va h.k.

289. Quyidagi taqqoslamalarni yeching:

1) $3x^3 + 4x^2 - 7x - 6 \equiv 0(mod15)$;

2) $6x^3 - 3x^2 - 13x - 10 \equiv 0(mod30)$; 7) $37x \equiv 17(\mathrm{mod}180)$

3) $x^4 - 33x^3 + 8x - 26 \equiv 0(mod35)$;

4) $x^5 - 3x^4 + 5x^3 + 9x^2 + 4x - 12 \equiv 0(mod42)$;

5) $x^5 + x^4 - 3x^3 + x^2 + 2x - 2 \equiv 0(mod77)$;

6) $3x^3 + 6x^2 + x + 10 \equiv 0\ (\mathrm{mod}15)$

290. Taqqoslamalarni yeching:

1) $4x^3 - 8x - 13 \equiv 0(mod27)$;

2) $x^4 - 3x^3 + 2x^2 - 5x - 10 \equiv 0(mod343)$;

3) $x^4 - 4x^3 + 2x^2 + x + 6 \equiv 0(mod25)$;

4) $9x^2 + 29x + 62 \equiv 0(mod64)$; 5) $6x^3 - 7x - 11 \equiv 0(mod125)$;

6) $x^3 + 3x^2 - 5x + 16 \equiv 0(mod125)$;

7) $x^4 + 4x^3 + 2x^2 + x + 12 \equiv 0(mod625)$; 8). $2x^4 + 5x - 1 \equiv 0(\mathrm{mod}\,27)$.

291. Taqqoslamalarni yeching:

1) $x^4 + 4x^3 + 2x^2 + x + 12 \equiv 0(mod45)$;

2) $x^4 - 3x^3 - 4x^2 - 2x - 2 \equiv 0(mod50)$;

3) $x^5 - 5x^4 - 5x^3 + 25x^2 + 4x - 20 \equiv 0(mod147)$;

4) $x^5 + 3x^4 - 7x^3 + 4x^2 + 4x - 10 \equiv 0(mod175)$;

5) $x^4 - 4x^3 + 2x^2 + x + 6 \equiv 0(mod135)$;

6) $4x^3 + 7x^2 - 7x - 10 \equiv 0(mod225)$;
7) $31x^4 + 57x^3 + 96x + 191 \equiv 0(mod225)$;
8) $2x^6 - 6x^4 - 7x^2 - 4 \equiv 0(mod441)$;
9) $2x^6 - 6x^4 - 7x^2 - 4 \equiv 0(mod1225)$.

IV.6-§. Ikkinchi darajali taqqoslamalar va Lejandr simvoli

1. **Ikkinchi darajali taqqoslamalar va ularning ikki noma'lumli ikkinchi darajali aniqmas tenglamalar bilan bog'liqligi.** Ikkinchi darajali taqqoslamaning umumiy ko'rinishi

$$Ax^2 + Bx + C \equiv 0 (\bmod M) \quad (1)$$

dan iborat. Bu ushbu ikki noma'lumli aniqmas tenglama

$$Ax^2 + Bx + C = My \quad (2)$$

ga teng kuchli. (1) ko'rinishdagi taqqoslamani yechishga ikkinchi darajali ikki noma'lumli aniqmas tenglamaning umumiy holi

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0$$

ham keltiriladi. Buni yechish esa o'z navbatida Pell tenglamasi $x^2 - ay^2 = c$ ning yechimi bilan ham bog'liqdir.

2. Ikki hadli taqqoslamaga keltirish. (1) ni hamma vaqt

$$x^2 \equiv a (\bmod m) \quad (3)$$

ko'rinishga keltirish mumkin. Buni quyidagicha amalga oshiriladi. (1) ning ikkala tomonini 4A ga ko'paytiramiz (modulini ham)

$$4A^2x^2 + 4ABx + 4AC \equiv 0 (\bmod 4AM). \quad (4)$$

(4) dan

$$(2Ax + B)^2 \equiv B^2 - 4AC (\bmod 4AM).$$

Bu yerda $y = 2Ax + B$, $D = B^2 - 4AC$ deb olsak, $y^2 \equiv D(mod4AM)$ hosil bo'ladi.

Agar (3) taqqoslamada $(a,m)=1$ bo'lib, u yechimga ega bo'lsa a ga m moduli bo'yicha *kvadratik chegirma*, agar yechimga ega bo'lmasa, *kvadratik chegirma emas* deyiladi. Shuningdek, agar $x^n \equiv a(\bmod m)$, $(a,m)=1$ taqqoslama yechimga ega bo'lsa, a ga *n – darajali chegirma*, aks holda esa a ga m moduli bo'yicha *n-darajali chegirma emas* deb ataladi.

(3) taqqoslamani yechish umumiy holda

1) $x^2 \equiv a(\bmod p)$, $p > 2$; 2) $x^2 \equiv a(\bmod p^\alpha)$, $\alpha > 1$; 3) $x^2 \equiv a(\bmod 2^\alpha)$, $\alpha > 1$.

taqqoslamalarni yechishga keltiriladi.

3. Yechimlari soni.Tanlash yo'li bilan yechimlarini topish. Kvadratik chegirmalar soni. Ushbu

$$x^2 \equiv a(\mathrm{mod}\, p), \qquad p > 2 \tag{5}$$

taqqoslama berilgan bo'lsin. Agar $p \backslash a$ bo'lsa, trivial hol bo'ladi, ya'ni $x \equiv 0(\mathrm{mod}\, p)$. Shuning uchun ham $x \equiv x_1(\mathrm{mod}\, p)$ deb hisoblaymiz. Tushunarliki, agar $x \equiv x_1(\mathrm{mod}\, p)$ (5) ning yechimi bo'lsa, $x \equiv -x_1(\mathrm{mod}\, p)$ ham (5) ning yechimi bo'ladi. $x \equiv -x_1(modp)$ dan $2x_1 \equiv 0\ (\mathrm{mod}\, p)$va $\delta > 2 \Rightarrow x_1 \equiv 0(\mathrm{mod}\, p)$ kelib chiqadi, u holda $(a, p) = 1$ ga ziddir. Shunday qilib (5) yechimga ega bo'lsa, u 2 ta har xil yechimga ega bo'lar ekan. (5) yechimlarini tanlash usuli bilan topish jarayoni umumiy holga nisbatan ancha sodda. Bu yerda biz p moduli chegirmalarning keltirilgan sistemasini absolyut qiymati jihatidan eng kichik sistema ko'rinishda yozib olib

$$\pm 1,\ \pm 2,\ \ldots,\ \pm \frac{p-1}{2} \tag{6}$$

musbat va manfiy chegirmalarning (5) ni qanoatlantirish yoki qanoatlantirmasligi bir vaqtda tekshirishimiz mumkin. Shuning uchun ham (5) da x ning o'rniga

$$1, 2, 3, \ldots, \frac{p-1}{2}$$

larni qo'yib tekshirish yetarli. Bunda chap tomonda:

$$1^2,\ 2^2,\ \ldots,\ \left(\frac{p-1}{2}\right)^2 \tag{7}$$

hosil bo'ladi. Bulardan birortasi, masalan k^2 soni a bilan *modp* bo'yicha taqqoslanuvchi bo'lsa, u holda $x \equiv \pm k(\mathrm{mod}\, p)$ ga ega bo'lamiz. Shu bilan birga, faqat $a(\mathrm{mod}\, p)$bo'yicha (7) da birorta son bilan taqqoslanuvchi bo'lgan (5) ko'rinishdagi taqqoslamalargina yechimga ega. Boshqacha so'z bilan aytganda (7) da $\mathrm{mod}\, p$ bo'yicha kvadratik chegirmalar yozilgan. Ularning barchasi har xil sinflarga tegishli. Haqiqatan ham, agar $1 \le k < l \le \frac{p-1}{2}$ bo'lib $k^2 \equiv l^2(\mathrm{mod}\, p)$ bo'lsa, u holda (5) 4 ta $x \equiv \pm k$ ââ $x \equiv \pm l\ (\mathrm{mod}\, p)$ yechimga ega bo'ladi. Buning bo'lishi mumkin emas. Shunday qilib, *modp* bo'yicha kvadratik chegirmalar soni $\frac{p-1}{2}$ ga teng va shuning uchun ham kvadratik chegirma emaslar son soni ham $\frac{p-1}{2}$ ga teng bo'ladi.

4. Eyler kriteriyasi. (5) ning yechimga ega yoki ega emasligini aniqlash uchun Eyler tomonidan taklif etilgan ushbu kriteriyadan foydalanish qulay: Agar a oni $modp$ bo'yicha kvadratik chegirma bo'lsa, $a^{\frac{p-1}{2}} \equiv 1 \pmod p$ bo'ladi. Agar a soni $modp$ bo'yicha kvadratik chegirma bo'lmasa, u holda $a^{\frac{p-1}{2}} \equiv -1 \pmod p$ bo'ladi. Haqiqatan ham, agar $(a,p)=1$ va $(a,2)=1$ bo'lsa, $a^{p-1} \equiv 1 \pmod p$ bo'ladi (Ferma teoremasi). Bundan $a^{p-1}-1 \equiv 0 \pmod p$ yoki $\left(a^{\frac{p-1}{2}}-1\right)\left(a^{\frac{p-1}{2}}+1\right) \equiv 0 \pmod p$.

Bu yerda bu qavslarning hech bo'lmasa birortasi p ga bo'linishi kerak. Ularning ikkalasi bir vaqtda p ga bo'linmaydi, aks holda ularning ayirmasi 2 ham p ga bo'linar edi, lekin $p>2$

Agar a kvadratik chegirma bo'lsa,

$$a^{\frac{p-1}{2}} \equiv 1 \pmod p \tag{8}$$

bajariladi. Bu yerdan agar $a \pmod p$ bo'yicha kvadratik chegirma emas bo'lsa, u holda

$$a^{\frac{p-1}{2}} \equiv -1 \pmod p$$

bajariladi.

5. Lejandr simvoli va uning xossalari. a sonining p moduli bo'yicha kvadratik chegirma yoki chegirma emasligini aniqlashda Eyler kriteriyasidan foydalanish p katta bo'lsa uncha ham qulay emas. Shuning uchun Lejandr simvoli $\left(\frac{a}{p}\right)$ qo'llaniladi. U quyidagicha aniqlanadi:

$$\left(\frac{a}{p}\right) = \begin{cases} 1, & \text{agar } a \text{ soni mod } p \text{ bo'yicha kvadratik chegirma bo'lsa;} \\ -1, & \text{agar } a \text{ soni mod } p \text{ bo'yicha kvadratik chegirma bo'lmasa.} \end{cases}$$

Lejandr simvoli ta'rifidan va Eyler kriteriyasidan

$$\grave{a}^{\frac{p-1}{2}} \equiv \left(\frac{\grave{a}}{\eth}\right) \pmod p \tag{9}$$

kelib chiqadi. Lejandr simvoli quyidagi xossalarga ega.

1^0. Agar $a \equiv a_1 \pmod p$ bo'lsa, $\left(\frac{a}{p}\right)=\left(\frac{a_1}{p}\right)$ bo'ladi.

Bundan $\left(\frac{a}{p}\right)=\left(\frac{a+pt}{p}\right)$, $t \in Z$.

$2^0.\ \left(\frac{1}{\delta}\right)=1,\quad 3^0.\left(\frac{-1}{p}\right)=(-1)^{\frac{\delta-1}{2}},\quad 4^0.\left(\frac{a\cdot b}{p}\right)=\left(\frac{a}{p}\right)\cdot\left(\frac{b}{p}\right),\quad 5^0.\left(\frac{2}{p}\right)=(-1)^{\frac{p^2-1}{8}},$

$6^0.\quad \left(\frac{p}{q}\right)=(-1)^{\frac{p-1}{2}\cdot\frac{q-1}{2}}\left(\frac{q}{p}\right).$

Lejandr simvolining qiymatini shu xossalardan foydalanib hisoblash mumkin. 6^0 – xossaga kvadratik chegirmalarning o'zgalik qonuni deyiladi.

292. Berilgan taqqoslamalarni ikkihadli taqqoslama ko'rinishiga keltirib, keyin yeching: 1) $2x^2+4x-1\equiv 0(\mathrm{mod}\,5)$; 2) $3x^2+2x\equiv 1(\mathrm{mod}\,7)$;
3) $2x^2-2x-1\equiv 0(\mathrm{mod}\,7)$; 4) $3x^2-x\equiv 0(\mathrm{mod}\,5)$; 5) $3x^2+7x+8\equiv 0(\mathrm{mod}\,17)$;
6) $3x^2+4x+7\equiv 0(\mathrm{mod}\,31)$; 7) $4x^2-11x-3\equiv 0(\mathrm{mod}\,13)$; 8) $x^2-5x+6\equiv 0(\mathrm{mod}\,24)$.

293. x ning qanday natural qiymatlarida quyidagi funksiyalar butun qiymatni qabul qiladi:

1) $\frac{x^2+2x+7}{55}$; 2) $\frac{x^2+3x+1}{25}$; 3) $\frac{x^2+3x+5}{15}$.

294.a).Eyler kriteriyasidan foydalanib 7 moduli bo'yicha eng kichik musbat chegirmalarning keltirilgan sistemasida qaysi sonlar shu modul bo'yicha kvadratik chegirma bo'ladi.

b). 17 moduli bo'yicha eng kichik musbat kvadratik chegirmalarni aniqlang.

295. Eyler kriteriyasidan foydalanib quyidagi modullar bo'yicha kvadratik chegirma sinflarini aniqlang: 1) 11; 2) 13; 3) 17.

296. Quyidagi taqqoslamalarni berilgan modul bo'yicha absolyut qiymati jihatidan eng kichik (noldan boshqa) chegirmalarni sinab ko'rish yo'li bilan yeching:

1) $x^2\equiv 2(\mathrm{mod}\,7)$; 2) $x^2\equiv 4(\mathrm{mod}\,7)$; 3) $x^2\equiv 3(\mathrm{mod}\,7)$; 4) $x^2\equiv 3(\mathrm{mod}\,13)$; 5) $x^2\equiv 4(\mathrm{mod}\,11)$.

297. Lejandr simvolining qiymatini hisoblang:

1) $\left(\frac{63}{131}\right)$; 2) $\left(\frac{35}{97}\right)$; 3) $\left(\frac{47}{73}\right)$; 4) $\left(\frac{29}{383}\right)$; 5) $\left(\frac{241}{593}\right)$; 6) $\left(\frac{257}{571}\right)$; 7) $\left(\frac{251}{577}\right)$; 8) $\left(\frac{342}{677}\right)$.

298. Lejandr simvolidan foydalanib quyidagi taqqoslamalardan qaysilari yechimga ega ekanligini aniqlang va yechimlarini toping:

1) $x^2\equiv 6(\mathrm{mod}\,7)$; 2) $x^2\equiv 3(\mathrm{mod}\,11)$; 3) $x^2\equiv 12(\mathrm{mod}\,13)$; 4) $x^2\equiv 3(\mathrm{mod}\,13)$;
5) $x^2\equiv 5(\mathrm{mod}\,11)$; 6) $x^2\equiv 13(\mathrm{mod}\,17)$; 7) $x^2\equiv 7(\mathrm{mod}\,19)$; 8) $x^2\equiv 5(\mathrm{mod}\,17)$.

299. Berilgan taqqoslamalar yechimga ega bo'ladigan a ning qiymatini toping:

1) $x^2 \equiv a(\mathrm{mod}\,5)$; 2) $x^2 \equiv a(\mathrm{mod}\,7)$; 3) $x^2 \equiv a(\mathrm{mod}\,11)$;

4) $x^2 \equiv a(\mathrm{mod}\,13)$; 5) $x^2 \equiv 5(\mathrm{mod}\,3)$.

300. $x^2+1 \equiv 0(\mathrm{mod}\,p)$ taqqoslama modulning $p = 4n + 1, (n = 1,2,3, \dots)$ qiymatida va faqat shundagina yechimga ega ekanligini isbotlang.

301. $(a,b)=1$ bo'lganda a^2+b^2 ko'rinishdagi sonning kanonik yoyilmasida faqat $p = 4n + 1, (n = 1,2,3, \dots)$ ko'rinishdagi tub sonlar qatnashishini isbotlang.

302. Ikki ketma-ket butun sonning ko'paytmasining 13 moduli bo'yicha 1 bilan taqqoslanuvchi bo'lmasligini isbotlang.

303. a ning $x(x+1) \equiv a(\mathrm{mod}\,13)$ taqqoslama yechimga ega bo'ladigan barcha qiymatlarini toping.

304. 300-masaladan foydalanib $p = 4n + 1, (n = 1,2,3, \dots)$ ko'rinishdagi tub sonlar sonining cheksiz ko'p ekanligini isbotlang.

305. Tenglamalarni butun sonlarda yeching (quyidagi egri chiziqlarda yotuvchi butun koordinatali nuqtalarni toping):

1) $4x^2-5y=6$; 2) $11y=5x^2-7$;

3) $x^2-10x-11y+5=0$; 4) $x^2-21x+110=13y$; 5) $15x^2-7y^2=9$.

306. Berilgan sonlar kvadratik chegirma(chegirma emas) bo'lgan modullarni toping: 1) $a=5$; 2) $a=-3$; 3) $a=3$; 4) $a=2$; 5) $a=-7$.

307. Berilgan taqqoslamalar yechimga ega bo'lgan barcha toq tub modullarni toping:

1). $x(x+1) \equiv 1(\mathrm{mod}\,p)$; 2). $x(x-1) \equiv 2(\mathrm{mod}\,p)$; 3). $x(x-1) \equiv 3(\mathrm{mod}\,p)$.

308. Lejandr simvolidan foydalanib quyidagi taqqoslamalar modul $p>2$ ning qiymatiga bog'liq bo'lmagan yechimga ega ekanligini isbotlang:

1) $(x^2-13)(x^2-17)(x^2-221) \equiv 0(\mathrm{mod}\,p)$;

2) $(x^2-3)(x^2-5)(x^2-7)(x^2-11)(x^2-1155) \equiv 0(\mathrm{mod}\,p)$.

V BOB. BOSHLANG'ICH ILDIZLAR VA INDEKSLAR

V.1-§.Ko'rsatkichga qarashli sonlar va boshlang'ich ildizlar

1.Ko'rsatkichga qarashli sonlar va boshlang'ich ildizlar.

Agar $(a,m)=1$ bo'lib δ>0

$$a^{\delta} \equiv 1(\mathrm{mod}\, m) \qquad (1)$$

ni qanoatlantiruvchi eng kichik butun son bo'lsa, u holda a soni m moduli bo'yicha δ *ko'rsatikichga tegishli* deyiladi. Shuni ham ta'kidlash kerakki, agar $(a,m)=d>1$ bo'lsa, (1) taqqoslama o'rinli bo'lmaydi, chunki uning o'ng tomoni d ga bo'linmaydi. Ma'lumki, $(a,m)=1$ bo'lsa, Eyler teoremasiga ko'ra

$$a^{\varphi(m)} = 1(\mathrm{mod}\, m) \qquad (2)$$

Demak, $0<\delta\leq\varphi\ (m)$. Agar δ=φ(m) bo'lsa, ya'ni a soni m moduli bo'yicha $\varphi(m)$ ko'rsatkichga tegishli bo'lsa, a va m moduli bo'yicha *boshlang'ich ildiz* deyiladi. Agar $m=p$ tub son bo'lsa, a soni p modul bo'yicha boshlang'ich ildiz bo'lishi uchun u p-1 ko'rsatkichiga tegishli bo'lishi kerak. a sonining m moduli bo'yicha tegishli bo'lgan ko'rsatkichini topish uchun quyidagicha yo'l tutish mumkin: $a, a^2, a^3, \ldots$ larni hisoblaymiz, toki birinchi $a^{\delta} \equiv 1(\mathrm{mod}\, m)$ shartni qanoatlantiruvchi δ ni hosil qilgunga qadar.

2. Endi ko'rsatkichga qarashli sonlarning ba'zi xossalarini qaraymiz.

$1^0. a_1 \equiv a(\mathrm{mod}\, m)$ bo'lsa, u holda a va a_2 lar m moduli bo'yicha bir xil ko'rsatkichga tegishli bo'ladi.

Demak, agar a soni m moduli bo'yicha δ ko'rsatkichga tegishli bo'lsa a bilan taqqoslanuvchi sonlar $a+mt$ ning barchasi shu δ ko'rsatkichga tegishli bo'lar ekan.

2^0. Agar a soni m moduli bo'yicha δ ko'rsatkichga tegishli bo'lsa, u holda

$$a^0, a, a^2, \ldots, a^{\delta-1} \qquad (3)$$

sonlari m moduli bo'yicha o'zaro taqqoslanmaydi. Bu xossadan kelib chiqadiki, agar $\delta=\varphi(m)$ bo'lsa, (3) sistema m moduli bo'yicha chegirmalarning keltirilgan sistemasini tashkil qiladi.

3^0. Agar a soni m moduli bo'yicha δ ko'rsatkichga tegishli bo'lsa, u holda

$$a^{\gamma} \equiv a^{\gamma_1} \pmod m \tag{4}$$

bo'lishi uchun $\gamma \equiv \gamma_1 \pmod \delta$ bo'lishi zarur va yetarlidir.

Natija.1). Agar a soni m moduli bo'yicha δ ko'rsatkichga tegishli bo'lib, $a^{\gamma} \equiv 1(mod m)$ bo'lishi uchun $\gamma \equiv 0 \pmod \delta$ bo'lishi zarur va yetarlidir.

2). Agar a soni m moduli bo'yicha δ ko'rsatkichga tegishli bo'lsa, $\delta \backslash \varphi(m)$.

2-natijadan foydalanib δ ni topish jarayonini biroz soddalashtirish mumkin, ya'ni δ bu $\varphi(m)$ ning bo'luvchilari orasida bo'ladi.

3). Agar a soni m moduli bo'yicha δ ko'rsatkichga tegishli bo'lsa, a^k soni $\frac{\delta}{(\delta,k)}$ ko'rsatkichga tegishli bo'ladi. Xususiy holda, agar $(k,\delta) = 1$ bo'lsa, γ=δ, ya'ni a^k soni ham δ ko'rsatkichga tegishli bo'ladi.

3. Ko'rsatkichga qarashli sinflarning mavjudligi va ularning soni. Biz bundan ilgari har bir $(a,m)=1$ shartni qanoatlantiruvchi sonining m moduli bo'yicha biror δ $(\delta \backslash \varphi(m))$ko'rsatkichga tegishli ekanligini ko'rdik. Buning teskarisi, ya'ni $\varphi(m)$ ning har bir bo'luvchisi m moduli bo'yicha biror sinfning ko'rsatkichi bo'ladimi? Xususan $\varphi(m)$ soni ham biror sinfning m moduli bo'yicha ko'rsatkichi bo'ladimi? Ya'ni ixtiyoriy m moduli bo'yicha boshlang'ich ildiz mavjudmi? Bu savolga faqat $m=p$ – tub son hamda m maxsus (ba'zi bir ko'rinishdagi butun sonlar uchun) ijobiy javob bor.

Lemma. p-1 sonining bo'luvchisi δ soni p moduli bo'yicha yoki birorta ham sinfning ko'rsatkichi bo'lmaydi yoki $\varphi(\delta)$ ta sinfning ko'rsatkichi bo'ladi.

(Bu lemmani boshqacha qilib quyidagicha aytish mumkin. Agar p moduli bo'yicha δ ko'rsatkichga tegishli biror sinf mavjud bo'lsa (bu yerda $\delta \mid p$-1), u holda shunday sinflar soni $\varphi(\delta)$ bo'ladi).

Agar p moduli bo'yicha δ ko'rsatkichga tegishli sinflar sonini ψ(δ) bilan belgilasak lemmani

$$\psi(\delta) = \begin{cases} 0 \\ \varphi(\delta) \end{cases}$$

ko'rinishda yozish mumkin. Bu agar δ ko'rsatkichga tegishli sonlar mavjud bo'lsa mod$p$ bo'yicha ularning soni $p-1$ga tengligini bildiradi. Lekin berilgan δ uchun p modul bo'yicha shu ko'rsatkichga tegishli son mavjud yoki mavjud emasligiga javob bermaydi. Bunga ushbu teorema javob beradi.

Teorema (Gauss). p tub modul bo'yicha p-1 ning har bir bo'luvchisi δ uchun shu δ ko'rsatkichga tegishli bo'lgan $\varphi(\delta)$ ta sinf mavjud. Xususan p moduli bo'yicha $\varphi(p-1)$ ta boshlang'ich ildiz mavjud.

Umuman boshlang'ich ildizlar $m=2,4,p^{\alpha}$ và $2p^{\alpha}$modullari bo'yichagina mavjud. Bu yerda $p>2$ tub son va $\alpha\geq 1$. I.M.Vinogradov p tub son bo'lsa, u holda $2^{2^{\kappa}}\sqrt{p}\ln p$ dan katta bo'lmagan boshlang'ich ildiz mavjud ekanligini isbotlagan, bu yerda κ soni p-1 ning har xil bo'luvchilari sonidir. Boshlang'ich ildizni topishning effektiv usuli esa hozirgacha topilgan emas. Qarab chiqilganlardan agar

$$g^{\frac{p-1}{\delta_1}}\neq 1,\quad g^{\frac{p-1}{\delta_2}}\neq 1,\ \ldots,\ g^{\frac{p-1}{\delta_k}}\neq 1$$

bo'lsa, u holda g ning p moduli bo'yicha boshlang'ich ildiz bo'lishi kelib chiqadi. Boshlang'ich ildizlarni aniqlashning ikkinchi bir usuli bu, agar p moduli bo'yicha boshlang'ich ildizlardan birortasi (yaxshisi eng kichigi) g ma'lum bo'lsa, qolgan barchasini $g^k(modp)$ ning eng kichik musbat chegirmasi sifatida aniqlash mumkin. Bunda $(k,p-1)=1$ va $1<k<p-1$.

309. 1) 2 soni 7 moduli bo'yicha tegishli bo'lgan daraja ko'rsatkichini toping.

2) 3 soni m=7 moduli bo'yicha tegishli bo'lgan daraja ko'rsatkichini toping.

3) 5 ning m=7 moduli bo'yicha qanday ko'rsatkichga tegishli ekanligini aniqlang.

310. Tanlash usuli bilan m moduli bo'yicha 2 dan $m-1$ gacha sonlar orasidan m bilan o'zaro tublari tegishli bo'lgan daraja ko'rsatkichlarini toping:

1). $m=5$; 2). $m=7$; 3). $m=8$; 4). $m=10$; 5) $m=11$; 6). $m=9$.

311. m moduli bo'yicha $m-1$ soni tegishli bo'lgan daraja ko'rsatkichini aniqlang.

312. Quyidagi modullar bo'yicha barcha boshlang'ich ildizlarni toping:

$1).p = 7;\quad 2)\, p = 11;\quad 3).p = 13;\quad 4).\ p = 17.$

313. Quyidagi modullar bo'yicha barcha boshlang'ich ildizlarning sonini va eng kichik boshlang'ich ildizni toping:

$1).p = 19;\quad 2)\, p = 23;\quad 3).p = 31;\quad 4).\ p = 37;$

$5).p = 43;\quad 6).53.$

314. Quyidagi modullarning har biri bo'yicha eng kichik boshlang'ich ildizni bilgan holda barcha boshlang'ich ildizlarni toping:

$1).p = 19;\quad 2)\, p = 23;\quad 3).p = 31.$

315. 6 moduli bo'yicha boshlang'ich ildizlarning barcha sinflarini toping.

316. $2, 2^2, 2^3, \dots, 2^{10}$ sonlarining 11 moduli bo'yicha chegirmalarning keltirilgan sistemasini tashkil etishini isbotlang.

317. $2^{2^n} + 1, (n = 1,2,\dots)$ sonining tub bo'luvchilari $k \cdot 2^{n+1} + 1$ ko'rinishda bo'lishini isbotlang.

318. $\varphi(a^m - 1) \equiv 0(mod m)$ ekanligini isbotlang. Bunda $a > 1$.

319. 8 moduli bo'yicha boshlang'ich ildizlarning mavjud emasligini isbotlang.

320. Quyidagi taqqoslamalar yechimga ega bo'ladigan b ning barcha qiymatlarini toping. $1).\, 5^x \equiv b(mod 9)$, $2).\, 4^x \equiv b(mod 9)$, $3).\, a^x \equiv b(mod m)$ taqqoslama yechimga ega bo'lmaydigan b ning barcha qiymatlarini sonini toping. Bunda $(a, m) = 1$.

V.2-§. Indekslar va ularning tatbiqlar

Boshlang'ich ildizlarning asosiy xossalari sonlar nazariyasiga logarifm tushunchasiga o'xshash yangi tushuncha, indekslar tushunchasini kiritish imkoniyatini beradi. Faraz etaylik g soni p tub moduli bo'yicha boshlag'ich ildiz bo'lsin. U holda

$$g^0,\ g^1,\ g^2,\ \dots,\ g^{p-1} \tag{1}$$

sonlari p moduli bo'yicha chegirmalarning to'la sistemasini tashkil etadi. Agar a, $(a,p)=1$ bo'lsa, u $modp$ bo'yicha (1) sistemadagi birorta $g^{\gamma_1}, 0 \le \gamma_1 \le p-1$ son bilan taqqoslanuvchi bo'lishi kerak, ya'ni

$$a \equiv g^{\gamma_1} (\mathrm{mod}\, p), \quad 0 \le \gamma_1 \le p-1 \tag{2}$$

Agar $(a,p)=1$ bo'lsa,

$$a \equiv g^{\gamma}(\mathrm{mod}\, p), \quad \gamma \geq 0 \qquad (3)$$

(3) shartni qanoatlantiruvchi γ soniga *a* sonining *p* moduli bo'yicha *g* asosga ko'ra indeksi deyiladi va $ind_g a$ ko'rinishda yoziladi. Demak, (3) dan

$$a \equiv g^{ind\, a}(\mathrm{mod}\, p)\ . \qquad (4)$$

Ta'rifdan *a* bilan *modp* bo'yicha taqqoslanuvchi barcha sonlar (4) da bitta indeksga ega:

$$0, 1, 2, \quad \ldots, p-2$$

Umuman har bir *a* soni (5) sistemada bitta indeksga ega. Lekin bir asosdan ikkinchi asosga o'tilsa indekslar umuman aytganda o'zgaradi. Ikkinchi tomondan esa berilgan *g* asosga ko'ra *a* soni cheksiz ko'p indekslar γ ga ega. (1) va (2) dan bular manfiy bo'lmagan butun sonlar bo'lib

$g^{\gamma} \equiv g^{\gamma_1}(\mathrm{mod}\, p)$ shartni qanoatlantirishi kerak. Bu yerda *g* soni *p* modul bo'yicha boshlang'ich ildiz bo'lganligi sababli, u *p-1* ko'rsatkichga tegishli. U holda ko'rsatkichga qarashli sonlarning xossalariga asosan yuqoridagi taqqoslama o'rinli bo'lishi uchun $\gamma \equiv \gamma_1(\mathrm{mod}\, p-1)$ bo'lishi kerak. Demak, *r* moduli bo'yicha *p* bilan o'zaro tub har bir chegirmalar sinfiga *p*-1 bo'yicha chegirmalarning biror sinfidagi manfiy bo'lmagan chegirmalardan iborat indekslar to'plami mos keladi va aksincha: $ind\, a \equiv ind\, b(\mathrm{mod}\, p-1)$ agarda $a \equiv b(\mathrm{mod}\, p)$ бўлса (4) га асосан

$$\gamma \equiv ind\, a(\mathrm{mod}\, p-1) \qquad (5)$$

Shuningdek indekslar quyidagi xossalarga ega:

1) ko'paytma $a \cdot b \cdot \cdots l$ ning indeksi *p*-1 moduli bo'yicha shu sonlar indekslari yig'indisi bilan taqqoslanuvchidir, ya'ni

$$ind(a \cdot b \cdot \ldots \cdot l) \equiv ind\, a + ind\, b + \ldots ind\, l(\mathrm{mod}\, p-1). \qquad (6)$$

2) $ind\, a^n \equiv n\, ind\, a(\mathrm{mod}\, p-1)$.

Shuningdek $ind 1 \equiv 0\,(\mathrm{mod}\, p-1)$, $ind\, g \equiv 1(\mathrm{mod}\, p-1)$

Indekslar jadvali. Indekslar jadvalini tuzish *p* tub modul bo'yicha berilgan songa ko'ra uning indeksi va aksincha berilgan indeksga ko'ra shu sonni topish imkoniyatini beradi. Bunda asos sifatida *p* modul bo'yicha boshlang'ich ildizlardan birortasi olinadi. Umuman indekslar jadvalini tub bo'lmagan boshlang'ich ildizlar mavjud bo'lgan *m* modul bo'yicha tuzish ham mumkin.

Indekslarning taqqoslamalarni yechishga tatbiqlari.

a) Ikki hadli taqqoslamalarni yechish. Ikki hadli bir noma'lumli tenglamaning umumiy ko'rinishi

$$ax^n \equiv b \pmod m \qquad (7)$$

Ma'lumki, murakkab m modul bo'yicha taqqoslamani tub modul bo'yicha taqqoslamani yechishga keltirish mumkin. Shuning uchun ham $m=p$ bo'lgan holni

$$ax^n \equiv b \pmod p, \qquad p \nmid a \qquad (8)$$

qaraymiz. $p>2$ deb olamiz. $p=2$ bo'lsa, 0 va 1 chegirmalarni sinab ko'rish yo'li bilan yechish mumkin. (8) dan $ind a + n ind x \equiv ind b (mod p-1)$ yoki bundan

$$n ind x = ind b - ind a \ (mod p-1). \qquad (9)$$

Demak,1) $(n, p-1)=1$ bo'lsa, u holda (9) va demak (8) ham yagona yechimga ega;

2) $(n, p-1)= d>1$ bo'lib, $d | ind\ b - ind a$ bo'lsa, (9) va demak (8) ham d ta yechimga ega;

3) $(n,p-1)=d>1$ bo'lib, $d \nmid ind\ b - ind a$ bo'lsa, (9) va demak (8) ham yechimga ega emas.

b). $x^n \equiv a \pmod p$ (10)

taqqoslamaning yechimga ega bo'lishi sharti. Bu taqqoslamani indekslasak

$$n\, ind\, x \equiv ind\, a \pmod{p-1}. \qquad (11)$$

Bu yerda $(n, \ p-1)=d$ bo'lsa, (11) ning yechimga ega bo'lishi uchun

$$ind\, a \equiv 0 \pmod d \qquad (12)$$

shartning bajarilishi zarur va yetarlidir. (12) shartni p va d ga bog'liq holda ifodalaymiz.

(12) ning ikkala tomonini va modulini $\frac{p-1}{d}$ ga ko'paytiramiz, u holda

$\frac{p-1}{d} ind\, a \equiv 0 \pmod{p-1}$ yoki $ind\, a^{\frac{p-1}{d}} \equiv 0 \pmod{p-1}$. Bundan esa

$$a^{\frac{p-1}{d}} \equiv 1 \pmod p \qquad (13)$$

Shunday qilib (10) ning yechimga ega bo'lishi uchun (13) shartning bajarilishi zarur va yetarlidir.

в) *Ko'rsatkichli taqqoslamalarni yechish.*

$$a^x \equiv b \pmod p. \qquad (14)$$

(14) dan $x ind a \equiv ind b (\mathrm{mod}\, p-1)$. Bu taqqoslamani esa osongina yechish mumkin.

321. Indekslar jadvalini tuzing: 1). 2 asosga ko'ra 29 moduli boyicha; 2). 5 asosga ko'ra 23 moduli bo'yicha.

322. 11 moduli bo'yicha indekslar jadvalini tuzing.

323. Quyidagi taqqoslamalardan δ ko'rsatkichni aniqlang:

1) $5^\delta \equiv 1(mod 7)$; 2)$5^\delta \equiv 1(mod 11)$; 3) $8^\delta \equiv 1(mod 13)$; 4) $12^\delta \equiv 1(mod 17)$; 5)$24^\delta \equiv 1(mod 31)$; 6) $10^\delta \equiv 1(mod 13)$ 7) $27^\delta \equiv 1(mod 17)$; 8)$18^\delta \equiv 1(mod 11)$; 9) $23^\delta \equiv 1(mod 41)$.

324. Indekslashdan foydalanib p tub moduli bo'yicha 2 dan p-1 gacha bo'lgan sonlar tegishli bo'lgan ko'rsatkichlarni toping: 1) $p = 5$; 2) $p = 7$; 3) $p = 11$.

325. Indekslashdan foydalanib quyidagi sonlarning 59 moduli bo'yicha boshlang'ich ildiz bo'lish bo'lmasligini aniqlang:

1) 2; 2) 3; 3) 6; 4) 8; 5) 12; 6) 13; 7) 14; 8) 19.

326. Quyidagi modullar bo'yicha barcha boshlang'ich ildizlarni toping: 1) $p = 17$; 2) $p = 19$; 3) $p = 23$.

327. Birinchi darajali taqqoslamalarni indekslardan foydalanib yeching:

1) $7x \equiv 23(mod 17)$; 2)$39x \equiv 84(mod 97)$;
3) $125x \equiv 7(mod 79)$;
4) $37x \equiv 25(mod 89)$; 5)$4x \equiv 13(mod 37)$;
6) $37x \equiv 5(mod 221)$;
7) $47x \equiv 13(mod 667)$; 8)$228x \equiv 317(mod 1517)$.

328. Ko'rsatkichli taqqoslamalarni indekslardan foydalanib yeching:

1) $2^x \equiv 7(mod 67)$; 2)$13^x \equiv 12(mod 47)$;
3) $16^x \equiv 11(mod 53)$;
4) $52^x \equiv 38(mod 61)$; 5)$12^x \equiv 17(mod 31)$;
6) $20^x \equiv 21(mod 41)$.

329. Ikki hadli taqqoslamalarni indekslardan foydalanib yeching:

1) $37x^{15} \equiv 62(mod 73)$; 2)$5x^4 \equiv 3(mod 11)$;
3) $2x^8 \equiv 5(mod 13)$;
4)$2x^3 \equiv 17(mod 41)$; 5)$27x^5 \equiv 25(mod 31)$;

$$6)\ 11x^3 \equiv 6(mod79);$$

$$7)\ 23x^3 \equiv 15(mod73); \quad 8) 8x^{26} \equiv 37(mod41);$$

$$9)\ 37x^8 \equiv 59(mod61); \ 10)\ 18x^8 \equiv 6(mod13).$$

330. Ikki hadli taqqoslamalarni indekslardan foydalanib yeching:

$$1)\ x^{12} \equiv 37(mod41); \quad 2) x^{55} \equiv 17(mod97);$$

$$3)\ x^{35} \equiv 17(mod67);$$

$$4)\ x^{30} \equiv 46(mod73); \quad 5) x^8 \equiv 23(mod41);$$

$$6)\ x^5 \equiv 74(mod71);$$

$$7)\ x^{27} \equiv 39(mod43); \quad 8) x^8 \equiv 29(mod13);$$

$$9)\ x^2 \equiv 59(mod67);$$

$$10)\ x^2 \equiv 59(mod83); \quad 11)\ x^2 \equiv 32(mod43);$$

$$12)\ x^2 \equiv -17(mod53);$$

$$13)\ x^2 \equiv -28(mod67); \quad 14)\ x^2 \equiv 56(mod41).$$

331. Eyler kriteriyasi va indekslardan foydalanib quyidagi sonlar 15, 16, 17, 18, 19, 20 dan qaysilari berilgan modul bo'yicha kvadratik chegirma bo'lishini aniqlang: 1) 23 moduli bo'yicha; 2) 29 moduli bo'yicha; 3) 41 moduli bo'yicha; 4) 73 moduli bo'yicha; 5) 97 moduli bo'yicha.

332. Berilgan modul bo'yicha indekslarning bir sistemasidan ikkinchi bir sistemasiga o'tish formulasini keltirib chiqaring.

333. a ning qanday butun qiymatlarida quyidagi munosabatlar o'rinli:

$$1)\ 3a^2 - 5 \vdots 7; \quad 2)\ 7a^2 + 13 \vdots 23; \quad 3)\ 13a^2 - 11 \vdots 29.$$

V.3-§. Taqqoslamalar nazariyasining ba'zi tatbiqlari

1.Berilgan songa bo'lishdan chiqqan qoldiqni hisoblash. Taqqoslamalar yordamida bo'linish belgilarini keltirib chiqarish.

A. Birinchi bo'lib fransuz matematigi B. Paskal berilgan N sonini m ga bo'lishdan chiqqan qoldiqni hisoblash qulay bo'ladigan qilib boshqa son bilan almashtirishning umumiy usulini ko'rsatgan. Biz bu usulni o'nlik sanoq sistemasida berilgan sonlar uchun qarab chiqamiz. Onlik sistemadagi N soni $N = a_0 + a_1 \cdot 10 + a_2 \cdot 10^2 + \dots + a_n \cdot 10^n$ ko'rinishda bo'lsin. 10^k ning m moduli bo'yicha absolyut qiymati jihatidan eng kichik chegirmasini r_k bilan belgilaylik, ya'ni $10^k \equiv r_k(modm)$, $k = 0,1,\dots,n$ va $r_0 = 1$ bo'lsin. U holda

$$N = a_0 r_0 + a_1 \cdot r_1 + a_2 \cdot r_2 + \dots + a_n \cdot r_n \qquad (1)$$

yoki

$$N \equiv R_m (mod m)$$

bajariladi. Bu yerda $R_m = a_0 r_0 + a_1 \cdot r_1 + a_2 \cdot r_2 + \ldots + a_n \cdot r_n$ yuqorida aytib o'tilgan almashtirishni ifodalaydi. (1)-taqqoslama Paskalning bo'linish belgisini ifodalaydi:

1. R_m va N ni m ga bo'lishdan bir xil qoldiq qoladi;

2. N ning m ga bo'linishi uchun R_m ning m ga bo'linishi zarur va yetarlidir.

Endi ba'zi bir xususiy hollarni qaraymiz:

1). Agar $m = 3$ bo'lsa, u holda $10 \equiv 1(mod 3)$ va $10^k \equiv 1(mod 3)$ bo'lgani uchun $R_3 = a_0 + a_1 + a_2 + \ldots + a_n$ bo'ladi. Bundan berilgan sonning 3 ga bo'linishi uchun uni tashkil etuvchi raqamlarining yig'indisining 3 ga bo'linishi zarur va yetarli degan tasdiq kelib chiqadi.

2). Shuningdek, agar $m = 9$ bo'lsa, u holda $10 \equiv 1(mod 9)$va $10^k \equiv 1(mod 9)$ bo'lgani uchun $R_9 = a_0 + a_1 + a_2 + \ldots + a_n$ bo'ladi. Bundan berilgan sonning 9 ga bo'linishi uchun uni tashkil etuvchi raqamlarining yig'indisining 9 ga bo'linishi zarur va yetarli degan tasdiq kelib chiqadi.

3). Agar $m = 11$ bo'lsa, u holda $10 \equiv -1(mod 11)$ va $10^k \equiv (-1)^k (mod 11)$ bo'lgani uchun $R_{11} = (a_0 + a_2 + \cdots) - (a_1 + a_3 + \cdots)$ bo'ladi. Bundan berilgan sonning 11 ga bo'linishi uchun uni tashkil etuvchi juft o'rindagi raqamlari yig'indisidan toq o'rindagi raqamlarini yig'indisining ayirmasi 11 ga bo'linishi zarur va yetarli degan tasdiq kelib chiqadi.

4). Agar $m = 7$ bo'lsa, u holda $10^0 \equiv 1(mod 7), 10 \equiv 3(mod 7), 10^2 \equiv 2(mod 7), 10^3 \equiv -1(mod 7)$,

$10^4 \equiv -3(mod 7), 10^5 \equiv -2(mod 7), 10^6 \equiv 1(mod 7)$ bo'lgani uchun $R_7 = (a_0 + 3a_1 + 2a_2) - a_3 - 3a_4 - 2a_5 + \cdots$ bo'ladi. Bu yerda endi ifoda biroz murakkab.

B. Endi 10 soni m moduli bo'yicha δ ko'rsatkichga qarashli bo'lgan holga to'xtalamiz: bu holda $10^\delta \equiv 1(mod m)$ bo'lgani uchun $r_\delta = 1$. Shuning uchun ham r_δ dan boshlab qoldiqlar takrorlanadi va $R_m = a_0 + a_1 \cdot r_1 + a_2 \cdot r_2 + \ldots + a_{\delta-1} \cdot r_{\delta-1} + a_\delta + a_{\delta+1} \cdot r_1 + \cdots$ bo'ladi. 3,7,9,11 modullari bo'yicha 10 soni mos ravishda 1, 6. 1, 2 ko'rsatkichlarga tegishli bo'lgani uchun bu modullar bo'yicha qoldiqlar mos ravishda r_1, r_6, r_1, r_2 lardan boshlab takrorlanadi. Buni biz yuqoridagi 1)-4)- misollarda ko'rdik.

2. Oddiy kasrni o'nlik kasrga aylantirishda hosil bo'ladigan kasrning davr uzunligini aniqlash. A. Ma'lumki, maxrajida 2 va 5 dan boshqa sonlar qatnashgan qisqarmas oddiy kasr $\frac{a}{b}$ ni o'nlik kasrga aylantirsak cheksiz davriy kasr hosil bo'ladi. Davrdagi raqamlar sonini aniqlash uchun avvalo qisqarmas oddiy kasr $\frac{a}{b}$ maxrajida 2 va 5 sonlari qatnashmagan, ya'ni $(10, b) = 1$ bo'lgan holni qaraymiz. Bunda $(a < b)$ bo'lgan holni ($\frac{a}{b}$ –to'g'ri kasrni) qarash bilan chegaralanish mumkin. Tushunarliki, bunday kasrning surati a soni $b$dan kichik va b bilan o'zaro tub bo'lgan $\varphi(b)$ ta quymatdan birini qabul qiladi. Oddiy kasrni o'nlik kasrga aylantirishdagi singari ish tutib m qadamdan keyin quyidagiga ega bo'lamiz:

$$\begin{cases} 10a = bq_1 + r_1, \\ 10r_1 = bq_2 + r_2, \\ \dots\dots\dots\dots\dots\dots\dots \\ 10r_{m-1} = bq_m + r_m, \end{cases} \qquad (1)$$

bu yerdagi barcha r_i qoldiqlar $0 < r_i < b$ shartni qanoatlantiradi. Shuningdek, $b > a$ bo'lgani uchun $q_1 < 10; b > r_1$ bo'lgani uchun $q_2 < 10$ va hokazo. Shunday qilib, barcha q_i lar raqamlardir. Misol uchun: $\frac{a}{b} = \frac{5}{13}$ bo'lsa, (1) quyidagicha bo'ladi: $10 \cdot 5 = 13 \cdot 3 + 11$,

$$10 \cdot 11 = 13 \cdot 8 + 6, 10 \cdot 6 = 13 \cdot 4 + 8, \ 10 \cdot 8 = 13 \cdot 6 + 2,$$
$$10 \cdot 2 = 13 \cdot 1 + 7,$$
$$10 \cdot 7 = 13 \cdot 5 + 5 \qquad (1')$$

lardan iborat bo'ladi.

(1) da $(10, b) = 1$ va$(a, b) = 1$ bo'lgani uchun $(10a, b) = 1$ bo'ladi bundan esa $(r_i, b) = 1$ ekanligi kelib chiqadi. Boshqacha qilib aytganda r_i lar b moduli bo'yicha chegirmalarning keltirilgan sistemasiga tegishli bo'ladi, ya'ni ularning soni $\varphi(b)$ tadan ko'p bo'la olmaydi. Shuning uchun ham ko'pi bilan $\varphi(b)$ qadamdan keyin qoldiqdagi va ular bilan birga bo'linmadagi raqamlar ham takrorlanadi. Bundan esa davrda $\varphi(b)$ tadan ko'p raqam bo'lmasligi kelib chiqadi.

B. Davrdagi raqamlar va davr haqida aniqroq ma'lumotga ega bo'lish uchun (1) dagi tengliklarni b moduli bo'yicha qaraymiz:

$$\begin{cases} 10a \equiv r_1(mod b), \\ 10r_1 \equiv r_2(mod b), \\ \dots\dots\dots\dots\dots\dots\dots \\ 10r_{m-1} \equiv r_m(mod b), \end{cases} \qquad (2)$$

bularni hadlab ko'paytirib va $(r_1 r_2 \cdots r_{m-1}, b) = 1$ bo'lgani uchun $r_1 r_2 \cdots r_{m-1}$ ga qisqartirib

$$10^m a \equiv r_m(mod b) \qquad (3)$$

ni hosil qilamiz. Endi m qandaydir bir son bo'lmasdan, balki 10 sonining b moduli bo'yicha tegishli bo'lgan daraja ko'satkichi bo'lsin, ya'ni m soni $10^m \equiv 1(mod b)$ taqqoslama o'rinli bo'lgan eng kichik ko'rsatkich bo'lsin. Bunday m lar uchun (3) dan $a \equiv r_m(mod b)$ kelib chiqadi. Bu yerda $0 < a < b$ va $0 < r_m < b$ bo'lgani uchun $a = r_m$ kelib chiqadi.

Shunday qilib, biz berilgan kasrning suratiga teng bo'lgan qoldiqni hosil qildik. Bu yerdan kelib chiqadiki, shu qadamdan boshlab qoldiqlar takrorlana boshlaydi: $r_{m+1} = r_1$, $r_{m+2} = r_2, \dots$. Tushunarliki, ana shunday aniqlangan m soni kasrning davridagi raqamlari sonini, davrning uzunligini bildiradi.

Demak, bu holda sof davriy kasrga ega bo'lamiz va bunda davrdagi raqamlar soni faqat berilgan kasrning maxrajiga bog'liq, suratiga bog'liq emas ekan. Bundan esa maxraji bir xil bo'lgan barcha oddiy kasrlarni o'nlik kasrga aylantirilganda bir xil davr uzunligiga ega degan xulosa kelib chiqadi.

C. Endi maxraji bir xil bo'lgan barcha oddiy kasrlarni o'nlik kasrga aylantirilganda davrda hosil bo'ladigan raqamlarni aniqlaymiz. (1) – tengliklardan $\frac{a}{b} = \frac{r_0}{b}$ kasrning davri $q_1 q_2 \cdots q_m$; $\frac{r_1}{b}$ kasrning davri $q_2 q_3 \cdots q_m q_1$; ... , $\frac{r_k}{b}$ kasrning davri $q_{k+1} \cdots q_k$ dan iborat ekanligi kelib chiqadi.

Shunday qilib, $\frac{r_0}{b}, \frac{r_1}{b}, \dots, \frac{r_{m-1}}{b}$ kasrlarning davrlari biridan raqamlarni doiraviy almashtirish natijasida hosil bo'lar ekan. Bunda $\frac{r_k}{b}$ kasrning davrini hosil qilish uchun $\frac{r_0}{b}$ kasrning davridagi k ta raqamni o'ng tomonga doiraviy almashtirish kerak bo'lar ekan.

Misol. $\frac{a}{b} = \frac{5}{13}$ bo'lsin. 10 soni 13 moduli bo'yicha 6 ko'rsatkichiga tegishli bo'lgani uchun davrda 6 ta raqam bo'lishi kerak. Shuning uchun ham (1') ga asosan

$$\frac{5}{13}=0,(384615),\quad \frac{11}{13}=0,(846153),\quad \frac{6}{13}=0,(461538),$$
$$\frac{8}{13}=0,(615384),\quad \frac{2}{13}=0,(153846),$$
$$\frac{7}{13}=0,(538461) \tag{4}$$
larga ega bo'lamiz.

D. Agar 10 soni b moduli bo'yicha boshlang'ich ildiz bo'lmasa ($b \neq p^{\alpha}, \alpha \geq 1$ bo'lganda bu albatta, shunday bo'ladi, chunki bunday b, $(b,10)=1$ modul bo'yicha boshlang'ich ildiz mavjud emas) u b moduli bo'yicha biror $m < \varphi(b)$ ko'rsatkichiga tegishli bo'ladi. Ma'lumki, bunda m soni $\varphi(b)$ ning bo'luvchisi bo'ladi, ya'ni $\varphi(b) = m \cdot d$, u holda maxraji b gat eng bo'lgan qisqarmas $\varphi(b)$ ta kasrlar $d$ta sistemaga bo'linadi. Bular:
$\frac{r_0}{b}, \frac{r_1}{b}, \dots, \frac{r_{m-1}}{b}; \frac{s_0}{b}, \frac{s_1}{b}, \dots, \frac{s_{m-1}}{b}; \dots; \frac{t_0}{b}, \frac{t_1}{b}, \dots, \frac{t_{m-1}}{b}$ lardan iborat bo'ladi. Bu yerda s_0 soni $r_0, r_1, \cdots, r_{m-1}$ lardan farql i $\frac{s_0}{b}$ kasrning surati.

Misol. 10 soni 13 moduli bo'yicha 6 ko'rsatkichiga tegishli bo'lgani uchun u boshlang'ich ildiz emas. Shuning uchun ham maxraji 13 ga teng bo'lgan to'g'ri kasrlar $d = \frac{\varphi(13)}{6} = \frac{12}{6}$ ta sistemaga ajraladi. Bulardan biri bilan biz yuqoridagi misolda tanishdik ((4) ga qarang). Ikkinchisini aniqlash maqsadida maxraji 13 ga teng bo'lgan surati esa (4) dagi kasrlarning suratidan farq qiluvchi biror kasrni olamiz. Masalan, $\frac{1}{13}$, u holda (4) ni tuzishdagi singari yo'l tutib
$$\frac{1}{13}=0,(076923),\quad \frac{10}{13}=0,(769230),\quad \frac{9}{13}=0,(692307),$$
$$\frac{12}{13}=0,(923076),\quad \frac{3}{13}=0,(230769),\quad \frac{4}{13}=0,(307692)$$
larni hosil qilamiz.

E. Endi b bilan 10 soni o'zaro tub bo'lmaganda $\frac{a}{b}$ kasrni o'nlik kasrga aylantirishni qaraymiz. Faraz etaylik $b = 2^{\alpha} \cdot 5^{\beta} \cdot b_1$ bo'lsin, bunda $(b_1, 10) = 1$. α va β sonlaridan eng kattasini n bilan belgilab olamiz. U holda

$$\frac{10^n a}{b} = \frac{2^{n-\alpha} \cdot 5^{n-\beta} \cdot a}{b_1} = \frac{a_1}{b_1}.$$

$\frac{a_1}{b_1}$, $(b, 10) = 1$ kasrni o'nlik kasrga aylantirib

$$\frac{10^n a}{b} = \frac{a_1}{b_1} = K, (q_1 q_2 \cdots q_m)$$

ni hosil qilamiz. Bundan $\frac{a}{b}$ ni topish uchun uni 10^n ga bo'lamiz. U holda vergulni n ta belgi chap tomonga surish kerak boladi. Buning natijasida $\frac{a_1}{b_1} = k, k_1 k_2 \cdots k_n (q_1 q_2 \cdots q_m)$ dan iborat aralash davriy kasrga ega bo'lamiz.

3. Arifmetik amallar natijasini tekshirish. Faraz etaylik N_1 sonini N_2 ga qo'shib N soni hosil qilingan bo'lsin:

$$N = N_1 + N_2 \tag{5}$$

U holda $N_1 \equiv r_1$, $N_2 \equiv r_2, N \equiv r(modm)$ deb yozish mumkin. (5) ga asosan

$$r_1 + r_2 \equiv r(modm) \tag{5'}$$

bajarilishi kerak. Shunga o'xshash

$$N = N_1 - N_2 \tag{6}$$

bo'lsa,

$$r_1 - r_2 \equiv r(modm); \tag{6'}$$

agarda

$$N = N_1 \cdot N_2 \tag{7}$$

bo'lsa,

$$r_1 \cdot r_2 \equiv r(modm); \tag{7'}$$

agarda

$$N = N_1 \cdot N_2 + N_3 \tag{8}$$

bo'lsa, ya'ni N ni N_1 ga bo'lsak N_2 tadan tegib N_3 qoldiq qolsa,

$$r_1 \cdot r_2 + r_3 \equiv r(modm) \tag{8'}$$

bajarilishi kerak. Tushunarliki, (5') – (8') shartlar (5) – (8) lardagi amallarning to'g'ri bajarilgan ekanligini tekshirishning zaruriy shartlari bo'lib ular yetarli shatlar bo'la olmaydi.

Amallar natijalarini tekshirish imkoni boricha ishonchli va bir vaqtning o'zida sodda bo'lishi uchun modul sifatida 9 soninni tanlash ma'qul, chunki sonni 9 ga bo'lishdan chiqqan qoldiq shu sonni tashkil etuvchi raqamlar yig'indisini 9 ga bo'lishdan chiqqan qoldiqga teng. Bu jarayonda berilgan sonning barcha raqamlari ishtirok etadi, shuning uchun ham ishonchlilik darajasi yuqori va jarayon sodda bo'ladi. Lekin $m = 10$ ni olsak jarayon yanada soddalashadi, lekin bunday tekshirishni ishonchli deb bo'lmaydi, chunki bu jarayonda berilgan sonning faqat oxirgi raqamigina istirok etadi. Agar tekshirishni $m = 11$ moduli bo'yicha bajarsak, ishonchlilik sezilarli darajada oshadi.

334. a sonini m ga bo'lishdan chiqqan qoldiqni toping:

1). $a = 2^{64}$, $m = 360$; 2). $a = 1532^5 - 1, m = 9$;

3). $a = (1271^{56} + 34)^{28}$, $m = 111$; 4). $a = 8!, m = 11$.

335. Agar $a^x \equiv 2(mod 13)$ va $a^{x+1} \equiv 6(mod 13)$ bo'lsa, a ni $m = 13$ bo'lishdan chiqqan qoldiqni toping.

336. Eyler teoremasini qo'llab a sonini m ga bo'lishdan chiqqan qoldiqni toping: 1). $a = 174^{249}, m = 13$; 2). $a = 18632^5 - 5, m = 10$; 3). $a = 2^{37\cdot 73-1}, m = 37 \cdot 73$.

337. Quyidagi sonlarning oxirgi ikkita raqamini toping:

1). 203^{20}; 2). 243^{402}; 3). $1812 \cdot 1941 \cdot 1965$; 4). $(116 + 17^{17})^{21}$.

338. Isbotlang: 1). $(2^{32} + 1) \vdots 641$; 2). $(222^{555} + 555^{222}) \vdots 7$;

3). $(220^{119^{69}} + 69^{220^{119}} + 119^{69^{220}}) \vdots 102$; 4). $(6^{2n+1} + 5^{n+2}) \vdots 31$.

339. $4^{\varphi(m)-1}$ sonini $m > 1$ toq soniga bo'lishdan chiqqan qoldiqni toping.

340. Indekslardan foydalanib berilgan a sonini m ga bo'lishdan chiqqan qoldiqni toping: 1). $a = 10^{10}, m = 67$; 2). $a = 178^{52}, m = 11$;

3). $a = 2017^{2018}, m = 11$.

341. Paskalning umumiy bo'linish belgisidan foydalanib 1)6 ga; 2) 8 ga;

3) 12 ga; 4) 15; 18; 45 ga bo'linish belgisini keltirib chiqaring.

342. 792 ga bo'linadigan $13xy45z$ ko'rinishidagi barcha sonlarni toping.

343. Quyidagi oddiy kasrlarni cheksiz o'nli kasrlarga aylantirmasdan davr uzunligini aniqlang:

1) $\frac{4}{21}$; 2) $\frac{9}{91}$; 3) $\frac{1}{43}$; 4) $\frac{a}{97}$, bunda $(a, 97) = 1$

344. Quyidagi oddiy kasrlarni o'nli kasrlarga aylantirganda hosil bo'ladigan davr uzunligini aniqlang:

1) $\frac{10}{17 \cdot 23}$; 2) $\frac{1}{53 \cdot 59}$; 3) $\frac{1}{7 \cdot 23 \cdot 31}$;

4) $\frac{1}{11 \cdot 13 \cdot 17}$; 5) $\frac{1}{13 \cdot 37}$.

345. Quyidagi oddiy kasrlarni o'nli kasrlarga aylantirganda hosil bo'ladigan davr uzunligini aniqlang:

1) $\frac{1}{14}$; 2) $\frac{7}{550}$; 3) $\frac{1}{5\cdot 23\cdot 31}$; 4) $\frac{1}{4\cdot 53\cdot 73}$; 5) $\frac{1}{10\cdot 37}$.

346. Taqqoslamalardan foydalanib quyidagi tengliklarning xato ekanligini ko'rsating: 1). $4237\cdot 27925 = 118275855$; 2). $42981:8264 = 5201$; 3). $1965^2 = 3761225$.

347. Taqqoslamalardan foydalanib quyidagi tengliklarning to'g'riligini tekshiring: 1). $25041 + 91382 = 116423$; 2). $42932 - 18265 = 24667$; 3). $13547 - 9862 = 3685$; 4). $235463 - 25376 = 210087$.

VI BOB. UZLUKSIZ KASRLAR VA ULARNING TATBIQLARI

VI.1 -§. Chekli uzluksiz kasrlar

Agar $\frac{a}{b}$ –qisqarmas (to'g'ri yoki noto'g'ri) oddiy kasr berilgan bo'lsa, uni Evklid algoritmi yordamida ko'rinishida ifodalash mumkin (I.2- paragrafga qarang).

$$\frac{a}{b} = q_0 + \cfrac{1}{q_1 + \cfrac{1}{q_2 + \ddots + \cfrac{1}{q_n}}} \qquad (1)$$

(1) ga $\frac{a}{b}$ - ratsional sonining chekli uzluksiz (zanjirli) kasrga yoyilmasa deyiladi. Bunda q_0 – butun son, $q_1, q_2, \dots, q_n$ lar natural sonlar, q_i larga chala bo'linmalar ham deyiladi. (1) yozuv o'rniga

$$\frac{a}{b} = (q_0, q_1, q_2, \dots, q_n) \qquad (2)$$

qisqa yozuv ham ishlatiladi. Agarda biz $q_n > 1$, bo'lishini talab qilsak (2) yagonadir. Aks holda yagona bo'lmaydi, chunki $q_n = (q_n - 1) + \frac{1}{1}$.

To'g'ri musbat kasrni uzluksiz kasrga yoysak $q_0 = 0$ bo'ladi. Agarda manfiy kasrni uzluksiz kasrga yoysak birinchi elementi $q_0 < 0$ bo'ladi, chunki manfiy sonning butun qismi manfiy, kasr qismi esa hamma vaqt musbat sondir.

Shuningdek har qanday butun sonni $m=(m)$ bir elementli uzluksiz kasr deb, har qanday $\frac{1}{m}$ ko'rinishdagi to'g'ri kasrni esa $\frac{1}{m} = (0, m)$ deb qarash mumkin.

Uzluksiz kasrlarning tatbiqlarida munosib kasrlar deb ataluvchi ushbu $\delta_0 = q_0,\ \delta_1 = q_0 + \frac{1}{q_1},\ \delta_2 = q_0 + \frac{1}{q_1 + \frac{1}{q_2}}, \ldots, \delta_n = q_0 + \frac{1}{q_1 + \ddots + \frac{1}{q_n}}$

yoki

$$\delta_0 = q_0, \quad \delta_1 = (q_0, q_1), \quad \delta_2 = (q_0, q_1, \quad q_2),$$
$$\ldots, \delta_n = (q_0, q_1, q_2, \quad \ldots, q_n)$$

kasrlar muhim hamiyatga ega. Tushunarliki.

$$\delta_n = (q_0, q_1, q_2, \ldots, q_n) = \frac{a}{b}.$$

δ_k ga odatda k-tartibli munosib kasr deyiladi. Endi $\delta_k = \frac{P_k}{Q_k}$ deb olsak uning surat va maxrajini hisoblash uchun quyidagi rekurent formula

$$\begin{cases} P_k = P_{k-1}q_k + P_{k-2} \\ Q_k = Q_{k-1}q_k + Q_{k-2} \end{cases}, \quad k = 0,1,2,\ldots$$

o'rinli. Bunda $P_{-2} = 0,\ P_{-1} = 1$ va $Q_{-2} = 1,\ Q_{-1} = 0$ deb olinadi. Munosib kasrlarni hisoblashda quyidagi javdal ancha qulay

q_i			q_0	q_1	...	q_{k-2}	q_{k-1}	q_k	...	q_n
P_i	$P_{-2} = 0$	$P_{-1} = 1$	$P_0 = q_0$	P_1	...	P_{k-2}	P_{k-1}	P_k	...	P_n
Q_i	$Q_{-2} = 1$	$Q_{-1} = 0$	$Q_0 = 1$	Q_1	...	Q_{k-2}	Q_{k-1}	Q_k	...	Q_n

Munosib kasrlar va berilgan $\frac{a}{b}$ kasr orasida quyidagi munosabatlar o'rinli:

$$\frac{P_0}{Q_0} < \frac{P_2}{Q_2} < \frac{P_4}{Q_4} < \ldots < \frac{a}{b} < \cdots < \frac{P_5}{Q_5} < \frac{P_3}{Q_3} < \frac{P_1}{Q_1}.$$

Bu yerdan ko'rinadiki, $\frac{a}{b}$ – kasr doimo ikkita qo'shni munosib kasr orasida joylashgan bo'ladi. Bunda munosib kasrlarning tartibi o'sishi bilan ular orasidagi interval kichrayib boradi. $\frac{a}{b}$ – kasrni $\frac{P_k}{Q_k}$ – munosib kasr bilan almashtirishdan hosil bo'ladigan xatoliknk baholash uchun

$$\left|\frac{a}{b}-\frac{P_k}{Q_k}\right| \le \frac{1}{Q_k Q_{k+1}}$$

munosabatdan foydalanamiz.

348. Berilgan kasrlarni uzluksiz kasrga yoying:

1) $\frac{127}{52}$, 2) $\frac{24}{35}$, 3) $1{,}23$, 4) $\frac{29}{37}$.

349. Berilgan chekli uzluksiz kasrlarga mos qisqarmas oddiy kasrni toping:

1) (1,1,2,1,2,1,2), 2) (0,1,2,3,4,5), 3) (5,4,3,2,1), 4) (a,a,a,a,a), 5) (a,b,a,b,a), 6) (2,1,1,3,1,2), 7) (1,1,2,3,4), 8) (2,5,3,2,1,4,2,3).

350. Quyidagi kasrlarni uzluksiz kasrlarga yoyishdan foydalanib qisqartiring:

1) $\frac{3587}{2743}$, 2) $\frac{1043}{3427}$, 3) $\frac{3653}{3107}$, 4) $\frac{11281}{6583}$, 5) $\frac{1491}{2247}$.

351. Tenglamalarni yeching: 1) $(x,2,3,4)=\frac{73}{30}$, 2) $7(xyz+x+z)=10(yz+1)$.

352. Berilgan kasrlarni uzluksiz kasrga yoying va uni $\frac{P_5}{Q_5}$ – munosib kasr bilan almashtirib xatoligini aniqlang hamda almashtirishni taqribiy tenglik yordamida xatoligini ko'rsatgan holda yozing:

1) $\frac{29}{37}$, 2) $\frac{163}{159}$, 3) $\frac{648}{385}$, 4) $\frac{1882}{1651}$.

353. Berilgan kasrlarni uzluksiz kasrga yoying va uni $\frac{P_5}{Q_5}$ – munosib kasr bilan almashtirib xatoliogini aniqlang hamda almashtirishni taqribiy tenglik yordamida xatoligini ko'rsatgan holda yozing:

1) $\frac{571}{359}$, 2) $\frac{2341}{1721}$.

354. Tishlari sonining nisbati $\frac{571}{359}$ ga teng bo'lgan ikkita shesterna yordamida tishli uzatma qurish talab etiladi. Tishlari sonining berilgan nisbatini surat maxraji eng kichik bo'lgan va xatoligi 0,001 dan oshmaydigan uzatmani qurish texnik jihatidan mumkinmi?

355. $(2,2,2,\ldots,2)$ uzluksiz kasrni 2 ga bo'lishdan hosil bo'lgan bo'linmani toping.

356. $(a, a, a, \dots, a)$ uzluksiz kasrni 2 ga bo'lishdan hosil bo'lgan bo'linmani toping.

357. Tenglikni isbotlang:

$$\left(\frac{P_{n+2}}{P_n}-1\right)\cdot\left(1-\frac{P_{n-1}}{P_{n+1}}\right)=\left(\frac{Q_{n+2}}{Q_n}-1\right)\left(1-\frac{Q_{n-1}}{Q_{n+1}}\right).$$

358. Agar P_i va Q_i lar $(q_1, q_2, \dots, q_n)$ – uzluksiz kasrning munosib kasrlarining elementlari bo'lib, $n \geq 1$ bo'lsa

$$\frac{P_n}{P_{n-1}}=(q_n, q_{n-1}, \dots q_1) \text{va } \frac{Q_n}{Q_{n-1}}=(q_n, q_{n-1}, \dots q_2)$$

ekanligini ko'rsating.

359. $\frac{P_n}{P_{n-1}}$ va $\frac{Q_n}{Q_{n-1}}$ larning qisqarmas kasr ekanligini isbotlang.

360. Isbotlang:

$$\left(\underbrace{2,2,2,\dots,2}_{nta}\right)=\frac{\left(1+\sqrt{2}\right)^{n+1}-\left(1-\sqrt{2}\right)^{n+1}}{\left(1+\sqrt{2}\right)^{n}-\left(1-\sqrt{2}\right)^{n}}$$

361. $P_nQ_{n-1}-Q_nP_{n-1}=(-1)^{n-1}$ munosabatdan foydalanib ikki noma'lumli birinchi darajali aniqmas tenglamalarni yechish usulini bayon qiling.

362. 361- misolda bayon qilingan usuldan foydalanib quyidagi tenglamalarni yeching: 1) $38x+117y=209$, 2) $122x+129y=2$, 3) $119x-68y=34$,4) $258x-175y=113$, 5) $41x+114y=5$, 6) $70x+33y=1$.

363. Agar a natural son bo'lsa, $\frac{a^4+3a^2+1}{a^3+2a}$ ning qisqarmas kasr ekanligini isbotlang.

364. Simmetrik uzluksiz kasr $(q_n=q_1,\ q_{n-1}=q_2, \dots)$ lar uchun $P_{n-1}=Q_n$ munosabatning o'rinli ekanligini isbotlang.

365. Agar $n \geq 2$ bo'lsa, $Q_n \geq 2^{\frac{n-1}{2}}$ ekanligini isbotlang.

366. $P_nQ_{n-1}-Q_nP_{n-1}=(-1)^n$ munosabatdan foydalanib $ax \equiv b(mod m)$ taqqoslamaning $(a,m)=1$ bo'lgandagi yechimini topish uchun formula keltirib chiqaring.

367. 366- misolda bayon qilingan usuldan foydalanib quyidagi taqqoslamalarni yeching: 1) $95x \equiv 59(mod 308)$, 2) $91x \equiv 1(mod 132)$.

VI.2 -§. Cheksiz uzluksiz kasrlarning yaqinlashuvchaniligi

Munosib kasrlar quyidagi xossalarga ega:

1^0. $\frac{P_k}{Q_k} - \frac{P_{k-1}}{Q_{k-1}} = \frac{(-1)^{k-1}}{Q_k Q_{k-1}}$ yoki $\Delta_k = P_k Q_{k-1} - P_{k-1} Q_k = (-1)^{k-1}$.

Bu yerdan $\frac{P_k}{Q_k}$ qisqarmas kasr degan xulosaga kelamiz, chunki $(P_k, Q_k) = 1$.

2^0. Munosib kasrlarning tartibining o'sib borishi bilan ularning juft tartiblilari o'sadi, toq tartiblilari esa kamayadi. Bunda har bir juft tartibli munosib kasr ixtiyoriy toq tartibli munosib kasrdan kichik bo'ladi.

3^0. $\alpha = (q_0, q_1, \dots, q_k, \alpha_{k+1}) = \frac{P_k \alpha_{k+1} + P_{k-1}}{Q_k \alpha_{k+1} + Q_{k-1}}$, $k = 1,2,\dots$ va $\alpha_{k+1} = (q_{k+1}, q_{k+2}, \dots)$.

4^0. $\alpha = (q_0, q_1, \dots, q_k, \dots)$ – irratsional soni uchun

$$\frac{P_0}{Q_0} < \frac{P_2}{Q_2} < \frac{P_4}{Q_4} < \dots < \alpha < \dots < \frac{P_5}{Q_5} < \frac{P_3}{Q_3} < \frac{P_1}{Q_1}$$

va

$$\alpha = \lim_{k\to\infty} \frac{P_k}{Q_k}$$

munosabatlar o'rinli. $\frac{P_k}{Q_k}$ – munosib kasr α – haqiqiy soni uchun eng yaxshi ratsional yaqinlashish bo'ladi, ya'ni maxraji $y \le Q_k$ shartni qanoatlantiruvchi birorta ham $\frac{x}{y}$ ratsional kasr α – haqiqiy soniga $\frac{P_k}{Q_k}$ – munosib kasrga qaragan yaqin bo'la olmaydi. $\frac{P_k}{Q_k}$ –kasr α – haqiqiy soniga $\frac{1}{Q_k Q_{k+1}}$ aniqlik bilan yaqinlashadi. α – haqiqiy soniga berilgan ε aniqlik bilan yaqinlashadigan munosib kasrni aniqlash uchun $Q_k > \sqrt{\frac{1}{\varepsilon}}$ bajariladigan qilib olish kerak bo'ladi. Shuni ham ta'kidlash kerakki, bunday aniqlikni kichikiroq tartibli munosib kasrlar ham ta'minlashi mumkin.

368. Quyidagi sonlarni 4-tartibli munosib kasrlar bilan almashtiring. va buning natijasida hosil bo'ladigan xatolikni baholang:

1). $\frac{587}{103}$, 2). 3,14159, 3). $\frac{-1+\sqrt{5}}{2}$, 4). $\frac{2-\sqrt{3}}{5}$,
5). $\frac{1+\sqrt{5}}{2}$, 6) $\frac{-1+\sqrt{2}}{2}$.

369. $\frac{1261}{881}$ – sonini imkoni boricha kichik maxrajli munosib kasr bilan almashtiringki, bunda xatolik 0,0001 dan katta bo'lmasin.

370. Berilgan sonlarga 0,001 gacha aniqlikdagi eng yaxshi yaqinlashishni toping: 1). $\sqrt{2}$, 2). $\sqrt{3}$, 3). $\sqrt{7}$, 4). $\sqrt{11}$.

371. Berilgan tenglamalarning ildizlariga 0,0001 gacha aniqlikdagi eng yaxshi yaqinlashishni toping: 1). $x^2-5x+2=0$, 2). $4x^2+20x+23=0$,
3). $x^2+9x+6=0$, 4). $2x^2-3x-6=0$.

372. Avvalo $\frac{P_n}{Q_n}$ va $\frac{P_n+P_{n+1}}{Q_n+Q_{n+1}}$ larning ikkalasi ham α ning bir tomonida yotishiga ishonch hosil qiling va $\left|\alpha-\frac{P_n}{Q_n}\right|>\frac{1}{Q_n(Q_n+Q_{n+1})}$ tengsizlikning o'rinli ekanligini isbotlang.

373. Agar q_n – chala bo'linma bir necha birlikka ortsa n-tartibli munosib kasr ortadimi yoki kamayadimi?

374. $n\geq 1$ bo'lsa quyidagi tengsizliklardan hech bo'lmasa bittasining o'rinli ekanligini isbotlang: $\left|\alpha-\frac{P_n}{Q_n}\right|<\frac{1}{2Q_n^2}$ yoki $\left|\alpha-\frac{P_{n-1}}{Q_{n-1}}\right|<\frac{1}{2Q_{n-1}^2}$.

VI.3 -§.Cheksiz uzluksiz kasrlar va kvadrat irratsionalliklar

Butun koeffitsiyentli kvadrat tenglamani qanoatlantiruvchi irratsionallikga kvadrat irratsionallik deyiladi. Kvadrat irratsionallikning umumiy ko'rinishi $\frac{a+\sqrt{b}}{c}$ dan iborat. Bunda $a,c\neq 0$ va $b>0$ – butun sonlar. Cheksiz davriy uzluksiz kasrlar (sof yoki aralash bo'lishidan qat'i nazar) kvadrat irratsionalliklar bilan yaqindan bog'langan. Bu bog'lanishlarni quyidagi teoremalar yordamida ifodalash mumkin:

1.Har bir cheksiz davriy uzluksiz kasrlar (sof yoki aralash bo'lishidan qat'i nazar) butun koeffitsiyentli kvadrat tenglamaning haqiqiy ildizi, ya'ni kvadrat irratsionallik bo'ladi.

2. Har bir butun koeffitsiyentli kvadrat tenglamaning haqiqiy irratsional ildizi cheksiz davriy uzluksiz kasr (sof yoki aralash bo'lishidan qat'i nazar) ga yoyiladi.

375. Quyidagi uzluksiz kasrlar bilan berilgan kvadrat irratsionalliklarni toping: 1). $(\overline{2,3})$, 2). $(\overline{1,1,2,2})$, 3). $(\overline{3,4,5})$, 4). $(1,2,3,\overline{4})$, 5). $(0,1,1,1,1,\overline{2,2,2,})$, 6). $(a, \overline{a,2a})$, 7). $(\overline{2,2,1,1})$.

376. Bir xilda chala bo'linmali cheksiz davriy uzluksiz kasrlarga yoyiladigan kvadrat irratsionalliklarning umumiy ko'rinishini toping.

377. Agar 1) $\frac{P_k}{Q_k}=\frac{10}{3}, \alpha_{k+1}=\sqrt{2}$; 2) $\frac{P_k}{Q_k}=\frac{37}{13}, \alpha_{k+1}=\frac{1+\sqrt{3}}{2}$ bo'lsa, α irratsionalliklarni toping.

378. Uzluksiz kasrlarga yoying va $\frac{P_3}{Q_3}$ ni aniqlang: 1) $\sqrt{x^2+1}$, 2) $\sqrt{a^4+2a}$.

379. $\sqrt{a^2+a+1}$ irratsionallik cheksiz davriy uzluksiz kasrlarga yoyilsa, $\frac{P_3}{Q_3}=\frac{2a+1}{2}$ bo'lishini isbotlang.

380. a va b lar natural sonlar bo'lsa, $bx^2-abx-a$ kvadrat uch hadning musbat ildizining sof cheksiz davriy uzluksiz kasrlarga yoyilishini isbotlang. Teskari teorema o'rinli bo'ladimi?

381. Quyidagi teoremani isbotlang: agar butun koeffitsiyentli kvadrat tenglamaning bitta ildizi $x=(\overline{a,b})$ bo'lsa, uning ikkinchi ildizi $-\frac{1}{(\overline{b,a})}$ bo'ladi.

382. Agar musbat kvadrat irratsionallik sof cheksiz davriy uzluksiz kasrga yoyilsa, unga qo'shma bo'lgan irratsionallikning $(-1,0)$ intervalga tegishli bo'lishini isbotlang.

383. Agar butun koeffitsiyentli kvadrat tenglamaning bitta ildizi $x=(a,\overline{b,c})$ bo'lsa, uning ikkinchi ildizi $a-(\overline{c,b})$ bo'lishini isbotlang.

384. $(\overline{a,b})$ va $(0,\overline{b,a})$ uzluksiz kasrlar ko'paytmasini toping.

385. $\alpha=(a,b,c)$ va $\beta=(c,b,a)$ sonlarining $x=(\overline{a,b,c})$ va $y=(\overline{c,b,a})$ sonlariga proporsional ekanligini isbotlang.

386. $\sqrt{n}$ – (n natural son) ko'rinishdagi irratsionallikning davri ikkinchi chala bo'linmadan boshlanuvchi cheksiz davriy uzluksiz kasrga yoyilishini isbotlang.

VI.4 -§. Algebraik va transsendent sonlar

Ushbu

$$a_0x^n + a_1x^{n-1} + \cdots + a_n = 0, \quad (a_0 \neq 0) \qquad (1)$$

ratsional koeffitsiyentli n-darajali tenglamaning ildizi α ga algebraik son deyladi. Aks holda α ga transsendent son deyladi. Boshqacha qilib aytganda algebraik bo'masagan sonlarga transsendent sonlar deyladi.

Ta'rifdan umuman olganda α algebraik son, bu kompleks son bo'lishi kelib chiqadi. Ma'lumki, ratsional koeffitsiyentli tenglamani hamma vaqt butun koeffitsiyentli tenglamaga keltirish mumkin.

Agar α

$$x^n + a_1x^{n-1} + \cdots + a_n = 0, \qquad (2)$$

ratsional koeffitsiyentli n-darajali bosh hadining koeffitsiyenti 1 ga teng bo'lgan tenglamaning ildizi bo'lsa, α ga butun algebraik son deyiladi. Agar α (1) tenglamaning ildizi bo'lib darajasi undan kichik bo'lgan algebraik tenglamaning ildizi bo'lmasa, α ga n-tartibli algebraik son deyiladi.

Agar α va β lar algebraik sonlar bo'lsa, u holda $\alpha + \beta, \alpha - \beta,\ \alpha \cdot \beta$ lar va agar $\beta \neq 0$ bo'lsa $\frac{\alpha}{\beta}$ ham algebraik son bo'ladi. Bundan tashqari quyidagi teoremalar o'rinli.

Liuvil teoremasi. Har bir haqiqiy n-tartibli α algebraik son uchun shunday c>0 soni mavjudki, α dan farqli barcha $\frac{a}{b}$ – ratsional sonlar uchun $\left|\alpha - \frac{a}{b}\right| > \frac{c}{b^n}$ munosabat o'rinli bo'ladi.

Natija. Agar $q_n > (Q_{n-1})^{n-1}, n = 1,2,\ldots$ bo'lsa, $\alpha = (q_0, q_1, q_2, \ldots)$ irratsional son transsendent son bo'ladi.

Gelfond teoremasi. Agar α soni 0 va 1 dan farqli algebraik son, β esa tartibi 2 dan kichik bo'lmagan algebraik son bo'lsa, u holda α^β – transsendent son bo'ladi.

Lindeman teoremasi. $x = 0$ va $y = 1$ dan boshqa hollarda $y = e^x$ tenglamada x va y sonlari bir vaqtda algebraik son bo'la olmaydi.

387. Quyidagi sonlarning algebraik sonlar ekanligini ko'rsating:

1). $\frac{3}{5}$; 2) $\sqrt{3}$; 3). $\sqrt[3]{3}$, 4). $1 + \sqrt{2}$; 5). $2 - \sqrt{2}$; 6). $1 + i$;

7). $\sqrt{3} + \sqrt{5}$; 8). $\sqrt[4]{4 - \sqrt[3]{2}}$; 9). $a + \sqrt[n]{b}$; 10). $a + i\sqrt{b}$

(bunda a va b lar ratsional sonlar); 11). $\cos\frac{\pi}{n} + i\sin\frac{\pi}{n}$; 12). $sin10^0$.

388. Quyidagi algebraik sonlarning tartibini aniqlang:
1). $a + bi$ (a va $b \neq 0$ lar ratsional sonlar); 2). $\sqrt[3]{3}$, 3). $\sqrt[3]{2} - 1$; 4). $\sqrt{2} - \sqrt{3}$; 5). $\sqrt{3} + \sqrt{5}$; 6). $2 + i$.

389. Berilgan tenglamalarning ildizlarining algebraik sonlar ekanligini isbotlang: 1). $x^3 + 2\sqrt{2}x^2 + 2 = 0$; 2). $x^2 + 2ix + 10 = 0$.

390. Quyidagi berilgan tenglamalarning ildizlarining tartibi berilgan tenglamaning tartibiga teng bo'lgan algebraik sonlar ekanligini isbotlang:
1). $x^3 + 2x^2 - 4x + 2 = 0$; 2). $2x^5 + 6x^3 - 9x^2 - 15 = 0$, 3). $x^4 - 5x^2 + 10x + 20 = 0$, 4). $x^5 - 3x^2 + 12x - 6 = 0$.

391. Liuvil metodidan foydalanib birorta transtsendent sonni quring.

392. Liuvil soni $\alpha = \frac{1}{10^{1!}} + \frac{1}{10^{2!}} + \frac{1}{10^{3!}} + \ldots$ ning transtsendent ekanligini isbotlang.

393. Gelfond teoremasidan foydalanib quyidagi sonlarning transtsendent ekanligini isbotlang:

1). $lg2$; 2) $log_2 10$; 3). $ln5$; 4). $3^{\sqrt{2}}$; 5). $5^{\sqrt{3}}$; 6). $2^{i\sqrt{3}}$; 7). 3^{1-i}; 8). $5^{2-i\sqrt{2}}$.

II qism. Javoblar

I.1-§.

1. 233. **2.** 1) $b = 7,\ 8$ va $r = 4,1$. 2) $b = 8,9$ va $r = 2,6$.
13. $n = 5q + 1$ va $n = 5q + 3$, $q = 0,1,2,\dots$.
23. $S_n = \frac{7}{81} \cdot (10^{n+1} + 9n - 10)$.

I.2-§.

27. 1)21. 2) 13. 3) 37.
28. $a)$21 va 6300.$b)$ 23 va 2799997. **33.** ha. **35.** $a)d$. $b)m$. $c)1$. $d)d$. **36.** $a)1$. $b)1$. $c)1$. **39.** $2a)23$. $2b)7$.
41. $(n, n+1, n+2) = 1$; $[n, n+1, n+2] = n(n+1)(n+2)$, agarda n toq son bo'lsa va $[n, n+1, n+2] = \frac{1}{2}n(n+1)(n+2)$, agarda n juft son bo'lsa.
42. nab ni $n - 1$ ta ko'rsatilgan ko'rinishda ifodalash mumkin.
43. $(899,493) = 29 = 899(-6) + 11 \cdot 49$ va $x = -6, y = 11$.
45. yo'q. **49.** $a)(30,120), (60,90), (90,60), (120,30)$. $b)x = 495$, $y = 315$. $c)(20,420)$, $(60,140), (140,60)$, $(420,20)$. $d)(140,252)0$. $e)(10,2), (2,10)$ **53.** Berilgan son 19 ga bo'linadi.

I.3-§.

55. $N = p_1 - 2$, bunda $p_1 -$ toq tub son. **58.** 1) 127 – tub son. 2)919 – tub son. 3) $7429 = 17 \cdot 437 -$ murakkab son. **59.** 1)101,103,107,109 lar tub sonlar. 2)191, 193, 197, 199 lar tub sonlar. 3) 211.4) 2647, 2657, 2659, 2663, 2671, 2677.
61. $21! + 2,\ 21! + 3,\ \dots,\ 21! + 20,\ 21! + 21$. **62.** $n, n+10, n+14$ sonlar bir vaqtda tub bo'ladigan n ning faqat 1 ta qiymati $n = 3$ mavjud.
63. $p = 3$ qiymatida $2p^2 + 1 = 19 -$ tub son bo'ladi.
64. $p = 5$. **67.** $2^{18} + 3^{18} = 13 \cdot 61 \cdot 37 \cdot 73 \cdot 181$.

II.1-§.

76. 1)$\pi(5)=3$. 2)$\pi(10) = 4$. 3) $\pi(25) = 9$. 4) $\pi(37) = 12$. 5) $\pi(200) = 46$. 6) $\pi(1000) = 168$. **77.** 1)$\pi(100) \approx 22, \omega = \frac{\Delta\pi(x)}{\pi(x)} = 12\%$. 2) $\pi(500) \approx 80, \omega \approx 16\%$. 3)$\pi(1000) \approx 145$, $\omega \approx 14\%$. 4) $\pi(3000) \approx 3,75$; $\omega \approx 12\%$.

II.2-§.

81. $a)3. b)11.$ $c)$ $1. d)2.$ $e)3.$ $i)2.$ $j)2.$ $f)-2. l)-1.$ $k)$ $7.$

89. $a)-\sqrt{3}<x\leq-\sqrt{2}$ va $\sqrt{2}<x\leq\sqrt{3}$. $b)$ $x=1$. $c)$ $x=0,$ $\frac{4}{3},$ $\frac{8}{3}$. $d)$ $x=0,1$.

91. $[-x]=\begin{cases}-[x] & \text{ga; agar } x \text{ butun son bo'lsa;}\\ -[x]-1 & \text{ga; agar } x \text{ kasir son bo'lsa.}\end{cases}$ **94.** 11450; **95.** 686.

96. 33. **97.** 502. **98.** $\frac{p^n-1}{p-1}$. **99.** 48. **100.** $11!=2^8\cdot3^4\cdot5^2\cdot7\cdot11$.

n**101.**148. **102.** $p=2$bo'lsa, $m+\sum_{i=1}^{k}\left[\frac{m}{2^i}\right]$ ga teng; $p>2$bo'lsa, $\sum_{i=1}^{s}\left[\frac{m}{p^i}\right]$ ga teng. **103.** $2m+1\leq x<2m+2,$ $m=0,1,2,\dots$.

104. $a>0$ bo'lganda $\left[-\frac{b^2-4ac}{4a}\right]\leq d;$ $a<0$ bo'lganda $\left[-\frac{b^2-4ac}{4a}\right]\geq d.$

105. $\sum_{k=a}^{b}([f(k)]+1)$. **106.** 136. **107.** 5631.

II.3-§.

108. 1).$\tau(375)=8, \sigma(375)=624$.2). $\tau(720)=30, \sigma(720)=2418$.3). $\tau(957)=8, \sigma(957)=1440$.4). $\tau(988)=12,$ $\sigma(988)=1960$.
5). $\tau(988)=24,$ $\sigma(990)=2808$.
6). $\tau(1200)=30,$ $\sigma(1200)=3844$.7). $\tau(1440)=36,$ $\sigma(1440)=4914$.8). $\tau(1500)=24,$ $\sigma(1500)=4368$.9). $\tau(1890)=32$, $\sigma(1890)=5760$. 10). $\tau(4320)=48,$ $\sigma(4320)=15120$. **109.** 1). 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360. Ularning jami soni 24 *ta*.2).1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15,16, 18, 20,24, 30,36, 40, 45, 48, 60, 72, 80,90,120,144,180,240,360,720. *Jami*: 30*ta*. 3). 1, 2, 3, 6, 9, 18, 53, 106, 159, 318, 477, 954. *Jami*: 12 *ta* .4). 1, 2, 4, 13, 19, 26, 38, 52, 76, 247, 494, 988. *Jami*: 12 *ta*. 5).1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24,25, 30, 40, 50,60, 75, 100,120150,200,300,600. *Jami*: 24 *ta*.

110.12. **111.**28. **115.** 1). $\tau(m)\tau(n)>\tau(mn)$. 2). $\sigma(m)\sigma(n)>\sigma(mn)$.

116. $\delta(m)=\sqrt{m^{\tau(m)}}$ va $\delta(10)=100$.

118. $\sigma_k(n) = \prod_{i=1}^{s} \frac{p^{k(\alpha_i+1)}-1}{p_i^k-1}$. **119.**1). $\sigma_2(12) = 210$. 2). $\sigma_2(18) = 455$. 3). $\sigma_3(36) = 6643$. 4). $\sigma_2(16) = 341$.5). $\sigma_3(8) = 585$.
123. 1). $n = 18$. 2). $n = 16875$. **124.** $\left[\frac{1+\tau(n)}{2}\right]$.
125. $N = 2^3 \cdot 5^2 \cdot 7 = 1400$. **126.** $N = 2^6 \cdot 3^5 \cdot 5^4 = 9720000$. **129.** $N = 2^3 \cdot 5 \cdot 3 = 120$.

II.4-§.

132. 1).100.2). 400.3). 48. 4). 64. 5). 384.6). 432.7). 1331;8). 506.9).64.10).6912. **133.** $\varphi(m)$. **134.** 88. **138.** *a)*. 3.b). 3. c).$p > 2$ bo'lsa tenglama yechimga ega emas. $p = 2$ da ixtiyoriy natural son x tenglamaning yechimi bo'ladi. d).$x = 2$; $y = 3$. **141**. $(m; n) > 1$ bo'lsa, $\varphi(m)\varphi(n) < \varphi(mn)$ bo'ladi. **144**. p^{α}. 16. $S = \frac{1}{2}m\varphi(m)$. **148.**1).$p = 2$ tenglama bitta $x = p = 2$ yechimga, $p > 2$ bo'lsa tenglama 2 ta p va $2p$ yechimga ega bo'ladi. 2). Tenglama yechimga ega emas. 3).$x = 15; 16; 20; 24; 30$. 4). $x = 5; 13; 21; 26; 28; 36; 42$. **149.** 1).$x = 2^{\alpha+1}$; $2^{\alpha-1} \cdot 5$; $2^{\alpha} \cdot 3$; 15; $2^{\alpha-2} \cdot 15$.2).$p = 3$ ixtiyoriy x qanoatlantiradi $p \neq 3$ da yechimi yoq. **150.** $m = 7875$. **151**. $x = 143$. **152.** 14161. **153.** *a)*.$p = 2$ da berilgan tenglamani x ning barcha toq qiymatlarini qanoatlantiradi; $p \geq 3$ bo'lsa tenglama yechimga ega emas. b). Agar $(x; p) = 1$ bo'lsa, yechim yo'q. Agar $x = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \dots . p_k^{\alpha_k}$ bo'lsa, berilgan tenglamani x ning p ga karra natural qiymatlari qanoatlantiradi. **154.** *a)*.$x = 2^{\alpha}$ tenglamaning yechimi ($\alpha \geq 1$). b). $x = 2^{\alpha} \cdot 3^{\beta}$. c). yechimga ega emas. **157**. $\varphi(2) + \varphi(3) + \dots + \varphi(n)$. **158**. 8 ta.

III.1-§.

159. Barcha butun sonlar 1 moduli bo'yicha o'zaro taqqoslanuvchi. **160**. Masalan 9,17 lar. **161.**$a), b)$. **165.** $x = 2 + 10t$, $t \in Z, x = 2, 12, 22, -8, -18$. **166.** *a)*.$x \equiv 0(mod3)$, $x = 3t, t \in Z$. b).$x \equiv 1(mod2), x = 1 + 2t$, $t \in Z$. **167.** *a)*.$m = 1,2,3,4,6,12$. b).$m = 1, 2, p, 2p$. **168.** $m = 1,2,4,8$. **169.** Misol uchun 1, 11, 101, 1001,... . **170.** a), b), c). **171**. $x = 2 + 5t_1$, t_1 - ixtiyoriy butun son.
181. 1).8 va 9. 2). 0 va 7.

III.2-§.

195. $x \equiv 0,1,2,\ldots,9(mod 10)$. **196.** 1).$1,2,3,4,\ \ldots\ ,9$ lar 9 moduli bo'yicha eng kichik musbat chegirmalarining to'la sistemasi. $-9,-8,-7,\ldots,-2,-1$ lar 9 moduli bo'yicha eng katta manfiy chegirmalarning to'la sistemasi; 0 ; ±1; ±2; ±3; ±4 lar 9 moduli bo'yicha absolyut qiymati jihatidan eng kichik chegirmalarining to'la sistemasi. Chegirmalarning keltirilgan sistemalar $1,2,4,\ 5,7,8$; c $-8,-7,-5,-4,-2,-1$; ±1; ±2; ±4 lardan iborat.

2). Chegirmalarining to'la sistemalari
$1,2,3,4,\ldots,8; -8,-7,-6,-5,\ldots,-2,-1;\quad \pm1;\ \pm2;\ \pm3;\ \pm4.$
Chegirmalarning keltirilgan sistemalari $1,3,5,7$; $-1,-3,-5,-7$; ±1; ±3 lardan iborat.

3). Chegirmalarining to'la sistemalari:
$1,2,3,4,\ldots,13; -13,-12,-11,\ldots,-2,-1; 0,\pm1,\pm2,\pm3,\pm4,\pm5,\pm6.$
Chegirmalarning keltirilgan sistemalari:
$1,2,3,4,\ldots,12;\ -12,-11,\ldots,-2,-1;$
$\pm1,\quad \pm2,\quad \pm3,\quad \pm4,\quad \pm5,\quad \pm6.$

4).Chegirmalarining to'la sistemalari:
$1,2,3,4,\ldots,12;\quad -12,-11,-10,\ldots,-2,-1;$
$\pm1,\pm2,\pm3,\pm4,\pm5,\pm6.$
Chegirmalarning keltirilgan sistemalari:
$1,5,7,11;\quad -1,-5,-7,-11;\ \pm1;\ \pm5.$

5). Chegirmalarining to'la sistemalari:
$1,2,3,4,5,6,7; -7,-6,-5,-4,-3,-2,-1;\quad 0,\pm1,\pm2,\pm3.$
Chegirmalarning keltirilgan sistemalari:
$1,2,3,4,5,6;\quad -7\ -6,-5,-4,-3,-2,-1;\quad \pm1,\pm2,\pm3.$

6).Chegirmalarining to'la sistemalari:
$1,2,3,4,\ldots,10; -10,-9,-8,\ldots,-2,-1;\ \pm1,\ \pm2,\ \pm3,\ \pm4,\ \pm5,$
Chegirmalarning keltirilgan sistemalari:
$1,3,7,9;\quad -9,-7,-3,-1; \pm1,\ \pm3.$

197. $x = 10q + r, 0 \le r < 10$ yoki $x = 10q$, $x = 10q + 1$, $x = 10q + 2$, $x = 10q + 3$, $x = 10q + 4$, $x = 10q + 5$ $x = 10q + 6, x = 10q + 7, x = 10q + 8$, $x = 10q + 9$.

198. *a).*$x \equiv 1,3,7,9(mod 10)$.*b).*$x \equiv 2,4,6,8(mod 10)$.c).$x \equiv 5(mod 10)$.d).$x \equiv 0(mod 10)$. **200.** Masalan:
$1,2,3,4,5,6,7,8,9,10; -10,-9,-8,-7,-6,-5,-4,-3,-2,-1; \pm1,$

$\pm 2,\ \pm 3,\ \pm 4,\ \pm 5,$ umumiy holda $x = 10q + r, 0 \le r < 10$ va $q \in Z$.
202. $m = 5$. **206.** 9,2,5,8,1,4,7,0,3,6. **207.** 0,1,2,3.
210. Masalan: $1,5;\ -5,5;\ -5,-1;\ 7,11;$

$13,17$. **211.** $(3,12) = 3$. **219.** $1,2,3,4,5,6,7,8,9-$ lar m=9 moduli boyicha musbat eng kichik chegirmalarning to'la sistemasi; $0,1,2,3,4,5,6,7,8-$lar $m = 9$ moduli bo'yicha manfiy bo'lmagan eng kichik chegirmalarning to'la sistemasi; $0, \pm 1, \pm 2, \pm 3, \pm 4-$ lar m=9 moduli boyicha absolyut qiymati jihatidan eng kichik chegirmalarning to'la sistemasi bo'ldi. $1,2,4,5,7,8$ – lar m=9 moduli bo'yicha musbat eng kichik chegirmalarning keltirilgan sistemasi; $1,2,4,5,7,8$ – lar m=9 moduli bo'yicha manfiy bo'lmagan eng kichik chegirmalarning keltirilgan sistemasi; $\pm 1, \pm 2, \pm 4-$ lar m=9 moduli bo'yicha absolyut qiymati jihatidan eng kichik chegirmalarning keltirilgan sistemasi bo'ladi.

III.3-§.

224.12. **225.**7. **227.**8. **228.**2. **229.**1. **230.**22. **235.**7 va 6. **236.**1. **236.**049. **246.**$p = 3$.

IV.1-§.

248. *a)*$x = 1 + 3t, t \epsilon Z$va $x = 2 + 3t, t \epsilon Z$.b)$x = 1 + 5t, t \epsilon Z$va $x = 2 + 5t, t \epsilon Z$.c) yechimga ega emas. d)$x = 3 + 5t, t \epsilon Z$. *e)*$x = 1 + 7t, x = 2 + 7t, t \epsilon Z$. *f)*$x = 11 + 15t, t \epsilon Z$. *i).*$x \equiv 1(mod7)$. **249.**Yechimga ega emas. **250.** *a)*$x = -4 + 15t, t \epsilon Z$. b) taqqoslamaning yechimi yo'q. c)$x = 1 + 6t,\ x = -2 + 6t, x = -1 + 6t, t \epsilon Z$.d) yechimga ega emas. **256.** $x \equiv 1,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 29,\ 31,\ 37,\ 41,\ 43, 47,\ 49,\ 53,\ 59\ (mod60)$.

IV.2-§.

257. *a)*$x = 3 + 6t, t \in Z$. b)taqqoslama yechimga ega emas. c)$x = 3 + 8t$ va $x = 7 + 8t, t \in Z$. *d)*$x = 5 + 7t, t \in Z$. e)taqqoslama yechimga ega emas. f) taqqoslama yechimga ega emas. g)$x \equiv 3 + 8t, t \in Z$. **258.** *a)*$x = 2 + 7t, t \in Z$.b)$x = -1 + 11t, t \in Z$. c) taqqoslama yechimga ega emas. d)$x = 3 + 13t, t \in Z$. e) $x = -3 + 12t,\ t \in Z$. f) $x = 2 + 9t, t \in Z$. g) $x \equiv 3(mod8)$. h) $x \equiv 3(mod15)$. **259.** *a)*$x = 9 + 19t, t \in Z$.b)$x = 11 + 58t, t \in Z$.c) taqqoslama yechimga ega emas. d)$x = 7 + 13t, t \in Z$. e)$x = 4 + 17t, t \in Z$. f)$x = 7 + 12t, t \in Z$. g)$x \equiv 6(mod11)$. **260.** *a)*$x = -2 + 27t, t \in Z$.b)$x = 7 + 117t,\ t \in Z$. c)$x = -46 + 311t, t \in Z$. d)$x = 51 + 360t, t \in Z$. e)$x = -5 + 93t, x =$

$26+93t,\ x=57+93t,\ t\in Z$. f) taqqoslama yechimga ega emas.g) $x=20+43t,\ t\in \mathbb{Z}$. **261**. *a)*$x=2+15t\ ,x=7+15t\ ,x=12+15t,t\in \mathbb{Z}$. b) taqqoslama yechimga ega emas. c)$x=3+25t\ ,x=8+25t\ ,x=13+25t,x=18+25t,x=23+25tt\in \mathbb{Z}$.*d)*$x=-1+7t\ ,x=6+7t,\ x=13+7t,\ x=20+7t,\ x=27+7t,\ t\in \mathbb{Z}$. e)$x=-5+93t\ ,x=26+93t\ ,x=57+93t,t\in \mathbb{Z}$. f)$x=9+138t,\ x=32++138t,\ x=55+138t,x=78+138t,\ x=101+138t,x=124+138t,t\in \mathbb{Z}$. g)$x=6+55t,\ x=17+55t,\ x=28+55t,\ x=39+55t,\ x=50+55t,t\in \mathbb{Z}$. **262.** *a)*$x=3+4t,y=-3-5t,\ t\in \mathbb{Z}$. b)$x=-3+13t,y=4-17t\ ,t\in \mathbb{Z}$. c)$x=1+4t,y=2+13t\ ,t\in \mathbb{Z}$. d)$x=2+3t,y=-2t,t\in \mathbb{Z}$.e)$x=2+3t,\ y=2+4t,t\in \mathbb{Z}$. f) $x=-2+7t,\ y=-1+3t,\ t\in \mathbb{Z}$. g)$x=5+6t,\ y=-4-7t,\ t\in Z$. **263.** *a)* 19 ta. b) 29 ta. **264.** *a)*$x=4+7t,t\in \mathbb{Z}$. b)$x=-2+15t\ ,t\in \mathbb{Z}$. c)$x=6+11t\ ,t\in \mathbb{Z}$. **265.** *a)* 2 ta 60 kg lik va 4 ta 80 kg lik qop yoki 6 ta 60 kg lik va 1 ta 80 kg lik qop kerak. b) markalarni 10 xilda turlicha qilib xarid qilish mumkin. $x=3+5t,\ y=28-3t,t\in Z$.

t	0	1	2	3	4	5	6	7	8	9
x	3	8	13	18	23	28	33	38	43	48
y	28	25	22	19	16	13	10	7	4	1

c)$x=5t,\ y=24-4t,t\in Z$.

t	0	1	2	3	4	5	6
x	0	5	10	15	20	25	30
y	24	20	16	12	8	4	0

266. *a)* 152 yoki 656. b) 13, 34,55, 76,97.

IV.3-§.

267. 1).$x=291+420t_3,\ t_3\epsilon Z$.2).$x=251+1260t_3\ ,\ t_3\epsilon Z$. 3).$x=-93+840t_3\ ,t_3\epsilon Z$. 4).$x=49+420t_3,\ t_3\epsilon Z$. 5). $x=93+560t_3\ ,t_3\in Z$. 6). Yechimga ega emas. 7). Yechimga ega emas. 8).Yechimga ega emas.9).$x=17+90t_3,\ \ t_3\in Z$. 10).$x=113+1001t_3,t_3\epsilon Z$. 11).$x_1=-3+825t_3,\ x_2=162+825t_3,\ x_3=327+825t_3,\ x_4=492+825t_3,\ x_5=657+825t_3,t_3\epsilon Z$. **268.** 1). $x\equiv 289(mod\ 462)$. 2).$x\equiv 93(mod\ 385)$. 3).$x\equiv 142(mod\ 765)$. 4).$x\equiv$

$381 (mod 1287)$. 5). $x \equiv 41 (mod\ 7735)$. 6). $x \equiv 37 (mod\ 1938)$. 7). $x \equiv 844 (mod\ 1386)$. 8). $x \equiv 622 (mod\ 2277)$. 9). $x \equiv 2671 (mod\ 3828)$. 10). $x \equiv 1680 (mod\ 24273)$. 11). $x \equiv -6 (mod\ 693)$. **269.** 1). 498. 2). 58. 3). 435. 4). 173. 5). 53. 6). 256. 7). 841. 8). 89. 9). 79. 10). 244. 11). 1546. **270.** 1). $a = 7k + 1, k \in Z$. 2). $\forall a \epsilon Z$. 3). a ning birorta ham qiymatida yechimga ega emas. 4). $a \equiv 6k + 1, k \in Z$. 5). $a = 3k + 1, k \in Z$. 6). $\forall a \epsilon Z$. 7). $a = 4k + 3, k \in Z$. 8). $\forall a \epsilon Z$. 9). $\forall a \epsilon Z$. 10). $a = 5k, k \in Z$. 11). $\forall a \epsilon Z$. **271.** 1). $-63 + 440t_3, t_3 \in Z$. 2). $291 + 819t_3, t_3 \in Z$. 3). $42 + 105t_3, t_3 \in Z$. 4). Masalaning shartini qanoatlantiruvchi nuqta ham mavjud emas. 5). $68 + 165t_3, t_3 \in Z$. 6). $-64 + 715t_3, t_3 \in Z$. 7). $508 + 728t_3, t_3 \in Z$. 8). $-53 + 315t_3, t_3 \in Z$. 9). $631 + 4403t_3, t_3 \in Z$. 10). Masala shartini qanoatlantiruvchi nuqtalar yo‘q. 11). $5 + 168t_3, t_3 \in Z$. **272.** *a)*. 428736, 498776, 468776.
b). 313138, 495138. c). 1380456. **273.** *a)*. $x = 3 + 7t_1, y = 3 + 7t_1, t_1 \in Z$. b). $\begin{cases} x \equiv 10 \\ y \equiv 3 \end{cases}; \begin{cases} x \equiv 10 \\ y \equiv 7 \end{cases}; \begin{cases} x \equiv 10 \\ y \equiv 11 \end{cases} (mod\ 12)$. c). Yechimga ega emas.

d). $\begin{cases} x \equiv 2 \\ y \equiv 11 \end{cases}; \begin{cases} x \equiv 6 \\ y \equiv 11 \end{cases}; \begin{cases} x \equiv 10 \\ y \equiv 11 \end{cases} (mod\ 12)$.

e). $\begin{cases} y \equiv 7 \\ x \equiv 0 \end{cases}; \begin{cases} y \equiv 7 \\ x \equiv 4 \end{cases}; \begin{cases} y \equiv 7 \\ x \equiv 8 \end{cases} (mod\ 12)$.

274. *a)*. $x \equiv 3 (mod\ 5), y \equiv 0 (mod\ 5)$. b). $x \equiv 1 (mod\ 5), y \equiv 2 (mod\ 5)$ c). $x \equiv 100 (mod\ 143), y \equiv 111 (mod\ 143)$. d). $x \equiv 0 (mod\ 5), y \equiv 2 (mod\ 5)$. e). $\begin{cases} x \equiv 5 (mod\ 6) \\ y \equiv 0 (mod\ 6) \end{cases}; \begin{cases} x \equiv 2 (mod\ 6) \\ y \equiv 3 (mod\ 6) \end{cases}$. *f)*. Yechimga ega emas. g). Sistemaning yechimlari to‘plami $x - y \equiv 2 (mod\ 6)$ taqqoslamaning yechimlari bilan bir xil. h). $\begin{cases} x \equiv 1 (mod\ 6) \\ y \equiv 2 (mod\ 6) \end{cases}, \begin{cases} x \equiv 4 (mod\ 6) \\ y \equiv 2 (mod\ 6) \end{cases}$. **275.** b). Berilgan sistemaning yechimga bo‘lmasligi sharti D_1 yoki D_2 larning birortasining $(D; m) = d$ ga bo‘linmasligidir.

IV.4-§.

276. *a)*. $x = 1 + 5t,\ t \in Z$. b). $x = -1 + 3t,\ t \in Z$. c). $x = 1 + 3t,\ x = 1 - 3t,\ t \in Z$ d). $x = -1 + 5t,\ x = -2 + 5t,\ t \in Z$. e). $x =$

$1+5t,\ t\in Z$. f). $x=1+5t,\quad t\in Z$. g). $x=2+5t,\quad t\in Z$. h). $\emptyset$. i). $x=1+5t,\ t\in Z$. j). $x=-1+5t,\quad t\in Z$.

277. *a)*.$(x-3)(mod5)$. b).$(x+2)^2(x-1)(x-2)(mod5)$. c).$(x-2)^2(x-3)(x+7)(mod11)$. d).$(x-1)(x+3)(mod5)$. *e)*. $(x+2)(3x^2-x+2)(mod5)$. f).$(x-2)(x-3)(x^2-2)(mod11)$. g).$(x+2)^2(x-2)^2(mod7)$. h).$(x-1)(2x^3+3x^2+2)(mod11)$. i). Ko'paytuvchilarga ajratib bo'lmaydi. j).$(x-2)(x-3)(x^2+5x+3)(mod7)$.

278. *a)*. Berilgan taqqoslama yechimga ega emas. b). $x\equiv 2(mod7)$. c). $x_1\equiv 2(mod11)$va$x_2\equiv 4(mod11)$. d).$x_1\equiv 3(mod11)$va$x_2\equiv 5(mod11)$. e). $x\equiv -2(mod5)$.

282. *a)*.$x_1\equiv 1,\ x_2\equiv 2,\ x_3\equiv 3(mod7)$. b). Taqqoslama yechimga ega emas. c). $x_1\equiv -1, x_2\equiv 2, x_3\equiv -3, x_4\equiv -4, x_5\equiv -5(mod11)$. d).$x_1\equiv -2(mod11),\ x_2\equiv 2(mod11)$. e).Taqqoslama yechimga ega emas. f). $x_1\equiv -2,\ x_2\equiv 2,\ x_3\equiv -3,\ x_4\equiv 3(mod13)$.

288. *a)*.$x_1\equiv 1(mod\ 5)$ va $x_2\equiv 2(mod\ 5)$. b). $x_1\equiv 1,\ x_2\equiv 2,\ x_3\equiv 3\ (mod7)$.

IV.5-§.

289. 1). $x\equiv 3,-3,-2,7\ (mod15)$. 2). $x\equiv -13,-10,-4,2,5,11\ (mod30)$.

3). $x\equiv 16(mod35)$.4). $x\equiv 3,24\ (mod42)$. 5).Taqqoslama yechimga ega emas.6).$x_1\equiv 5,\ x_2\equiv 2, x_3\equiv 11(mod15)$. 7).$x\equiv -19(mo180)$.**290.**1. $x\equiv 8(mod\ 27)$. 2). $x\equiv 143(mod\ 343)$. 3).$x_1\equiv 2(mod\ 25),\ x_2\equiv 3(mod\ 25)$. 4). $x\equiv 22(mod\ 64),\ x\equiv 53(mod\ 64)$. 5). $x\equiv -4(mod\ 125)$. 6). $x\equiv 66(mod\ 125)$. 7). $x=256+625t_4, x=-3+625t_4,\ t_4\in Z$. 8).$x=13+27t,\ t\in Z$. **291.**1).$x\equiv 6,24,42(mod\ 45)$. 2). $x\equiv 12,24,37,49\ (mod\ 50)$. 3). $x\equiv -50,-47,-2,-1,1,2,47,50\ (mod\ 147)$. 4). $x\equiv -10,11,15,32,36.\ 40,57,61\ 82,(mod\ 175)$. 5). $x\equiv 2,3,57,83(mod135)$. 6). $x\equiv 70;124;223(mod225)$. 7). $x\equiv -103,-49,22,76(mod225)$. 8). $x\equiv -47,\ -2,\ 2,\ 47\ (mod\ 441)$.9).$x\equiv -586,-198,\ -2,\ 2,198,\ 439,\ 586,786(mod\ 1225)$.

IV.6-§.

292.1). $x\equiv -3,1\ (mod5)$. 2). $x\equiv -1,-2(mod7)$. 3). Taqqoslama yechimga ega emas. 4). $x\equiv 0,2(mod5)$. 5). $x\equiv 2,7(mod17)$. 6). $x\equiv$

$-5, 14(mod31)$. 7). $x \equiv 3(mod13)$. 8). $x \equiv 7(mod17)$.**293.**1). $x = 6 + 55t$, $x = 17 + 55t$, $x = 36 + 55t$, $x = 47 + 55t, t \in Z$. 2). Berilgan ifoda butun qiymat qabul qiladigan x ning natural qiymatlari mavjud emas. 3).$x = 2 + 15t_2$, $x = 5 + 15t_2$, $x = 7 + 15t_2$, $x = 10 + 15t_2, t_2 \in Z$.**294.** 1,2,4 sonlari 7 modul bo'yicha kvadratik chegir-ma, qolganlari, ya'ni 3,5,6 lar esa kvadratik chegirma emas. **295.** 1). $1 + 11k, 3 + 11k, 4 + 11k, 5 + 11k, 9 + 11k, k \in Z$. 2). $1 + 13k, 3 + 13k, 4 + 13k, 9 + 13k, 10 + 13k, 12 + 13k, k \in Z$. 3). $1 + 17k, 2 + 17k, 4 + 17k, 9 + 17k, 9 + 17k, 13 + 17k, 15 + 17k, 16 + 17k\ k \in Z$.
296. 1). $x \equiv \pm 3(mod7)$. 2). $x \equiv \pm 2(mod7)$. 3). Taqqoslama yechimga ega emas. 4). $x \equiv \pm 4(mod13)$. 5). $x \equiv \pm 2(mod11)$.

297. 1). 1. 2). 1. 3). -1. 4). -1. 5).-1. 6). 1 7).1. 8).1.

298.1).Berilgan taqqoslama yechimga ega emas. 2).Berilgan taqqoslama yechimga ega va uning yechimlari $x \equiv \pm 5(mod11)$ dan iborat. 3). Berilgan taqqoslama yechimga ega va uning yechimlari $x \equiv \pm 5(mod13)$ dan iborat. 4).Berilgan taqqoslama yechimga ega va uning yechimlari $x \equiv \pm 4(mod13)$ dan iborat. 5).Berilgan taqqoslama yechimga ega va uning yechimlari $x \equiv \pm 4(mod11)$ dan iborat. 6). Berilgan taqqoslama yechimga ega va uning yechimlari $x \equiv \pm 8(mod17)$ dan iborat. 7). Berilgan taqqoslama yechimga ega emas.

299. 1). $a = \pm 1 + 5t,\ t \in Z$.

2). $a = -3 + 5t, a = 1 + 5t, a = 2 + 5t, t \in Z$. 3).$a = 1 + 11t, a = 3 + 11t, a = 4 + 11t, a = 5 + 11t, a = 9 + 11t, t \in Z$. 4). $a = 1 + 13t, a = 3 + 13t, a = 4 + 13t, a = 9 + 13t, a = 10 + 13t, a = 10 + 13t, t \in Z$. 5). $a = 1 + 3t, t \in Z$.**303.** $a = 13t, a = 2 + 13t, 3 + 13t, 4 + 13t, 6 + 13t, 7 + 13t, 12 + 13t, t \in Z$. **305.** 1). $(\pm 2 + 5t, 2 \pm 16t + 20t^2), t \in Z$. 2). $\emptyset$.3).$(2 + 11t, 11t^2 - 6t - 1)$ va $(8 + 11t, 11t^2 + 6t - 1), t \in Z$. 4). $((-2 + 13t, 13t^2 - 25t + 12)$ va $(10 + 13t, 13t^2 - t), t \in Z$. 5).Berilgan tenglama yechimga ega emas.

306. 1). $a = 5$ soni $p = 5k + 1$ va $p = 5k + 4$ ko'rinishdagi tub modullar bo'yicha kvadratik chegirma, $p = 5k + 2$ va $p = 5k + 3$ ko'rinishdagi tub modullar bo'yicha kvadratik chegirma emas bo'ladi. 2). $a = -3$ soni $p = 3k + 1$ ko'rinishdagi tub modul bo'yicha kvadratik chegirma, $p = 3k + 2$ ko'rinishdagi tub modul bo'yicha kvadratik chegirma emas bo'ladi. 3). 3 soni $p = 12q + 1, p = 12q + 11$ ko'rinishdagi tub modullar bo'yicha kvadratik chegirma, $p = 12q + 5, p = 12q + 7$ ko'rinishdagi tub modullar bo'yicha kvadratik chegirma emas bo'ladi.4).2 soni $p = 8k + 1, p = 8k + 7$modullar bo'yicha kvadratik chegirma, $p = 8k + 3, p = 8k + 5$, modullar bo'yicha kvadratik chegirma emas bo'ladi. 5). $a = -7$ soni $p = 1 + 7k$, $p = 2 + 7k, p = 4 + 7k$ modullar bo'yicha kvadratik chegirma, $p = 3 + 7k$, $p = 5 + 7k, p = 6 + 7k$ modullari bo'yicha kvadratik chegirma emas bo'ladi.

307. 1). $p = 1 + 5k$, $p = 4 + 5k$ modullar bo'yicha berilgan taqqoslama yechimga ega, $p = 2 + 5k, p = 3 + 5k$ modullar bo'yicha taqqoslama yechimga ega. 2). Ixtiyoriy $p > 2$ modul bo'yicha berilgan taqqoslama yechimga ega. 3). $p = 1 + 13k$, $p = 3 + 13k$, $p = 4 + 13k$, $p = 9 + 13k, p = 10 + 13k$, $p = 12 + 13k$ va $p = 13$ modullar bo'yicha taqqoslama yechimga ega. $p = 2 + 13k$, $p = 5 + 13k$, $p = 6 + 13k$, $p = 7 + 13k, p = 8 + 13k, p = 11 + 13k$ modullar bo'yicha berilgan taqqoslama yechimga ega emas.

V.1-§.

309.1). $P_7(2) = 3$. 2). $P_7(3) = 6$. 3). $P_7(5) = 6$.**310.** 1). $P_5(2) = 4, P_5(3) = 4$, $P_5(4) = 2$. 2). $P_7(2) = 3$, $P_7(3) = 6$, $P_7(4) = 3$, $P_7(5) = 6$,, $P_7(6) = 2$. 3).$P_8(3) = 2, P_8(5) = 2, P_8(7) = 2$. 4). $P_{10}(3) = 4$, $P_{10}(7) = 4$, $P_{10}(9) = 2$. 5).$P_{11}(2) = 10, P_{11}(3) = 5, P_{11}(4) = 5$, $P_{11}(5) = 5$, $P_{11}(6) = 10$, $P_{11}(7) = 10, P_{11}(8) = 10$, $P_{11}(9) = 5, P_{11}(10) = 2$. 6). $P_9(2) = 6, P_9(4) = 3, P_9(5) = 6$, $P_9(7) = 3$, $P_9(8) = 2$.

311. $P_m(m-1) = \begin{cases} 1, & \text{agar } m = 2 \text{ bo'lsa,} \\ 2, & \text{agar } m \geq 2 \text{ bo'lsa} \end{cases}$**312.**1). 3, 5.2). 2,6,7,8.
3). 2,6,7, 11. 4). 3,5,6,7,10, 11,12, 14.**313.** 1). 6 va 2. 2). 10 va 2.
3). 8va 3. 4). 12 va 2. 5). 12 va 3. 6). 24 va 2.**314.** 1). 2, 3, 10,13, 14, 15. 2). 5, 7, 10,13, 14, 15, 17, 19, 20, 21.3). 3, 11,12, 13,17, 21, 22, 24.
315.$x \equiv 5(mod 6)$.**320.** 1). b ning $(b, 9) = 1$ shartni qanoatlantiruvchi barcha qiymatlari. 2). $b \equiv 1, 4, 7(mod 9)$ qiymatlari.
3). $\varphi(m) - P_m(a)$.

V.2-§.

321. 1).

N	0	1	2	3	4	5	6	7	8	9
0		0	1	5	2	22	6	12	3	10
1	23	25	7	18	13	27	4	21	11	9
2	24	17	26	20	8	16	19	15	14	

2).

N	0	1	2	3	4	5	6	7	8	9
0		0	2	16	4	1	18	19	6	10
1	3	9	20	14	21	17	8	7	12	15
2	5	13	11							

322.

N	0	1	2	3	4	5	6	7	8	9
0		0	1	8	2	4	9	7	3	6
1	5									

323. 1). $\delta = 6$. 2). $\delta = 5$. 3). $\delta = 4$. 4). $\delta = 16$. 5). $\delta = 30$. 6). $\delta = 6$. 7). $\delta = 16$. 8). $\delta = 10$. 9). $\delta = 10$. **324.**1).$4, 4, 2$. 2).$3, 6, 3,6,2$. 3).$10, 5, 5,5,10,10,10,5,2$. **325.**1). Bo'ladi. 2). Bo'lmaydi. 3).Bo'ladi. 4).Bo'ladi. 5). Bo'lmaydi. 6). Bo'ladi. 7). Bo'ladi. 8). Bo'lmaydi. **326.** 1). $3,5,6,7,10,11,12,14$. 2). $2,3,10,13,14,1$

3). $5,7,10,11,14,15,17,19,20,21$. **327.** 1). $x \equiv 13(mod 17)$.

2). $x \equiv 32(mod 97)$. 3). $x \equiv 74(mod 79)$. 4). $x \equiv 56(mod 89)$. 5). $x \equiv 31(mod 37)$. 6). $x \equiv 30(mod 221)$. 7). $x \equiv 128(mod 667)$. 8). $x \equiv 873(mod 1517)$. **328.** 1). $x = 23 + 66t, t \in Z$. 2). $x = 26 + 46t, t \in Z$. 3). Yechimga ega emas. 4). Yechimga ega emas. 5). $x = 13 + 30t, t \in Z$. 6). $x = 11 + 40t,\ x = 31 + 40t, t \in Z$. **329.** 1). $x = 17 + 73t,\ x = 63 + 73t, x = 66 + 73t,\ t \in Z$. 2). $x = 2 + 11t,\ x = 9 + 11t,\ t \in Z$. 3). $x = 2 + 13t,\ x = 3 + 13t, x = 10 + 13t, x = 11 + 13t, t \in Z$. 4). $x = 22 + 41t,\ t \in Z$. 5). Taqqoslama yechimga ega emas. 6). $x = 6 + 79t,\ x = 14 + 79t, x = 59 + 79t,\ t \in Z$. 7).$x = 13 + 73t,\ x = 29 + 73t, x = 31 + 73t, t \in Z$. 8). $x = 19 + 41t,\ x = 22 + 41t,\ t \in Z$. 9). $x = 25 + 61t,\ x = 30 + 61t, x = 31 + 61t, x = 36 + 61t,\ t \in Z$.
10). $x = 2 + 13t,\ x = 3 + 13t, x = 10 + 13t, x = 11 + 13t,\ t \in Z$.
330.1).$x = 2 + 41t,\ x = 18 + 41t, x = 23 + 41t, x = 39 + 41t,\ t \in Z$. 2). $x = 58 + 97t, t \in Z$. 3). $x = 33 + 67t, t \in Z$. 4). $x = 7 + 73t, x = 10 + 73t,\ x = 17 + 73t,\ x = 56 + 73t,\ x = 63 + 73t,\ x = 66 + 73t, t \in Z$. 5).Taqqoslama yechimga ega emas. 6). Taqqoslama yechimga ega emas. 7). $x = 20 + 43t, x = 32 + 43t,\ x = 34 + 43t, t \in Z$. 8). $x = 4 + 13t, x = 6 + 13t,\ x = 7 + 13t, x = 9 + 13t, t \in Z$. 9). $x \equiv \pm 27(mod 67)$. 10). $x \equiv \pm 15(mod 83)$. 11).

Taqqoslama yechimga ega emas. 12). $x \equiv \pm 6(mod 53)$. 13). $x \equiv \pm 21(mod 67)$. 14). Taqqoslama yechimga ega emas. **331.** 1). 16 va 18. 2). 16 va 20. 3). 16,18 va 20. 4). 16,18 va 19. 5). 16 va 18. **332.**$ind_{a_2} b \equiv \left(ind_{a_1} a_2\right)^{\varphi(p-1)-1} ind_{a_1} b (mod p-1)$. **333.**1). $a \equiv \pm 8(mod 17)$.
2). $a \equiv \pm 10(mod 23)$. 3). Bunday qiymatlar mavjud emas.

V.3-§.

334.1). 16. 2). 4. 3). 70. 4). 5. **335.** 3. **336.**1). 5. 2). 8. 3). 1.
337. 1). 0 va 1. 2). 4 va 9. 3).8 va 0. 4). 9 va 3.
339. Agar $m = 4q + 1$ ko'rinishida bo'lsa, $\frac{3m+1}{4}$ ga va agar r $m = 4q - 1$ ko'rinishida bo'lsa, $\frac{m+1}{4}$ ga teng.
340. 1). 23. 2). 4. 3). 9. **342.**1380456. **343.** 1).6. 2). 6. 3). 21. 4).96. **344.** 1). 176. 2). 734. 3).330. 4).48. 5).6. **345.** 1). 6. 2). 2. 3).330. 4).104. 5). 32

VI.1 -§.

348. 1).(2,2,3,1,5). 2). (0,1,2,5,2). 3). (1,4,2,1,7).
4). (0,1,3,1,1,1,2). **349.** 1). $\frac{71}{41}$. 2).$\frac{157}{225}$. 3). $\frac{225}{43}$.
4). $\frac{a^5+4a^3+3a}{a^4+3a^2+1}$. 5).$\frac{a^3b^2+4a^2b+3a}{a^2b^2+3ab+1}$. 6). $\frac{64}{25}$. 7). $\frac{73}{43}$. 8). $\frac{4163}{1902}$.
350.1).$\frac{17}{13}$. 2). $\frac{7}{23}$. 3). $\frac{281}{239}$. 4). $\frac{389}{227}$. 5).$\frac{71}{107}$.
351.1). $x = 2$. 2). $x = 1, y = 2, z = 3$.
352. 1). $\frac{29}{37} \approx \frac{4}{5}(-0{,}02)$. 2).$\frac{163}{159}(\pm 0)$. 3). $\frac{648}{385} \approx \frac{32}{19}(-0{,}0013)$. 4). $\frac{1882}{1651} \approx \frac{57}{50}(-0{,}000103)$. **353.**1).$\frac{571}{359} \approx \frac{27}{17}(+0{,}0027)$. 2). $\frac{2341}{1721} \approx \frac{34}{25}(+0{,}00029)$. **354.**Mumkin. **355.** Agar $n = 2k -$ juft son bo'lsa izlanayotgan bo'linma $(\underbrace{1,4,1,4,\ \dots\ ,1,4}_{2k\ ta})$ dan iborat va $n = 2k + 1 -$ toq son bo'lsa $(\underbrace{1,4,1,4,\ \dots\ ,1,4,1,5}_{2k+1\ ta})$bo'ladi. **356.**agar $n = 2k -$ juft son bo'lsa izlanayotgan bo'linma $(\underbrace{1,a^2,1,a^2,\ \dots\ ,1,a^2}_{2k\ ta})$ dan iborat va $n = 2k + 1$ –toq son bo'lsa $(\underbrace{1,a^2,1,a^2,\ \dots\ ,1,a^2 1,a^2+1}_{2k+1\ ta})$bo'ladi.
362. 1). $x = -8360 + 117t,\ y = 2717 - 38t, t \in Z$. 2). $= -74 + 129t,\ y = 70 - 122t, t \in Z$. 3). $x = -2 - 4t,\ y = -4 - 7t, t \in Z$.

4). $x=-8814+175t,\ y=-12995-258t, t\in Z$. 5). $x=-125+114t,\ y=45-41t,\ t\in Z$. 6). $x=-8+33t,\ y=17-70t,\ t\in Z$. **367.** 1). $x\equiv 153(mod308)$. 2). $x\equiv 103(mod132)$.

VI.2-§.

368. 1). $\frac{57}{10}$, $\Delta\alpha=\frac{1}{1030}$. 2).$\frac{355}{113}$, $\Delta\alpha=0{,}00001$. 3).$\frac{2}{3}$, $\Delta\alpha=0{,}05$. 4).$\frac{2}{3}$, $\Delta\alpha=0{,}05$. 5).$\frac{5}{3}$, $\Delta\alpha=0{,}05$. 6).$\frac{5}{24}$, $\Delta\alpha=0{,}002$. **369.** $\frac{73}{51}$.

370. 1). $\frac{41}{29}$. 2).$\frac{71}{41}$. 3).$\frac{82}{31}$. 4).$\frac{63}{19}$. **371.** 1).$x_1=\frac{5+\sqrt{17}}{2}\approx\frac{593}{130}(-0{,}0001)$; $x_2=\frac{5-\sqrt{17}}{2}\approx\frac{57}{130}(+0{,}0001)$. 2).$x_1=\frac{-5+\sqrt{2}}{2}\approx-\frac{251}{140}(-0{,}0001)$; $x_2=\frac{-5-\sqrt{2}}{2}\approx-\frac{449}{140}(-0{,}0001)$. 3).$x_1=\frac{-9+\sqrt{57}}{2}\approx-\frac{211}{291}(+0{,}0001)$; $x_2=\frac{-9-\sqrt{57}}{2}\approx-\frac{2408}{291}(-0{,}0001)$. 4).$x_1=\frac{3+\sqrt{57}}{4}\approx\frac{662}{251}(+0{,}0001)$; $x_2=\frac{3-\sqrt{57}}{4}\approx-\frac{331}{291}(-0{,}0001)$.

373. Juft tartibli munosib kasrlar ortadi, toq tartiblilari esa kamayadi.

VI.3-§.

375.1). $1+\sqrt{\frac{5}{3}}$. 2).$\frac{9+\sqrt{201}}{14}$. 3).$\frac{32+\sqrt{1297}}{13}$. 4).$\frac{18-\sqrt{5}}{11}$. 5).$\frac{10-\sqrt{2}}{14}$. 6).$\sqrt{a^2+2}$. 7).$\frac{-9+\sqrt{221}}{10}$. **376.**$\frac{a+\sqrt{a^2+4}}{2}$. **377.**1). $\alpha=\frac{57-\sqrt{2}}{17}$. 2). $\alpha=\frac{166+\sqrt{3}}{59}$. **378.**1). $\alpha=(x,\overline{2x})$va$\frac{P_3}{Q_3}=\frac{4x^3+3x}{4x^2+1}$. 2). $\alpha=\left(a^2,\overline{a,2a^2}\right)va\frac{P_3}{Q_3}=\frac{2a^5+3a^2}{2a^3+1}$. **384.**$\frac{a}{b}$.

VI.4-§.

388.1).2. 2).3. 3). 3. 4).4. 5). 4. 6).2.

III qism. Misollarning yechimlari

I.1-§.

1. Qoldiqli bo'lish haqidagi teoremaga asosan: $m = 13 \cdot 17 + r, 0 \leq r < 13$ bo'lgani uchun $r = 12$ da eng katta m soni hosil bo'ladi va bu holda

$$m = 13 \cdot 17 + 12 = 170 + 51 + 12 = 233.$$

2. 1). Qoldiqli bo'lish haqidagi teoremadan foydalanamiz $a = b \cdot q + r, 0 \leq r < b$ bizda $a = 25$ $q = 3$ va $b, r = ?$ Shuning uchun ham $r = 25 - 3b \to 0 \leq 25 - 3b < b \to 3b \leq 25 < 4b \to b = 7; 8$ va $r = 4{,}1$.

2). Qoldiqli bo'lish haqidagi teoremadan foydalanamiz $a = b \cdot q + r, 0 \leq r < b$ bizda $a = -30$ $q = -4$ va $b, r = ?$ Shuning uchun ham $r = -30 + 4b \to 0 \leq -30 + 4b < b \to -4b \leq -30 < -3b \to b = 8, 9$ va $r = 2, 6$.

3. *a)* $N = 2n + 1,\ N^2 = (2n + 1)^2 = 4n^2 + 4n + 1 = 4n(n + 1) + 1$. Bu yerda $n(n + 1) \vdots 2$ bo'linganligi uchun $N^2 = 8q + 1$.

b) $x = n^2 + (n + 1)^2 = 2n(n + 1) + 1$. Bu yerda $n(n + 1) \vdots 2$ bo'linganligi uchun $x = 4q + 1$.

4. Bizda $p \geq 5$ –tub son. Ma'lumki, N natural sonni 6 ga bo'lganda $N = 6q + r,\ r = 0{,}1{,}2{,}3{,}4{,}5$ bo'ladi. $r = 0{,}2{,}3{,}4$ bo'lganda N tub son bo'lmaydi yoki 5 dan kichik tub son bo'ladi. Demak, $p \geq 5$ –tub son $p = 6q + 1$ yoki $p = 6q + 5$ ko'rinishida bo'lishi mumkin.

5. 4-misolga asosan $p \geq 5$ tub son $p = 6q + 1$ yoki $p = 6q + 5$ ko'rinishida bo'lishi mumkun. Agar $p = 6q + 1$ ko'rinishda bo'lsa, u holda $p^2 = 36q^2 + 12q + 1 = 12q(3q + 1) + 1 = 12q(q + 1 + 2q) + 1 = 12q(q + 1) + 24q^2 + 1 = 24Q + 1$.

Agar $p = 6q + 5$ bo'lsa, u holda $p^2 = 36q^2 + 60q + 25 = 12q(3q + 5) + 25 = 12q(q + 1 + 2q + 4) + 25 = 12q(q + 1) + 24q(q + 2) + 24 + 1 = 24Q + 1$.

6. Misolning shartiga asosan

$$\begin{cases} a = mq_1 + 1 \\ b = mq_2 + 1 \end{cases}$$

bo'lgani uchun $ab = (mq_1 + 1)(mq_2 + 1) = m^2q_1q_2 + +m(q_1 + q_2) + 1 = m(mq_1q_2 + q_1 + q_2) + 1 = mQ + 1$.

7. $3m + 2 = x^2$ tenglamaning natural sonlarda yechimga ega emasligini isbotlashimiz kerak. Buning uchun $x = 3q$, $x = 3q + 1$, $x = 3q + 2$ larni tenglamaga qo'yib tekshirib ko'ramiz.

$x = 3q$ bo'lsa, $3m + 2 = 9q^2$ bajarilmaydi;

$x = 3q + 1$ bo'lsa, $3m + 2 = 9q2 + 6q + 1 = 3q + 1$ bajarilmaydi;

$x = 3q + 2$ bo'lsa, $3m + 2 = 9q2 + 12q + 4 = 3T + 1$ bajarilmaydi.

Demak, tenglama natural sonlarda yechimga ega emas.

8. $15^n = 7q_n + 1$ ekanligini matematik induksiya usuli orqali isbotlaymiz. $n = 1$ uchun $15 = 7 \cdot 2 + 1$ tasdiq to'g'ri. Endi faraz qilaylik $n = k$ da isbotlangan tasdiq o'rinli bo'lsin, ya'ni $15^k = 7q_k + 1$tenglik bajarilsin u holda$15^{k+1} = 15^k \cdot 15 = (7q_k + 1) \cdot 15 = (7q_k + 1)(7 \cdot 2 + 1) = 98q_k + 7 \cdot q_k + 7 \cdot 2 + 1 = 7(15q_k + 2) + 1 = 7Q + 1$.Demak, matematik induksiya metodiga ko'ra isbotlangan tasdiq ixtiyoriy n natural son uchun o'rinli.

9. $2^{2^n} + 1 = x_n$ni qaraymiz matematik induksiya usulidan foidalanib $n = 2$ da $x_2 = 2^{2^2} + 1 = 2^4 + 1 = 17$ tasdiq o'rinli.Endi faraz qilaylik $n = k$ da tasdiq o'rinli bo'lsin, ya'ni $x_k = 2^{2^k} + 1 = 10q + 7$ soni 7 raqami bilan tugasin. U holda

$$x_{k+1} = 2^{2^{k+1}} + 1 = 2^{2^k \cdot 2} + 1 = (2^{2^k})^2 + 1 = (10q + 6)^2 + 1 =$$
$$= 100q^2 + 120q + 36 + 1 = 10(10q^2 + 12q + 3) + 7 =$$
$$= 10l + 7.$$

Endi $y_n = 2^{4^n} - 5$ $(n = 1,,2,3 \dots\dots n)$ ni qaraymiz.$n = 1$ da$y = 11 = 10 + 1$. Faraz qilaylik, $y_k = 2^{4^k} - 5 = 10q + 1$ bo'lsin. U holda

$$y_{k+1} = 2^{4^{k+1}} - 5 = 2^{4^k \cdot 4} - 5 = (2^{4^k})^4 - 5 = (10q + 6)^4 - 5$$
$$= 10t + 36 - 5 = 10t_1 + 1.$$

Demak, matematik induksiya prinsipiga asosan isbotlanayotgan tasdiq ixtiyoriy n natural soni uchun o'rinli.

10. $l = 2n + 1$va $m = 2s + 1$ lar toq sonlar berilgan bo'lsin. $l^2 + m^2$ yig'indini qaraymiz: $l^2 + m^2$=$(2n + 1)^2 + (2s + 1)^2 = 4n^2 + 4n + 1 + 4s^2 + 4s + 1 = 4(n^2 + n + s^2 + s) + 2 = 4M + 2 = 2(2M + 1)$, bunda $M = n^2 + n + s^2 + s$ va $(2M + 1)$-toq son biror butun sonning kvadrati bo'lsa ham 2 soni esa butun sonning kvadratiga teng bo'la olmaydi. Shuning uchun ham $l^2 + m^2 = 2(2M + 1)$ soni butun sonning kvadratiga teng bo'lmaydi.

11. Qaralayotgan uchburchakning katetlari x, y va gipotenuzasini z bilan belgilaylik. Ikkala katet ham 3 ga bo'linmasa ularning har biri $3q + 1$ yoki

$3q + 2$ ko'rinishdabo'ladi. Bundan agar $x = 3q + 1,\ y = 3q + 1$ bo'lsin, u holda $x^2 + y^2 = (3q + 1)^2 + (3q + 1)^2 = 3Q_1 + 1 + 3Q_2 + 1 = 3Q + 2$.

Agar $x = 3q + 1,\ y = 3q + 2$ bo'lsa, u holda $x^2 + y^2 = (3q + 1)^2 + (3q + 2)^2 = 3Q_1 + 1 + 3Q_2 + 4 = 3(Q_1 + Q_2 + 1) + 2 = 3Q + 2$.

Agar $x = 3q + 2,\ y = 3q + 2$ bo'lsa , u holda

$$x^2 + y^2 = (3q + 2)^2 + (3q + 2)^2 = 3Q_1 + 4 + 3Q_2 + 4$$
$$= 3(Q_1 + Q_2 + 2) + 2 = 3Q + 2.$$

$x^2 + y^2 = z^2$ bo'lgani uchun gipotenuzaning kvadrati z^2 ham va 2 ning o'zi ham 3 ga bo'linmaydi, ya'ni $z^2 = 3Q + 2$. Lekin bu holda z^2 ni 3 ga bo'lsak 2 emas *1* qoldiq qolish kerak (7- masalaga qarang) shuning uchun ham $x \vdots 3$ yoki $y \vdots 3$.

12. Agar x katet 5 ga bo'linmasa uni $x = 5q + r, 1 \le r \le 4$ deb yoza olamiz. Bundan $x^2 = 5Q_1 + r^2,\ \ r = 1,2,3,4$;

$$\begin{cases} r = 1\ da\ x^2 = 5Q_1 + 1 \\ r = 2\, da\ \ x^2 = 5Q_1 + 4 \\ r = 3\, da\ \ x^2 = 5Q_2 + 4 \\ r = 4\, da\ \ x^2 = 5Q_3 + 1 \end{cases}$$

ya'ni $x^2 = 5Q + 1$ yoki $x^2 = 5Q + 4$ bo'lar ekan.Endi agar y katet ham *5* ga bo'linmasa u holda uni ham $y^2 = 5T + 1$ yoki $y^2 = 5T + 4$ ko'rinishda ifodalash mumkin bo'ladi va bulardan

$$x^2 + y^2 = 5K + r,\ \ r = 0, 2, 3 \qquad (*)$$

ni hosil qilamiz. Agar z gipotenuza 5 ga bo'linmasa, uning kvadrati z^2ni 5 ga bo'lishdan quyidagicha qoldiqlar hosil bo'ladi:

$z = 5q + 1$, $z = 5q + 2$, $z = 5q + 3$,$z = 5q + 4$, $z^2 = 5l_1 + 1$, $z^2 = 5l_2 + 4$, $z^2 = 5l_3 + 4$, $z^2 = 5l_3 + 1$ ya'ni z^2 ni *5* ga bo'lishdan 1 yoki 4 qoldiq qoladi shuning uchun, $(*)$ da r uchun faqat bizda $r = 0$ imkoniyat mavjud.U holda $x = 5q$, ya'ni x katet 5 ga bo'linadi. Birinchi x katet va z gipotenuza 5 ga bo'linmasligidan ikkinchi y katetning 5 ga bo'linishi ham shunga o'xshash isbotlanadi.

13. $S_n = 1 + 2 + 3 + \cdots + n = \frac{(1+n)}{2} \cdot n$ dan foydalanamiz. Agar $n = 5q$ ko'rinishda bo'lsa, u holda $S_n = \frac{(5q+1)5\cdot q}{2} = 5Q$ bo'ladi.

Agarda $n = 5q + 1$ ko'rinishda bo'lsa, u holda

$$S_n = \frac{(5q+2)(5q+1)}{2} = \frac{25q^2 + 15q + 2}{2}$$
$$= \frac{5q(5q+3) + 2}{2} \frac{5q\big((q+1) + (4q+2)\big) + 2}{2}$$
$$= \frac{5q(q+1)}{2} + 5q(2q+1) + 1$$
$$= 5\left(\frac{q(q+1)}{2} + q(2q+1)\right) + 1 = 5Q + 1$$

bo'ladi.

Agarda $n = 5q + 2$ ko'rinishda bo'lsa, u holda

$$S_n = \frac{(5q+3)(5q+2)}{2} = \frac{25q^2 + 25q + 6}{2} = \frac{25q(q+1)}{2} + 3$$
$$= 5Q + 3$$

bo'ladi. Agar $n = 5q + 3$ bo'lsa, u holda

$$S_n = \frac{(5q+4)(5q+3)}{2} = \frac{25q^2 + 35q + 12}{2} = \frac{5q(5q+7)}{2} + 6$$
$$= \frac{5q\big((q+1) + 4q + 6\big)}{2} + 6$$
$$= \frac{5q(q+1)}{2} + \frac{5q(2q+3)}{1} + 5 + 1$$
$$= 5\left(\frac{q(q+1)}{2} + q(2q+3) + 1\right) + 1 = 5Q + 1$$

bo'ladi. $n = 5q + 4$ bo'lsa, u holda

$$S_n = \frac{(5q+5)(5q+4)}{2} = \frac{5(q+1)\big(q + 4(q+1)\big)}{2}$$
$$= \frac{5q(q+1)}{2} + 10(q+1)^2 = 5\left(\frac{q(q+1)}{2} + 2(q+1)^2\right)$$
$$= 5Q$$

Demak, $n = 5q + 1$ va $n = 5q + 3$, $q = 0,1,2,...$, ko'rinishidagi n lar uchun S_n yig'indini 5 ga bo'lsak 1 qoldiq chiqar ekan.

14. $ax - by$ ifodaga bx qo'shib va ayirib quyidagicha yozib olamiz: $ax - by = ax - by + bx - bx = (a - b)x + b(x - y)$. Shartga ko'ra bu tenglikning chap tomoni m ga bo'linadi, ya'ni $ax - by = m \cdot k$; shuning uchun o'ng tomoni ham m ga bo'linadi. Ya`ni $a - b = m \cdot l$va $(b, m) = 1$ bo'lganda $b(x - y) = (ax - by) + (b - a)x = mk - ml = m(k - l)$va demak $(x - y) \vdots m$ kelib chiqadi.

15. Berilgan ifodalarda quyidagicha shakl o'zgartirish qilamiz:

$$4^n + 15n - 1 = (1 + 3)^n + 15n - 1 = 1 + n \cdot 3 + 3^2 \cdot \frac{n(n-1)}{2!} + \frac{n(n-1)(n-2)}{3!} \cdot 3^3 + \cdots + 3^n + 15n - 1 = 18n + 9Q = 9 \cdot (2n + Q)$$

Demak, bu tenglikning o'ng tomoni 9 ga bo'linadi, demak, chap tomoni ham 9 ga bo'linishi kerak.

16. 1) $f(n) = 10^n + 18n - 1$ifodaning 27 ga bo'linishini ko'rsatamiz. Buning uchun matematik induksiya metodidan foydalanamiz. $f(1) = 27 \vdots 2$.

Endi n=k uchun $f(k) \vdots 27$, ya`ni $f(k) = 27q$ bo'lsin. $f(k + 1)$ ni qaraymiz: $f(k + 1) = 10^{k+1} + 18(k + 1) - 1 = 10 \cdot 10^k + 18k + 17 = (10^k + 18k - 1) + 9 \cdot 10^k + 18 = 27q + 9(10^k + 2)$, bu yerda $10^k + 2$ ifoda k ning natural qiymatlarida 3 ga bo'linadi. Shuning uchun ham oxirgi tenglikning o'ng tomoni 27 ga va demak chap tomoni, ham 27 ga bo'linadi. Shunday qilib matematik induksiya prinsipiga ko'ra istalgan natural n uchun $f(x) \vdots 2$.

2). Endi $F(n) = 3^{2n+3} + 40n - 27$ ning 64 ga bo'linishini isbotlaymiz. $f(1) = 243 + 40 - 27 = 256 \vdots 64$. Faraz qilaylik $f(k) \vdots 64$, ya'ni $f(k) = 64q$ bo'lsin. U holda

$$\begin{aligned} f(k + 1) &= 3^{2(k+1)+3} + 40(k + 1) - 27 = 3^{2k+3} \cdot 3^2 + 40k + 13 \\ &= (3^{2k+3} + 40k - 27) + 8 \cdot 3^{2k+3} + 40 \\ &= 64q + 8(3^{2k+3} + 5) \end{aligned}$$

tenglik o'rinli. Endi $g(k) = 3^{2k+3} + 5$ ning 8 ga bo'linishini ko'rsatamiz. $g(1) = 3^{2\cdot1+3} + 5 = 248$ bo'lib bu son 8 ga bo'linadi, ya'ni $g(1) \vdots 8$.

$g(k) \vdots 8$ bo'lsin deb faraz qilaylik, ya'ni $g(k) = 8l$ bo'lsin, u holda $g(k + 1) = 3^{2(k+1)+3} + 5 = 3^{2k+3} \cdot 3^2 + 5 = (3^{2k+3} + 5) + 8 \cdot 3^{2k+3} = 8l + 8 \cdot 3^{2k+3} = 8(l + 3^{2k+3}) = 8s$, demak, ixtiyoriy $k \in N$

uchun $g(k) = 3^{2k+3} + 5$ ifoda 8 ga bo'linadi. Shuning uchun ham $F(k+1) \vdots 64$. Shunday qilib ixtiyoriy $k \in N$ uchun $F(k) \vdots 64$.

17. 1) $f(n) = \frac{n}{2n^2+1}$ kasrni qaraymiz. Bu kasr qisqarmas kasr, chunki $(n;\ 2n^2 + 1) = 1$. Kasr sof davriy kasrga yoyilishi uchun uning maxrajida 2 va 5 sonlarning ko'paytuvchi sifatida qatnashmasligi kerak. Shuni tekshiramiz: $2n^2 + 1$ ifoda 2 ga bo'linmaydi (2 ga bo'lsa 1 qoldiq qoladi).

Endi maxrajda 5 ko'paytuvchi sifatida qatnashmasligini kop'rsatamiz. $n = 5q + r,\ r = 0,1,2,3,4$ deb olamiz, u holda $g(n) = 2n^2 + 1$ dan

$g(5q + r) = 2(5q + r)^2 + 1 = 2(25q^2 + 10qr + r^2) + 1 = 5Q + g(r)$, (*)

bunda $r = 0,1,2,3,4$. r ning bu qiymatlarda $g(r)$ ning 5 ga bo'linmasligini ko'rsatamiz. Buni bevosita tekshirish orqali amalga oshirish mumkin.

$$g(0) = 1, \quad g(1) = 3, \quad g(2) = 9, \quad g(3) = 19, \quad g(4) = 33$$

larning birortasi ham 5 ga bo'linmaydi. (*) dan $g(n)$ ning 5 ga bo'linmasligi kelib chiqadi.

2). Endi $f(n) = \frac{n}{n^2+n+1}$ ni qaraymiz. Bu yerda ham $(n;\ n^2 + n + 1) = 1$ va $g(n) = n^2 + n + 1 = n(n+1) + 1 = 2q + 1$, ya'ni maxraj 2 ga bo'linmaydi. Bu yerda $g(5q + r) = (5q + r)^2 + 5q + r + 1 = 25q^2 + 10qr + r^2 + 5q + r + 1 = 5Q + r^2 + r + 1 = 5Q + g(r)$. (*)

Bunda $g(r)$ ifoda $(r = 0,1,2,2,3,4)$ 5 ga bo'linmaydi. Haqiqatan ham, $g(0) = 1,\ g(1) = 3,\ g(2) = 7,\ g(3) = 13, g(4) = 21$ sonlarning birortasi ham 5ga karrali emas. (*) dan berilgan kasrning maxrajida 5 soni ko'paytuvchi sifatida qatnashmaydi degan xulosa kelib chiqadi. Demak $f(n)$ kasr son davriy kasrga yoyiladi.

18. $N_1 = \overline{a_1a_2a_3}$, $N_2 = \overline{b_1b_2b_3}$ lar uch xonali sonlar bo'lsin, u holda $M = \overline{a_1a_2a_3b_1b_2b_3} = \overline{a_1a_2a_3} \cdot 10^3 + \overline{b_1b_2b_3} = N_1 \cdot 10^3 + N_2 = (N_1 + N_2) + 999N_1 = (N_1 + N_2) + 37 \cdot 27N_1$ va masalaning sharti bo'yicha $(N_1 + N_2) \vdots 37$, ya'ni $N_1 + N_2 = 37q$. Shuning uchun ham $M = 37q + 37 \cdot 27N_1 = 37(q + 27N_1)$ va demak $M \vdots 37$.

19. *a)*.Birinchi usul. $m^5 - m = m(m^4 - 1) = m(m^2 + 1)(m^2 - 1) = m(m^2 + 1)(m - 1)(m + 1) = (m - 1)m(m + 1)(m^2 - 4 + 5) = (m - 2)(m - 1)m(m + 1)(m + 2) + 5(m - 1)m(m + 1) = 5! \cdot \frac{(m-2)(m-1)m(m+1)(m+2)}{5!} + 5(m - 1)m(m + 1) = 5\left(4! \cdot C_{m+1}^5 + (m - 1)m(m + 1)\right),$

Bu yerda C_{m+1}^5 butun son bo'lgani uchun $(m^5 - m) \vdots 5$.

Ikkinchi usul. $m = 5x + y, 0 \le y \le 4$ deb olsak, $m^5 - m = (5x + y)^5 - 5x - y = 5Q + (y^5 - y)$ ga ega bo'lamiz. Bu tenglikning o'ng tomonidagi birinchi qo'shiluvchi 5 ga bo'linadi. Ikkinchi qo'shiluvchu $g(y) = (y^5 - y)$ ning 5 ga bo'linishini esa $y = 0,1,2,3,4$ qiymatlarda bevosita tekshirib ko'rish mumkin: $g(0) = 0$, $g(1) = 0$, $g(2) = 30$, $g(3) = 240$, $g(4) = 1020$ bularning barchasi 5 ga bo'linadi. Demak, $(m^5 - m) \vdots 5$.

b). $m = 6x + y, 0 \le y \le 5$ deb olsak, $m(m^2 + 5) = (6x + y)((6x + y)^2 + 5) = (6x + y)(36x^2 + 12xy + y^2 + 5) = 6Q + (y^3 + 5y)$ ga ega bo'lamiz. Bu tenglikning o'ng tomonidagi birinchi qo'shiluvchi 6 ga bo'linadi. Ikkinchi qo'shiluvchu $g(y) = (y^3 + 5y)$ ning 5 ga bo'linishini esa $y = 0,1,2,3,4,5$ qiymatlarda bevosita tekshirib ko'rish mumkin: $g(0) = 0$, $g(1) = 6$, $g(2) = 18$, $g(3) = 42$, $g(4) = 84$ bularning barchasi 6 ga bo'linadi. Demak, $m(m^2 + 5) \vdots 6$.

c). $f(m) = m(m + 1)(2m + 1) = m(m + 1)((m + 2) + m - 1) =$
$= m(m + 1)(m + 2) + m(m - 1)(m + 1) = 6(C_{m+2}^3 + C_{m+1}^3)$.

Bu yerda ikkala had ham 3 ta ketma-ket natural sonlar ko'paytmasidan iborat.

C_{m+2}^3, C_{m+1}^3 lar mos ravishda $m + 2$ elementdan 3 tadan, $m + 1$ elementdan 3 tadan tuzilgan guruhlashlar sonini bildirgani uchun ular natural sonlar.

Demak $f(m) \vdots 6$.

20. $S = l + (l + 1) + \cdots + (l + 2n)$ yig`indini qaraymiz. Arifmetik progressiya hadlari yig`indisi topish formulasiga asosan

$$S = \frac{l + l + 2n}{2}(2n + 1) = (l + n)(2n + 1)$$

Bundan $S \vdots (2n + 1)$ ekanligi kelib chiqadi.

21. *a)* $N = 1000q + r$ sonini qaraymiz. Bundan $N = 100q + r - q = 7 \cdot 11 \cdot 13q + (r - q)$. Demak berilgan N sonning 7,11 yoki 13 ga bo'linishi uchun uning mingliklar soni q va N ni 1000 ga bo'lishidan chiqqan qoldiq r ning ayirmasi $r-q$ ning 7,11 13 ga bo'linishi zarur va yetarlidir.

b) $N = 368312 = 368\cdot 1000 + 312;\ \ 368 - 312 = 56$ .56 soni 7 ga bo'linadi. Demak, berilgan 368312 soni ham 7 ga bo'linadi. 56 soni 11 ga ham 13 ga ham bo'linmaydi shuning uchun ham $N = 368312$ soni 11ga ham13 ga ham bo'linmaydi.

22. $N_1 = \overline{a_n a_{n-1} \dots a_1 a_0}$, $N_2 = \overline{b_m b_{m-1} \dots b_1 b_0}$ sonlarni qaraymiz. Shart bo'yicha

$$\sum_{i=0}^{n} a_i = \sum_{j=0}^{m} b_j.$$

Bundan

$$N_1 = a_n\cdot 10^n + a_{n-1}\cdot 10^{n-1} + \dots + a_1\cdot 10^1 + a_0 = a_n\cdot(9+1)^n +$$
$$+a_{n-1}\cdot(9+1)^{n-1} + \dots + a_1\cdot 10^1 + a_0 = 9Q + a_n + a_{n-1} + \dots$$
$$\dots + a_1 + a_0 = 9Q + \sum_{i=0}^{n} a_i.$$

Shuningdek

$$N_2 = 9T + \sum_{j=0}^{m} b_j.$$

U holda $N_1 - N_2 = 9Q - 9T = 9(Q-T)$, ya'ni $(N_1 - N_2) \vdots 9$.

23. $7 + 77 + 777 + \dots + \underbrace{777\dots77}_{n\ ta} = 7\cdot(1 + 11 + 111 + \dots +$

$\underbrace{111\dots11}_{n\ ta}) = 7\cdot\left(\frac{10-1}{9} + \frac{10^2-1}{9} + \frac{10^3-1}{9} + \dots + \frac{10^n-1}{9}\right) = \frac{7}{9}\cdot(10 +$
$10^2 + 10^3 +$

$$+\dots + 10^n - n) = \frac{7}{9}\cdot\left(\frac{10(1-10^n)}{1-10} - n\right) = \frac{7}{81}\cdot(10^{n+1} + 9n - 10).$$

Bu yerda biz geometrik progressiya hadlari yig'indisini topish formulasi

$$S_n = \frac{b_1(1-q^n)}{1-q}$$

dan foydalandik.

24. $N = \underbrace{444\ldots44}_{n\ ta} \cdot \underbrace{888\ldots88}_{n\ ta}$ sonini qaraymiz.

$$N = \underbrace{444\ldots44}_{n\ ta} \cdot 10^n + \underbrace{888\ldots88}_{n\ ta}$$
$$= (4 \cdot 10^{n-1} + 4 \cdot 10^{n-2} + \cdots + 4 \cdot 10 + 4) \cdot$$
$$\cdot 10^n + (8 \cdot 10^{n-1} + 8 \cdot 10^{n-2} + \cdots + 8 \cdot 10 + 8)$$
$$= 4 \cdot \frac{10^n - 1}{9} \cdot 10^n +$$
$$+8 \cdot \frac{10^n - 1}{9} = \frac{4}{9} \cdot \frac{10^n - 1}{9} \cdot (10^n + 2)$$
$$= \left[\frac{2}{3} \cdot (10^n - 1)\right] \cdot \left[\frac{2}{3} \cdot (10^n - 1 + 3)\right] \cdot$$

Bu yerda

$$10^n - 1 = 9 \cdot (10^{n-1} + 10^{n-2} + \cdots + 10 + 1)$$

bo‘lgani uchun

$$\frac{2}{3} \cdot (10^n - 1) = \underbrace{666\ldots66}_{n\ ta} \quad \text{va} \quad \frac{2}{3}(10^n - 1 + 3) = \underbrace{666\ldots68}_{n\ ta}$$

Demak, $N = \underbrace{666\ldots66}_{n\ ta} \cdot \underbrace{666\ldots68}_{n\ ta}$.

25. $N = \underbrace{111\ldots1}_{n\ ta}\underbrace{55\ldots5}_{n\ ta}6 = \frac{10^{n+1} - 1}{9} \cdot 10^{n+1} + 5 \cdot 10 \cdot \frac{10^n - 1}{9} + 6 =$

$$\frac{(10^{n+1})^2 + 40 \cdot 10^n - 50}{9} + 6 = \frac{(10^{n+1} + 2)^2}{3^2} = \left(\frac{(10^{n+1} - 1)}{3} + 1\right)^2 =$$
$$\left(\underbrace{333\ldots3}_{n\ ta} + 1\right)^2.$$

26. $S_n = (n + 1)(n + 2) \ldots (n + n)$ ifodani n ga ko‘paytirib bo‘lamiz:

$$S_n = \frac{1 \cdot 2 \cdot 3 \cdots n(n + 1)(n + 2) \cdots 2n}{1 \cdot 2 \cdot 3 \cdots n}$$
$$= \frac{(2 \cdot 4 \cdot \cdots \cdot 2n) \cdot \big(1 \cdot 3 \cdot 5 \cdot \cdots (2n - 1)\big)}{1 \cdot 2 \cdot 3 \cdots n}$$
$$= 2^n \cdot (2n - 1)!!.$$

Demak, $S_n \,\vdots\, 2^n$.

I. 2-§.

27. Berilgan a va b sonlaridan foydalanib Evklid algoritimini tuzib olamiz:

1) $a = 546$ va $b = 231$

$$546 = 231 \cdot 2 + 84$$
$$231 = 84 \cdot 2 + 63$$
$$84 = 63 \cdot 1 + 21$$
$$63 = 21 \cdot 3$$

$(546; 231) = 21;$ $J: 21.$

2) $a = 6253$ va $b = 1001$

$$6253 = 1001 \cdot 6 + 247$$
$$1001 = 247 \cdot 4 + 13$$
$$247 = 13 \cdot 19$$

$(6253; 1001) = 13;$ $j: 13.$

3) $2257 = 1517 \cdot 1 + 740, 1517 = 740 \cdot 2 + 37, 740 = 37 \cdot 20.$ $J: 37.$

28. Berilgan sonlarni tub ko'paytuvchilarga ajratib yozib olamiz:

a)

420	2
210	2
105	3
35	5
7	7
1	

126	2
63	3
21	3
7	7
1	

525	3
175	5
35	5
7	7
1	

bo'lgani uchun $420 = 2 \cdot 2 \cdot 5 \cdot 3 \cdot 7;$ $126 = 2 \cdot 3 \cdot 3 \cdot 7;$ $525 = 3 \cdot 5 \cdot 5 \cdot 7$ va bulardan $(420; 126; 525) = 3 \cdot 7 = 21;$ $[420; 126; 525] = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 = 6300. J: 21 \ va \ 6300.$

b)

529	23
23	23
1	

1541	23
67	67
1	

1817	23
79	79
1	

bo'lgani uchun $529 = 23 \cdot 23;$ $1541 = 23 \cdot 67;$ $1817 = 23 \cdot 79$ va $(529; 1541; 1817) = 23;$ $[529; 1541; 1817] = 23 \cdot 23 \cdot 67 \cdot 79 = 529 \cdot 67 \cdot 79 = 2799997.$ $j:$ 23 va 2799997

29. *a*). Bunda$(6; 35) = (6; 143) = (35{,}143) = 1,$ ya'ni bu sonlar $6, 35{,}143$ juft-juft bilan o'zaro tub. Shuning uchun ham ularning EKUKi berilgan sonlarning ko'paytmasiga teng, ya'ni $[6; 35; 143] = 6 \cdot 35 \cdot 14.$

b) n va $n+1$ sonlari o'zaro tub $(n; n + 1) = 1$ shuning uchun ham $[n, n + 1] = n(n + 1).$

30. $2n$ va $2n+2$ lar ikkita ketma-ket keluvchi juft sonlar bo'lsin. U holda

$$(2n; 2n+2) = 2(n; n+1) = 2.$$

Endi $2n+1$va $2n+3$ lar ikkita ketma-ket keluvchi toq sonlar bo'lsin. U holda

$$2n+3 = (2n+1)\cdot 1 + 2, \qquad 2n+1 = 2n+1$$

lardan Evklid algoritmiga asosan $(2n+1, 2n+3) = 1$ ekanligi kelib chiqadi.

31.$(cb; bc; ca) = c(b; b; a) = c(a; b)$ tenglik o'rinli. Ikkinchi tomondan esa$(a; b; c) = ((a; b); c) = d$ => $(a; b) = dx$ va $c = dy$. Shuning uchun ham $(cb; bc; ca) = d^2xy$ => $(cb; bc; ca) \vdots (a; b; c)^2$.

32. $(a+b; a-b) = d$ bo'lsin. U holda $a+b = dx$ $a-b = dy$ deb yoza olamiz.Bundan $2a = d(x+y), 2b = d(x-y)$, ya'ni d soni $2a$ va $2b$ larning umumiy bo'luvchisi. Shartga asosan $(a; b)$=1 bo'lgani uchun $(2a; 2b) = 2$. Shuning uchun ham $2 \vdots d$ $d = 1$ yoki $d = 2$.

33. Aytaylik $(a, a+b) = d$ bo'lsin, u holda $a = dxa + b = dy$ yoki $dx + b = dy$ bundan esa $b \vdots d$ bo'lishini va $d = UB\,(a, b)$, (UB-umumiy bo'luvchi) ekanligini topamiz $\frac{a}{b}$ qisqarmas kasr bo'lganidan $d=1$. Demak $\frac{a}{a+b}$ kasr qisqarmas kasr.

34. a va b lar toq sonlar va $a - b = 2^n$ bo'lsin. U holda $(a; b) = d$ –toq son bo'ladi. $a = dx,\ b = dy,\ (x; y) = 1$ deb olsak $a - b = d(x-y) = 2^n$ => $2^n \vdots d \Rightarrow d = 1$

35. a)$(d, m) = (d; [dx : dy]) = d(1; [x; y]) = d$.

b)$(ab, m) = (dm, m) = m(d; 1) = m$. Bunda $d = (a, b)$ $m = [a, b]$.

c) $(a+b; ab) = x$ $(a; b) = 1$. Faraz etaylik p soni $a+b$ va ab ning umumiy bo'luvchisi bo'lsin. U holda $a \vdots p$ yoki $b \vdots p$. U holda $(a+b) \vdots p$ bajarilgani uchun p soni a va b sonining umumiy bo'livchisi bo'ladi. $(a, b) = 1$ bo'lgani uchun $p = 1$ va demak $(a+b; ab) = x = 1$.

d) $(a+b; m) = ?$. $m = [a; b]$va$(a, b) = d$ bo'lsin u holda $a = dx$, $b = dy$ va $(x; y) = 1$. Bulardan $(a+b; m) = (d(x+y); [dx; dy]) = (d(x+y); d[x; y]) = d(x+y; xy)$.

b)misolga asosan $(x+y; xy) = 1$ va demak $(a+b; m) = d$, ya'ni $(a+b; [a; b]) = (a; b)$.

36. $a)$ $(n; 2n+1) = (n; n+(n+1))$, $d = (n; 2n+1)$, $n = dx$

$$d = (n; 2n+1) = (dx; 2dx+1) \Rightarrow d = 1.$$

$b)$ $(10n+9; n+1) = d,\quad 10n+9 = dx,\quad n+1 = dy,\quad (x;y) = 1.$

$$10(dy-1)+9 = dx \Rightarrow 10dy - 1 = dx \Rightarrow 10dy - dx = 1$$
$$\Rightarrow d(10y - x) = 1 \Rightarrow d = 1.$$

$c)$ $(3n+1; 10n+3) = d$ bo'lsin, u holda

$$\begin{cases} 3n+1 = dx \\ 10n+3 = dy \end{cases} \Rightarrow 10dx - 3dy = 1 \Rightarrow d(10x-3y) = 1 \Rightarrow d = 1.$$

Eslatma: a) va c) misollarni Evklid algoritmidan foydalanib ham ishlash mumkin.

37. Agar $x = [a;b] = \dfrac{ab}{(a;b)}$ bo'lsin u holda $\left(\frac{x}{a};\frac{x}{b}\right) = \left(\frac{b}{(a,b)};\frac{a}{(a,b)}\right)$, bunda $(a;b) = d$ va $b = db_1$ $a = da_1$va demak $\left(\frac{x}{a};\frac{x}{b}\right) = 1$. Aksincha agar $\left(\frac{x}{a};\frac{x}{b}\right) = 1$ bo'lsa $x = [a;b]y$ deb olib

$$1 = \left(\frac{x}{a};\frac{x}{b}\right) = \left(\frac{[a;b]y}{a};\frac{[a;b]y}{b}\right) = y\left(\frac{[a;b]}{a};\frac{[a;b]}{b}\right) = \left(\frac{b}{(a,b)};\frac{a}{(a,b)}\right)$$
$$= y,$$

ya'ni $y = 1$. Bundan $x = [a,b]$ni hosil qilamiz.

38. a, b, c - toq sonlar bo'lib $(a;b;c) = D$ bo'lsin. U holda $a = Dx,\ b = Dy, c = Dz$ va x, y, z lar toq sonlar bo'ladi. U holda $\frac{a+b}{2} = D\cdot\frac{x+y}{2};\ \frac{a+c}{2} = D\cdot\frac{x+z}{2};\ \frac{b+c}{2} = D\cdot\frac{y+z}{2}$ bo'lib $\frac{x+y}{2};\ \frac{x+z}{2};\ \frac{y+z}{2}$ lar butun sonlar bo'ladilar.

Bu yerdan D ning $\frac{a+b}{2}, \frac{a+c}{2}, \frac{b+c}{2}$ sonlarining umumiy bo'luvchisi ekanligi kelib chiqadi. Endi agar $\left(\frac{a+b}{2}, \frac{a+c}{2}, \frac{b+c}{2}\right) = d$ desak $d \vdots D$. Ikkinchi tomondan esa d soni a, b, c larning umumiy bo'luvchisi, ya'ni $D \vdots d$. Bulardan $D = d$. Haqiqatan ham $\frac{a+b}{2} = dx \Rightarrow a + b = 2dx$. Shuningdek $a + c = 2dy$ va $b + c = 2dz$. Oxirgi tenglikdan $c = 2dz - b$ ga egamiz. Buni $a + c = 2dy$ tenglikka olib borib qo'ysak $a + 2dz - b = 2dy \Rightarrow a - b = 2dy - 2dz$ hosil bo'ladi. U holda bundan va birinchi tenglikdan

$$\begin{cases} a + b = 2dx \\ a - b = 2dy - 2dz \end{cases} \Rightarrow a = d(x + y - z) \Rightarrow a \vdots d$$

va $b = d(x - y + z) \Rightarrow b \vdots d$. Bulardan $c = 2dz - b_1 d = d(2z - b_1) \Rightarrow c \vdots d$. d soni a, b, c larning umumiy bo'luvchisi.

39. 1) agar

$$a = cq + r; \quad b = cq + r_1 \qquad (1)$$

bo'lsa $(a, b, c) = (c; r; r_1)$ ekanligini isbotlashimiz kerak.
$(a; b; c) = d$ deb olaylik. U holda (1) dan $r \vdots d$ va $r_1 \vdots d$ ekanligi kelib chiqadi. Biz $d = (c; r; r_1)$ ekanligini ko'rsatamiz. $(c; r; r_1) = D$ bo'lsin. U holda (1) dan $a \vdots D$ va $b \vdots D$ kelib chiqadi. Shuningdek $c \vdots D$. Demak $D = (a; b; c)$ va $D = d$.

2) *a*) $667 = 299 \cdot 2 + 69$ va $391 = 299 \cdot 1 + 92$ bo'lgani uchun 1)-misolga asosan $(299, 391, 667) = (299, 69, 92) = (23 \cdot 13;\ 23 \cdot 3;\ 23 \cdot 4) = 23 \cdot (13; 3; 4) = 23$.

b). $(588; 2058; 2849) = (588; 497; 294) = (2^2 \cdot 3 \cdot 7^2;\ 7 \cdot 71;\ 2 \cdot 3 \cdot 7^2) = 7$, bunda $2889 = 588 \cdot 4 + 497$ va $2058 = 588 \cdot 3 + 294$; $497 = 71 \cdot 7$ ekanligidan foydalandik.

40. Faraz qilaylik $(a, b) = d$ bo'lsin, unda $a = dx,\ b = dy$ bo'lib, bu yerda $(x, y) = 1$ bo'ladi. U holda $5a + 3b = d(5x + 3y)$ va $13a + 8b = d(13x + 8y)$ bo'lib, bundan esa $UB(5a + 3b, 13a + 8b) = d$ ekanligini topamiz. Endi $(5x + 3y, 13x + 8y) = 1$ ekanligini isbotlash kerak. Aytaylik

$(5x + 3y, 13x + 8y) = \delta$ va $5x + 3y = \delta u, 13x + 8y = \delta v$

bo'lsin u holda

$x = \delta(8u - 3v)\ vay = \delta(5v - 13u)$ bo'ladi, ammo $(x, y) = 1$ edi, shuning uchun $\delta = 1$. Shunday qilib $(5a + 3b, 13a + 8b) = d$.

41. $(n, n + 1, n + 2)$va $[n, n + 1, n + 2]$ larni topish kerak.

$$(n, n + 1, n + 2) = \big((n, n + 1), n + 2\big) = (1, n + 2) = 1$$

$$[n, n + 1, n + 2] = \big[[n, n + 1], n + 2\big] = [n(n + 1), n + 2] =$$

$$\frac{n(n + 1)(n + 2)}{\big(n(n + 1); (n + 2)\big)} = \frac{n(n + 1)(n + 2)}{\big(n; (n + 2)\big)},$$

chunki $(n + 1; n + 2) = 1$. Bu yerda

$$(n; n + 2) = \begin{cases} 1, \text{agar } n \text{ toq son bo'lsa;} \\ 2, \text{agar } n \text{ juft son bo'lsa.} \end{cases}$$

Haqiqatan ham, $n = 2k$ – juft son bo'lsin. U holda $(n; n + 2) = (2k; 2k + 2) =$

$2(k;k+1)=2$; Endi agar $n=2k+1$ – toq son bo'lsa, u holda $(n;n+2)=(2k+1;2k+3)=1$, chunki Evklid algoritmiga asosan $2k+3=(2k+1)\cdot 1+2,\quad 2k+1=2\cdot k+\boxed{1},\quad 2=1\cdot 2+0.$

Shunday qilib,

$$[n,n+1,n+2]=\begin{cases} n(n+1)(n+2), & \text{agar } n \text{ toq son bo'lsa;} \\ \frac{1}{2}n(n+1)(n+2), & \text{agar } n \text{ juft son bo'lsa.} \end{cases}$$

42. $\quad nab=ax+by,\ \ (a,b)=1 \qquad (*)$

bo'lsa, $x=bk\ (k=1,2,3,4.\ldots,)$ deb yoza olamiz. U holda (*) dan $nab=abk+by$. Bundan $y=na-ak=a(n-k)$. Bu yerda a,b,x,y lar natural sonlar bo'lgani uchun $k=1,2,3,4,\ldots\ldots n-1(n>1)$, shunday qilib $x=bk,y=a(n-k),\ \ k=1,2,3,4,\ldots\ldots,n-1$. Demak, nab ni $n-1$ ta ko'rsatilgan ko'rinishda ifodalash mumkin ekan.

43. Evklid algoritmidan foydalanamiz. U holda quyidagilarga ega bo'lamiz:

$899=493\cdot 1+406,\ 493=406\cdot 1+87,\ 406=87\cdot 4+58,$ $87=58\cdot 1+\boxed{29},\ 58=29\cdot 2$, bundan $(899,493)=29$. Shuning uchun ham

$$29=87-58\cdot 1=87-(406-87\cdot 4)\cdot 1=5\cdot 87-406=$$
$$=5(493-406\cdot 1)-406=5\cdot 493-6\cdot 406$$
$$=5\cdot 493-6(899-493)=11\cdot 493-6\cdot 899.$$

Shunday qilib, $29=899(-6)+11\cdot 493 \qquad x=-6,y=11.$

44. $a=cq+r$ bo'lsin. U holda $(a,c)=(c,r)=1$ bo'ladi. Shuningdek $b=c\cdot q_1+r_1$, bo'lsa $(b,c)=(c,r_1)$ bo'ladi.Bulardan $ab=cQ+rr_1$ ga ega bo'lamiz.Oxirgi tenglikdan $(ab;c)=(c;rr_1)=1$.

45.$m>n$ va $d=(m,n)$ bo'lsin. U holda $m=dm_1,n=dn_1$ va $m_1-n_1>0$ bo'ladi. Agar $d>m-n=d(m_1-n_1)$ yoki $0<m_1-n_1<1$bo'lishi kerak m_1-n_1 –butun son bo'lgani uchun ham bunday bo'lisi mumkin emas. Demak $(m,n)\le m-n\ \ (m>n)$.

46. Tushunarliki, $ab\ \vdots\ (ab,c)$ va $bc\ \vdots\ (ab,c)$ bajariladi. Shuning uchun ham $(ab,bc)\ \vdots\ (ab,c)$. Lekin shartga ko'ra $(a,c)=1$ bo'lgani uchun $(ab;bc)=b(a,c)=b$ va bundan $b\ \vdots\ (ab;c)$.

47. 20-masaladan $c \vdots (ac; b)$ va $(a; b) = 1$ dan$(c, b) \vdots \vdots(ac; b)$. Ikkinchi tomondan $(ac; b) \vdots (c; b)$. Shunday qilib $(ac; b) = (c; b)$ bo'ladi.

48. Faraz qilaylik $(mn, mk, nk) = d$ bo'lsin. U holda

$$mnk = dx. \qquad (1)$$

Bundan

$$x = \frac{mnk}{d} = m \cdot \frac{nk}{d} = n \cdot \frac{mk}{d} = k \cdot \frac{mn}{d}.$$

Demak, x soni m, n, k larning umumiy karralisi va $x = [m, n, k] \cdot q, q \geq 1$ deb yoza olamiz.$(*)$dan

$$\frac{x}{m} = \frac{nk}{d}, \quad \frac{[m, n, k]q}{m} = \frac{nk}{d}, \quad \frac{[m; n; k]q}{n} = \frac{mk}{d},$$

$$\frac{[m; n; k]q}{k} = \frac{mn}{d}.$$

Bulardan q soni $\frac{nk}{d}, \frac{mk}{d}, \frac{mn}{d}$ sonlarning umumiy bo'luvchisi. Faramizga ko'ra

$\left(\frac{nk}{d}, \frac{mk}{d}, \frac{mn}{d}\right) = 1$, shuning uchun ham $q = 1$ va $x = [m, n, k]$. Endi (1) dan isbotlanish talab etilgan tenglik kelib chiqadi.

49. *a*) berilgan sistemadagi ikkinchi tenglama

$\begin{cases} x = 30u \\ y = 30v \\ (u, v) = 1 \end{cases}$ sistemaga teng kuchli, shu sababli sistemaning birinchi tenglamasi $u + v = 5$ ko'rinishda bo'ladi, bundan esa $u = 1,2,3,4$ (yoki $v = 1,2,3,4$) bo'lishi mumkinligini ko'ramiz. u (v)ning topilgan bu qiymatlari bo'yicha $x = 30,60,90,120$ (yoki $y = 30, 60, 90, 120$) bo'lishini topamiz. y ning x ra mos qiymatlarini $y = 150 - x$ tenglikdan topamiz.

Shunday qilib, sistemaning yechimlari $(30,120), (60,90), (90,60), (120,30)$ juftliklardan iborat ekan.

b) berilgan sistema $\begin{cases} (x, y) = 45 \\ \frac{x}{y} = \frac{11}{7} \end{cases}$ dagi birinchi tenglama

$\begin{cases} x = 45u \\ y = 45v \\ (u, v) = 1 \end{cases}$ sistemaga teng kuchli, shu sababli sistemaning ikkinchi tenglamasidan $u = 11$ va $v = 7$ bo'lishini topamiz. Demak $x = 495$, $y = 315$ bo'lar ekan.

$$c) \begin{cases} xy = 8400 \\ (x, y) = 20 \end{cases} \Rightarrow \begin{cases} x = 20x_1 \\ y = 20y_1 \\ (x_1, y_1) = 1 \\ 400x_1y_1 = 8400 \end{cases} \Rightarrow \begin{cases} x_1 \cdot y_1 = 21 \\ (x_1, y_1) = 1. \end{cases}$$

)

Bundan

$x_1 = 1, 3, 7, 21$ va $x = 20, 60, 140, 420$ $y_1 = 21, 7, 3, 1;$ $y = 420, 140, 60, 20.$

$$d) \begin{cases} \frac{x}{y} = \frac{5}{9} \\ (x, y) = 28 \end{cases} \left| \begin{matrix} x = 28x_1 \\ y = 28y_1 \\ (x_1; y_1) = 1 \end{matrix} \right| \Rightarrow \begin{cases} \frac{x_1}{y_1} = \frac{5}{9} \\ (x_1; y_1) = 1 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{5}{9} y_1 \\ (x_1; y_1) = 1 \end{cases},$$

$y_1 = 9$, $y = 29 \cdot 9 = 252$ va$x_1 = 5$, $x = 28 \cdot 5 = 140$.

e)

$$\begin{cases} xy = 20 \\ [x, y] = 10 \end{cases} \Rightarrow \left| [x, y] = \frac{xy}{(x, y)} \right| \Rightarrow 10 = \frac{20}{(x, y)} \Rightarrow (x, y) = 2$$

$$\left| \begin{matrix} x = 2x_1 \\ y = 2y_1 \\ (x_1, y_1) = 1 \end{matrix} \right| \to \begin{cases} x_1 y_1 = 5 \\ (x_1, y_1) = 1 \end{cases} \to \left| \begin{matrix} x_1 = 5; 1 \\ y_1 = 1; 5 \end{matrix} \right| \to \left| \begin{matrix} x = 10; 2 \\ y = 2; 10 \end{matrix} \right|.$$

50. $m = 10q + 1$ dan $(m, q) = 1$ kelib chiqadi. $N = 10a + b$ ning ikkala tomonini q ga ko'paytiramiz. U holda

$$Nq = 10aq + bq = (m - 1)a + bq = am - (a - bq)$$

hosil bo'ladi. Agar $(a - bq) \vdots \vdots m$ bo'lsa, $(m, q) = 1$ bo'gani uchun N sonining m bo'linishi kelib chiqadi.

51. $N = 10a + b$ ning ikkala tomonini $q + 1$ ga ko'paytiramiz va $m = 10q + 9$ ekanligidan foydalanamiz. U holda $(q + 1)N = 10a(q + 1) + b(q + 1) = (10q)a + 10a + bq + b = (m - 9)a + 10a + bq + b = ma + a + b(q + 1)$. 10.b)-misolga asosan $(q + 1; m) = (q + 1, 10q + 9) = 1$ bo'lgani uchun $a + b(q + 1)$ soni m ga bo'linsa N soni m ga bo'linadi.

52. $10N_1$ sonini qaraymiz. $10N_1 = a_n a_{n-1} \cdots a_2 a_1 \cdot 10 + 20a_0 = N + 19a_0$, $(19,10) = 1$. Endi agar $N_1 \vdots 19$ bo'lsa, $N \vdots 19$ va aksincha $N \vdots 19$ bo'lsa, $N_1 \vdots 19$ bo'ladi.

53.26-misoldagi qoidani N=3086379 soniga tatbiq etamiz.

$$N_1 = 308637 + 18 = 308655, \qquad N_2 = 30865 + 10 = 30875$$
$$N_3 = 3087 + 10 = 3097, N_4 = 309 + 14 = 323,$$
$$N_5 = 32 + 6 = 38 \ va\ 38 : 19 = 2.$$

Demak 3086379 soni ham 19 ga bo'linadi.

I.3-§.

54. $6n + 1 = p_1 - p_2$ bo'lsin, u holda: *a)*agar $p_2 = 2$ bo'lsa $p_1 = 6n + 3 = 3(2n + 1)$ bo'lishi kerak. $p_1$tub son bo'lgani uchun bunday bo'lishi mumkin emas.

b) p_2 toq tub son bo'lsin, ya'ni $p_2 = 2k + 1$, bu holda $p_1 = 6n + 2k + 2 = 2(3n + k + 1)$bo'ladi. Bunday bo'lishi ham mumkin emas.$6n + 1$ ni 2 ta tub sonning ayirmasi ko'rinishida ifodalab bo'lmaydi.

55. $N = 2k + 1 = p_1 - p_2$dan tub sonlarning bitta juft son bo'lishi kerak ekanligi kelib chiqadi, $p_2 = 2$ desak $N = p_1 - 2$, bunda p_1 tub son.

56. Faraz kilaylik $N^2 = n^2 + p$ bo'lsin, u holda $N^2 - n^2 = p$ bo'lib bundan haqli ravishda $(N - n)(N + n) = p$ tenglikni yoza olamiz va bundan esa $N - n = 1, N + n = \text{p}$ kelib chiqadi. Demak $2N = \text{p} + 1$ yoki $\text{p} = 2N - 1 = 6m + 3$ bo'lib, bu esa p tub son deb qilingan farazimizga ziddir, demak, farazimiz noto'g'ri va $N = 3m + 2$ $(m = 1,2, \dots)$ sonning kvadratini natural son kvadrati va tub sonning yig'indisi ko'rinishida ifodalash mumkin emas ekan.

57. 1-usul. $a = p \cdot a_1$va $p < a_1$ bo'lsin.Agar $p > \sqrt{a}$bo'lsa, $a_1 > \sqrt{a}$ bo'ladi va bu tengsizliklarni hadlarni ko'paytirsak $p \cdot a_1 > a$ bo'lar edi. Demak $p \leq \sqrt{a}$.

2-usul. $a = p \cdot a_1$va $p < a_1$ bo'lsin. U holda $p^2 < p \cdot a_1$ yoki $p^2 < a$. Bundan $p < \sqrt{a}$.

Agar a tub son bo'lsa, bu teorema o'rinli emas, chunki bu holda a ning eng kichik tub bo'luvchisi ham $a = p$ bo'ladi.

58. 1)$11 < \sqrt{127} < 12$ bo'lgani uchun 127 ni ketma-ket 2,3,5,7,11 tub sonlariga bo'lib ko'ramiz. Agar shularning birortasi ham bo'linmasa

127 soni tub son bo‘ladi, aks holda tub son bo‘lmaydi. 127 soni 2,3,5,7,11 larning birortasiga ham bo‘linmaydi. Demak 127 tub son.

2)$30 < \sqrt{919} < 31$ bo‘lgani uchun 919 ni ketma-ket 2, 3, 5, 7, 11,13,

17,19, 23, 29 tub sonlariga bo‘lib ko‘ramiz.Agar shularning birortasi ham bo‘linmasa 919 soni tub son bo‘ladi, aks holda tub son bo‘lmaydi. 919 soni bu sonlarning birortasiga ham bo‘linmaydi. Demak, 919 tub son.

3) $86 < \sqrt{7429} < 87$ va 7429 soni 2,3,5,7,11,13 larga bo‘linmaydi, lekin 17 ga bo‘linadi, ya’ni $7429 = 17 \cdot 437$. Shuning uchun ham u murakkab son.

59. 1) 100 va 110, bu yerda $10 < \sqrt{110} < 11$. Shuning uchun ham berilgan sonlar orasidagi 2,3,5,7 ga karralilarni o‘chirib chiqamiz. 2 ga karralilarini tushirib qoldirsak: 101,103,105 ,107,109 lar qoladi.Bular orasidan 3 ga bo‘lina diganlarini o‘chirsak:101,103,107,109 lar qoladi.Bular orasida 5 ga va 7ga karralisi yo‘q. Shuning uchun ham 101,103,107,109 lar qaralayotgan oraliqdagi tub sonlar.

2) 190 va 200, bu yerda $14 < \sqrt{200} < 15$ bo‘lgani uchun 1)- misoldagi singari ish tutib berilgan sonlar orasidagi 2,3,5,7,11,13 ga karralilarni o‘chirib chiqamiz. 2 ga karrali sonlarni tushirib qoldirsak 191, 193, 195,197, 199 lar qoladi. Bular orasidan 3, 5 ga bo‘linadiganlarini tushirib qoldirsak 191, 193, 197, 199 lar qoladi. Bu sonlar orasida 7 ga, 11 ga yoki 13 ga bo‘linadiganlari yo‘q. Shuning uchun ham bu sonlar tub sonlardir.

3)200 va 220, $14 < \sqrt{220} < 15$ bo‘lgani uchun 2)- misoldagi singari ish tutib berilgan sonlar orasidagi 2,3,5,7,11,13 ga karralilarni o‘chirib chiqamiz. 2 ga karrali sonlarni tushirib qoldirsak 201,203, 205, 207,209,211, 213, 215, 217, 219 lar qoladi. Bular orasidan 3, 5 ga bo‘linadiganlarini tushirib qoldirsak 203, 209, 211, 217 lar qoladi. Bu sonlar orasida 7 ga bo‘linadiganlarini tushirib qoldirsak 209, 211 lar qoladi. Bulardan 209 soni 11 ga karrali. 211 esa 11 ga ham 13 ga ham bo‘linmaydi. Shuning uchun ham 211 qaralayotgan oraliqdagi yagona tub sondir.

4)2640 va 2680. Bu yerda $51 < \sqrt{2680} < 52$. Bo‘lgani uchun berilgan oraliqdagi sonlar orasidan 2 dan 47 gacha bo‘lgan tub sonlarga bolinadiganlarini tushirib qoldiramiz. U holda $2647, 2657, 2659, 2663, 2671, 2677$ sonlarining berilgan oraliqdagi tub sonlar ekanligiga ishonch hosil qilamiz.

60. Faraz qilaylik p soni $n! - 1$ sonining tub bo'luvchisi bo'lsin. U holda $p \leq n! - 1$ bajariladi. Bundan $p < n!$. Ikkinchi tomondan $n!$ soni p ga bo'linmaydi, shuning uchun ham $p > n$. Bulardan $n < p < n!$. Isbotdan tub sonlar sonining cheksiz ko'p ekanligi kelib chiqadi.

61. $21! + 2, 21! + 3, \dots, 21! + 20, 21! + 21$ larning barchasi murakkab sonlar.

62. $n = 1$da$1,11,15$ bo'lib faqat 11 tub son. $n = 2$ da $2,12,16$ bo'lib bularning birortasi ham tub son emas. $n \geq 3$ bo'lsa, uni $n = 3k + r, r = 0,1,2$ deb yozish mumkin. $r = 0$ bo'lsa, $3k$, $3k + 10, 3k + 14$. Bularning uchalasi tub bo'ladigan faqat 1 ta k=1 qiymati mavjud. Bu holda $n = 3$ va $3,13,17$ tub sonlari hosil bo'ladi. $r = 1$ bo'lsa $3k + 11$, $3k + 15$ va $3k + 15$ tub emas. $r = 2$ da $3k + 2, 3k + 12, 3k + 16$ va $3k + 12$ soni tub son emas. Demak, $n, n + 10, n + 14$ sonlar bir vaqtda tub bo'ladigan n ning faqat 1 ta qiymati $n = 3$ mavjud ekan.

63. Tub sonlarni 3 ta sinfga $p = 3q, p = 3q + 1, p = 3q + 2$ bo'lamiz. 1-sinfda faqat 1 ta $p = 3$ tub soni$(q = 1)$mavjud. Bu holda $2p^2 + 1 = 2 \cdot 9 + 1 = 19$-tub son bo'ladi.

Agar $p = 3q + 1$ ko'rinishda tub son bo'lsa, (2 va 3- sinflar cheksiz ko'p tub sonlar mavjud), $2p^2 + 1 = 2(9q^2 + 6q + 1) + 1 = 18q^2 + 12q + 3 = 3(6q^2 + 4q + 1)$ – murakkab son bo'ladi.

Endi, agar $p = 3q + 2$ ko'rinishdagi tub son bo'lsa u holda $2p^2 + 1 = 2(9q^2 + 12q + 4) + 1 = 18q^2 + 24q + 9 = 3(6q^2 + 8q + 3)$, ya'ni bu holda ham murakkab son bo'ldi. Shunday qilib, p ning faqat bitta $p = 3$ qiymatida $2p^2 + 1 = 19$ tub son bo'lar ekan.

64. Barcha natural sonlarni 5 ga bo'lib qoldiqlari bo'yicha $N = 5n + r, 0 \leq r \leq 4$ deb yoza olamiz. Agar $r = 0$ bo'lsa $N = 5n$ bo'lib faqat $n = 1$ da tub son $p = 5$ bo'ladi va bu holda $4p^2 + 1 = 4 \cdot 25 + 1 = 101$, $6p^2 + 1 = 6 \cdot 25 + 1 = 151$ lar ham tub son bo'ladi. Qolgan hollarda $p = 5n + r$ n=1,2,3,4 tub son bo'lsa u holda $4p^2 + 1 = 4(25n^2 + 10nr + r^2) + 1 = 5(20n^2 + 8nr) + 4r^2 + 1$ va $6p^2 + 1 = 6(25n^2 + 10nr + r^2) + 1 = 5(30n^2 + 12nr + 6r^2 + 1)$, bu yerda $4r^2 + 1$ ifoda $r = 1$ da 5, $r = 4$ da 65, $r = 2$ da $6r^2 + 1 = 25$, $r = 3$ da 55 ga teng qiymat qabul qiladi, ya'ni $p = 5n \pm 1$ ko'rinishda bo'lsa $4p^2 + 1$ ifoda

5 ga bo'linadi, agarda $p = 5n \pm 2$ ko'rinishidagi tub son bo'lsa, u holda $6p^2 + 1$ ifoda 5 ga bo'linadi. Demak, izlanayotgan qiymat bitta $p = 5$.

65. 1) $p + 5$ va $p + 10$ lar uchun $p = 2$ da $p + 10 = 12$ murakkab son. Agar $p = 2k + 1$ toq tub son bo'lsa, $p + 5 = 2k + 6 = 2(k + 3)$ murakkab son bo'ladi.

2) $p, p + 2, p + 5$ uchun $p = 2$ da $p + 2 = 4$ murakkab son. $p = 2k + 1$ bo'lsa, $p + 5 = 2k + 6 = 2(k + 1)$ murakkab son.

3) $2^n - 1$; $2^n + 1, (n > 2)$ uchun sonlarning $3q + r$; $r = 0,1,2$ ko'rinishidagi yozuvidan foydalanamiz. Bunda quyidagi uchta holni qaraymiz:

1)$2^n = 3q$ 2)$2^n = 3q + 1$ 3)$2^n = 3q + 2$.

Birinchi holda $2^n = 3q$ bo'lishi mumkin emas. Ikkinchi hol $2^n = 3q + 1$ bo'lsa, $2^n - 1 = 3q$bo'ladi. $q = 1$ da $2^n - 1 = 3$ tub son bo'ladi va $n = 2$ da $2n + 1 = 5$ ham tub son, lekin misolning shartida $n > 2$. Uchinchi holda $2^n = 3q + 2$ bo'lsa, $2^n + 1 = 3(q + 1)$bo'ladi. Shunday qilib $2^n - 1$ va $2^n + 1$, $n > 2$ bo'lganda bir vaqtda tub son bo'lmas ekan.

66. Agar $p = 3$bo'lsa, p va $8p^2 + 1 = 73$ lar tub sonlar hamda $8p^2 + 2p + 1 = 8 \cdot 9 + 6 + 1 = 79$ tub son bo'ladi, $p = 3k + 1$ bo'lsa, $8p^2 + 1 =8(9k^2 + 6k + 1) + 1 = 72k^2 + 48k + 9 = 3(24k^2 + 16k + 3)$ murakkab son bo'ladi. Shuningdek, agarda $p = 3k + 2$ bo'lsa, $8p^2 + 1 =8(9k^2 + 12k + 4) + 1 = 72k^2 + 96k + 33 = 3(24k^2 + 32k + 11)$ murakkab son bo'ladi.

67. $2^{18} + 3^{18} = (2^6)^3 + (3^6)^3 = (2^6 + 3^6)(2^{12} - 2^6 \cdot 3^6 + 3^{12}) = (2^2 + 3^2)(2^4 - 2^2 \cdot 3^2 + 3^4)(2^{12} - 2^6 \cdot 3^6 + 3^{12}) = 13 \cdot 61 \cdot 488881 = 13 \cdot 61 \cdot 37 \cdot 73 \cdot 181$.

68. $a > 3$; $m = 3q_1 + 1$; $n = 3q_2 + 2$ bo'lsin.

a)$a = 3q; q > 1$ bo'lsin, u holda a murakkab son ekanligi ma'lum.

b) $a = 3q + 1$ bo'lsin u holda $a + m = 3q + 1 + 3q_1 + 1 = 3(q + q_1) + 2$ va $a + n = 3q + 3q_2 + 2 + 1 = 3(q + q_2 + 1)$ bo'ladi. Bunda $a + n$ murakkab son;

c) $a = 3q + 2$ ko'rinishda bo'lsin, u holda $a + m = 3(q + q_1 + 1)$ murakkab son. Demak, berilgan shartlarda $a > 3, a + m$; $a + n$ lar bir vaqtda tub sonlar bo'la olmas ekan.

69. 1) $n^4+4=n^4-4n^2+4n^2+4=(n^2+2)^2-4n^2=(n^2+2+2n)(n^2+2-2n)=((n+1)^2+1)((n-1)^2+1)$ bo‘lib, bu esa $n>1$ da murakkab son bo‘lishligini anglatadi.

2) $n^4+n^2+1=(n^2+1)^2-n^2=(n^2-n+1)\cdot(n^2+n+1), n>1$ da murakkab son bo‘ladi.

70. $p, p+2, p+4, (p>3)$ sonlarni qaraymiz. $p>3$ tub sonlar $3q+1; 3q+2$ ko‘rinishlarida bo‘ladi. $p=3q+1$ $(q=2,4,..)$ deb olsak, u holda $p+2=3(q+1)$ murakkab son bo‘ladi, agarda $p=3q+2, (q=1,3,5 ...)$ bo‘lsa, u holda $p+4=3(q+2)$ murakkab son bo‘ladi. $p=3q$ bo‘lsa, $q=1$ da tub son $p=3$; $p+2=5$; $p+4=7$, yagona egizak tub sonlar uchligini hosil bo‘ladi.

71. $p=3n+2$ ko‘rinishidagi tub son bo‘lsin.U holda $N=3\cdot 5\cdot 7\cdots p+2$ sonini qaraymiz.N soni ham $3n+2$ ko‘rinishidagi son, chunki $N=3(5\cdot 7\cdot\ldots\cdot p)+2$ deb yoza olamiz. N sonining kanonik yoyilmasida p dan katta murakkab son qatnashadi va ularning orasida albatta $3n+2$ko‘rinishidagi tub son mavjud, agar N ning barcha bo‘luvchilari $3n+1$ ko‘rinishida bo‘lsa, N ham shunday ko‘rinishda bo‘lishi kerak bo‘lar edi. Demak, p qanday bo‘lishidan qat’i nazar p dan katta $3n+2$ ko‘rinishidagi tub son mavjud ekan.

72. $\prod_{i=1}^{n} p_i-1=p_k\cdot q(k>n, q\geq 1)$ bo‘lib, bundan $p_k\leq \prod_{i=1}^{n} p_i-1$ va $p_k\leq \prod_{i=1}^{n} p_i$. Shunday ekan $p_{n+1}\leq \prod_{i=1}^{n} p_i$.

73. Ma'lumki $p_5=11$, ya'ni beshinchi tub son 11 ga teng va $2\cdot 5=10$ bo‘lib, 11>10 bo‘ladi. Agar $p_n>2n, (n=5,6,7\ldots)$ bo‘lsa, u holda $p_{n+1}-p_n\geq 2$ ekanligidan $p_{n+1}-2n>2$ yoki $p_{n+1}>2(n+1)$ kelib chiqadi, bu esa isbotlanishi talab qilinayotgan tengsizlikni beradi.

74. $p_n\leq 2^{2^{n-1}}$dan $n=1$ da $p_1\leq 2.(p_1=2$ bajariladi). $n=2$ da $p_2=3<4$; $n=3$ da $p_3=5<16$ bajariladi. Endi faraz qilaylik $n=k$ da $(k=2,3,\ldots n)$ $p_k<2^{2^{k-1}}$ tengsizlik o‘rinli bo‘lsin. U holda

$$p_{n-1}\leq \prod_{k=1}^{n} p_k+1\leq 2\cdot 2^2\cdot 2^{2^2}\cdots 2^{2^{n-1}}+1=2^{2^n}\cdot 2^{-1}+1<2^{2^n}.$$

Demak, berilgan munosabat ixtiyoriy n natural soni uchun o‘rinli.

75. Faraz qilaylik 2^n-1 tub son bo‘lib n murakkab son bo‘lsin u holda $n=n_1\cdot n_2$ $(n_1>1, n_2>1)$ deb yoza olamiz. Bundan $2^n-1=2^{n_1n_2}-1=(2^{n_1})^{n_2}-1$ murakkab son bo‘ladi. 2^n-1 tub son degan teskari tasdiq hamma vaqt ham o'rinli emas. Masalan: $2^{11} - 1 = 2048 - 1 = 2047 = 23 \cdot 89$.

II.1-§.

76. 1) $\pi(5) = 3$; 2) $\pi(10) = 4$; 3) $\pi(25) = 9$; 4) $\pi(37) = 12$; 5) $\pi(200) = 46$; 6) $\pi(1000) = 168$.

77. 1) $\pi(100) = \frac{100}{ln100} = \frac{100}{ln4+ln25} = \frac{100}{2ln10} = \frac{50}{2{,}3026} \approx 22.$

Nisbiy xatolikni hisoblaymiz:

$$\omega = \frac{\Delta\pi(x)}{\pi(x)} = \frac{25-22}{25} = \frac{3}{25} = 0{,}12 = 12\%.$$

2)

$$\pi(500) = \frac{500}{ln500} = \frac{5 \cdot 100}{ln5 + ln100} = \frac{500}{1{,}6094 + 4{,}6052} = \frac{500}{6{,}2146} \approx 80;$$

$$\omega = \frac{95-80}{95} = \frac{15}{95} = \frac{3}{16} \approx 0{,}16 = 16\%.$$

3)

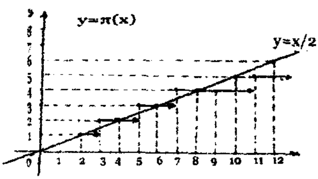
$$\pi(1000) = \frac{1000}{ln1000} = \frac{1000}{3ln10} = \frac{1000}{3 \cdot 2{,}3026} = \frac{1000}{6{,}9078} \approx 145;$$

$$\omega = \frac{168-145}{168} = \frac{23}{168} \approx 0{,}14 = 14\%.$$

4)

$$\pi(3000) = \frac{3000}{ln3000} = \frac{3000}{ln3 + ln1000} = \frac{3000}{8{,}0064} \approx 3{,}75;$$

$$\omega = \frac{427-375}{427} = \frac{52}{427} \approx 0{,}12 = 12\%.$$



78. *1-shakl.*

79. Chebishyev tengsizligidan

$$\frac{a}{lnx} < \frac{\pi(x)}{x} < \frac{6}{lnx}$$

Bu tengsizlikning ikkala tomonidan $x \to +\infty$ limitga otsak:

$$\lim_{x\to\infty} \frac{a}{lnx} = a \cdot \lim_{x\to\infty} \frac{1}{lnx} = a \cdot 0 = 0$$

va

$$\lim_{x\to\infty} \frac{b}{lnx} = 0$$

larga ega bo'lamiz. Bulardan

$$\lim_{x\to\infty} \frac{\pi(x)}{x} = 0$$

ekanligi kelib chiqadi .

Isbotlanganidan xulosa qilish mumkinmi, $\pi(x)$f unksiya x ga qaraganda sekin o'sadi. $\frac{\pi(x)}{x}$ nisbatni L.Eyler $[1, x]$ kesmadagi tub sonlarning o'rtacha zichligi deb atagan.

80. Tushunarliki $\pi(p) < p$. Bunda $-p < -\pi(p)$. Oxirgi tengsizlikning ikkala tomoniga $p\pi(p)$ ni qo'shamiz. U holda

$$p\pi(p) - p < (p-1)\pi(p)$$

hosil bo'ladi. Buni

$$\frac{\pi(p)-1}{p-1} < \frac{\pi(p)}{p}$$

ko'rinishida yozish mumkin. $\pi(p) - 1 = \pi(p-1)$ bo'lgani uchun

$$\frac{\pi(p-1)}{p-1} < \frac{\pi(p)}{p}$$

ni hosil qilamiz. m murakkab son bo'lsa, $\pi(m-1) = \pi(m)$ bo'lgani uchun

$$\frac{\pi(m-1)}{m} = \frac{\pi(m)}{m}$$

bo'ladi. Bundan

$$\frac{\pi(m)}{m} < \frac{\pi(m-1)}{m-1}$$

kelib chiqadi.

II.2-§.

81. *a)* $[-2{,}7] = -2{,}7 - \{-2{,}7\} = -2{,}7 - 0{,}3 = -3.$

b) $[2+\sqrt[3]{987}]$ hisoblang. Bu yerda $9 < \sqrt[3]{987} < 10$ bo'lgani uchun $[\sqrt[3]{987}] = 9$ va demak $[2+\sqrt[3]{987}] = 2 + [\sqrt[3]{987}] = 2 + 9 = 11.$

c) $\sqrt{21} = 4 + \alpha, 0 < \alpha < 1$ bo'lgani uchun $\left[\frac{7-\sqrt{21}}{2}\right] = \left[\frac{7-(4+\alpha)}{2}\right] = \left[\frac{3-\alpha}{2}\right] = 1$

bo'ladi.

d) $\frac{10}{3+\sqrt{3}} = \left[\frac{10(3-\sqrt{3})}{9-3}\right] = \frac{30-10\sqrt{3}}{6} = \frac{30-(17+\alpha)}{6} = \frac{13-\alpha}{6} = 2.$

e) $\left[1,(3) + 2\,\mathrm{tg}\frac{\pi}{4}\right] = [1,(3) + 2] = [1,(3)] + 2 = 1 + 2 = 3.$

i) $\left[3 + \sin\frac{13\pi}{7}\right] = \left[3 + \sin\left(2\pi - \frac{\pi}{7}\right)\right] = \left[3 - \sin\frac{\pi}{7}\right] = 3 + \left[-\sin\frac{\pi}{7}\right] = 3 - 1 = 2.$

j) $\left[3 - 2cos\frac{90\pi}{181}\right] = [3 - \alpha] = 2,$ chunki $0 < cos\frac{90\pi}{181} < \frac{1}{2}.$

f). Bu yerda $\lg 2512 = x \Rightarrow 2512 = 10^x \quad 3 < x < 4$ ya'ni $x = 3 + \alpha, 0 < \alpha < 1$ bo'lgani uchun $[2 - \log_{10} 2512] = [2 - (3 + \alpha)] = [-1 - \alpha] = -2.$

l). $log_{10}\overline{abcd} = x \Rightarrow \overline{abcd} = 10^x \Rightarrow 3 < x < 4$ bo'lgani uchun agar $\overline{abcd} > 1000$ bo'lsa, $[2 - log_{10}\overline{abcd}] = 2 - [(3+\alpha)] = 2 - 4 = -2$ va agar $\overline{abcd} = 1000$ bo'lsa, u holda $[2 - log_{10}\overline{abcd}\,] = [2 - 3] = -1;$

k) $\sqrt{30} + \sqrt[3]{10} = (5+\alpha) + (2 + \beta);\ 0 < \alpha < 0{,}5; 0 < \beta < 0{,}2.$ $[\sqrt{30} + \sqrt[3]{10}] = [7+\alpha+\beta] = 7;$ chunki $0<\alpha+\beta < 1.$

82. Berilgan tenglikning chap tomoni $[\pi]^{[e]} + [e] = 3^2 + 2 = 11$ o'ng tomoni $[e]^{[\pi]} + [\pi] = 2^3 + 3 = 8 + 3 = 11.$ Bu tengliklarning o'ng tomonlari teng, Shuning uchun ham chap tomonlari ham teng bo'lishi kerak.

83. $p = 4k + 1$ yoki $p = 4k + 3$ ko'rinishida deb olishimiz mumkin. $p = 4k + 1$ ko'rinishda bo'lsa, $\left[\frac{p}{4}\right] = \left[\frac{4k+1}{4}\right] = \left[k + \frac{1}{4}\right] = k$ va $\frac{p-1}{4} = \frac{4k+1-1}{4} = k;$ ya'ni $\frac{p}{4} = \frac{p-1}{4};$ agar $p = 4k + 3$ ko'rinishida bo'lsa, $\left[\frac{p}{4}\right] = \left[k + \frac{3}{4}\right] = k = \frac{p-3}{4}.$

84. $a=mq+r;\ 0\le r<m$ deb yozib olsak, $\left[\frac{a}{m}\right]=q+\frac{r}{m};\ 0\le\frac{r}{m}<1$ bo'ladi. Bundan $\left[\frac{a}{m}\right]=q=\frac{a-r}{m}$.

85. Berilgan munosabat $[nx]\le nx<[nx]+1, n=1,2,...$ munosabatga teng kuchli. Buning to'g'ri ekanligi esa butun qism funksiyasi ta'rifidan bevosita kelib chiqadi.

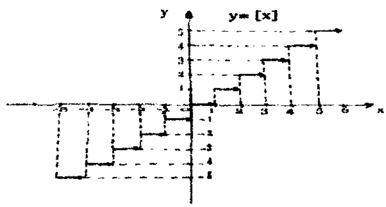
86. $\frac{x+y}{n}=\frac{x}{n}+\frac{y}{n}=\left[\frac{x}{n}\right]+\alpha+\left[\frac{y}{n}\right]+\beta;\ 0\le\alpha<1$ va $0\le\beta<1$. Bundan $\left[\frac{x+y}{n}\right]=\left[\frac{x}{n}\right]+\left[\frac{y}{n}\right]+[\alpha+\beta]$; bunda $0\le\alpha+\beta<2$. Shuning uchun ham $[\alpha+\beta]=0$ yoki 1. Birinchi holda $\left[\frac{x+y}{n}\right]=\left[\frac{x}{n}\right]+\left[\frac{y}{n}\right]$ bo'ladi. Ikkinchi holda esa $\left[\frac{x+y}{n}\right]=\left[\frac{x}{n}\right]+\left[\frac{y}{n}\right]+1$.

87. *1-usul*. m toq son bo'lsa, $m=2q+1$ deb yoza olamiz va

$$\left[\frac{m}{2}\right]=\left[\frac{2q+1}{2}\right]=\left[q+\frac{1}{2}\right]=q=\frac{m-1}{2}.$$

2-usul. $\left[\frac{m}{2}\right]=\frac{m-1}{2}$ tenglik $\frac{m-1}{2}\le\frac{m}{2}<\frac{m-1}{2}+1$ ga, ya'ni $\frac{m-1}{2}\le\frac{m}{2}<\frac{m+1}{2}$ ga teng kuchli. Bundan $\frac{m-1}{2}-\frac{m}{2}\le 0<\frac{m+1}{2}-\frac{m}{2}$ yoki $-\frac{1}{2}\le 0<\frac{1}{2}$, ya'ni $-1\le 0\le 1$, doimo bajariladigan munosabat kelib chiqadi.

88. a) $y=[x]$ ning grafigini (2-shakl) chizamiz $(0\le x<1; y=0)$; $(1\le x<2; y=1)$; $(2\le x<3; y=2)$ va xokazo $(n\le x<n+1; y=n)$. Bularni Dekart koordinatalar sistemasida tasvirlaymiz:

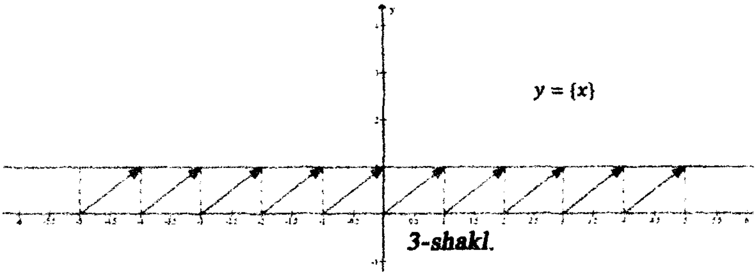


2-shakl.

b). $y = \{x\}$ ning grafigini chizmiz.

$$\begin{pmatrix} 0 \le x < 1 \\ 0 \le y < 1 \end{pmatrix}; \begin{pmatrix} 1 \le x < 2 \\ 0 \le y < 1 \end{pmatrix}; \begin{pmatrix} 2 \le x < 3 \\ 0 \le y < 1 \end{pmatrix}; \dots; \begin{pmatrix} n \le x < m \\ 0 \le y < 1 \end{pmatrix}$$

Bularni Dekart koordinatalar sistemasida tasvirlab berilgan funksiyaning grafigiga



3-shakl.

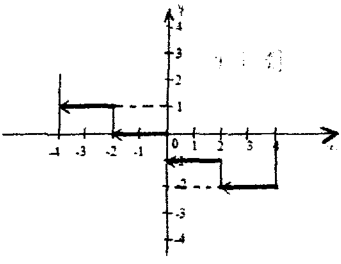
ega bo'lamiz (3-shakl).

c) $y = \left[-\frac{m}{2}\right]$ ning grafigini chizmiz.

$$\begin{pmatrix} 0 < x \le 2 \\ y = -1 \end{pmatrix}; \begin{pmatrix} -2 < x \le 0 \\ y = 0 \end{pmatrix}; \begin{pmatrix} -4 < x \le -2 \\ y = 1 \end{pmatrix}; \dots;$$

$$\begin{pmatrix} -2n < x \le -2(n-1) \\ y = n - 1 \end{pmatrix}$$

Bularni Dekart koordinatalar sistemasida tasvirlab berilgan funksiyaning grafigiga ega bo'lamiz (4-shakl).

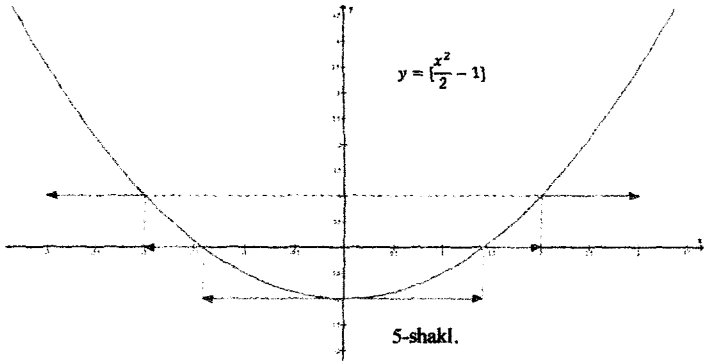


4-shakl

d) $y = \left[\frac{x^2}{2} - 1\right]$ ning grafigini chizmiz. Bu yerda $\frac{x^2}{2} - 1 = 0 \Rightarrow$ $x^2 = 2 \Rightarrow x_1 = -\sqrt{2},\ x_2 = +\sqrt{2}$. Bundan

$$\begin{pmatrix} -\sqrt{2} < x < \sqrt{2} \\ y = -1 \end{pmatrix}; \begin{pmatrix} \sqrt{2} \le x < 2 \\ y = 0 \end{pmatrix}; \begin{pmatrix} 2 \le x < \sqrt{6} \\ y = 1 \end{pmatrix}; \dots$$

$x_1 = -\sqrt{2}$, $x_2 = +\sqrt{2}$.

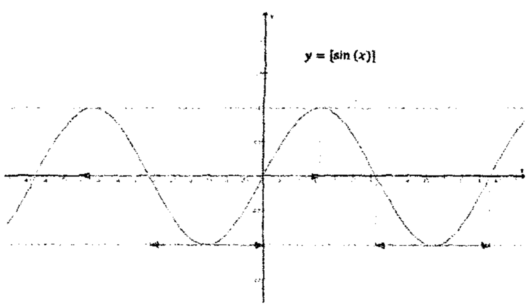


5-shakl.

e) $y = [\sin x]$. Bu yerda $y = [\sin x] =$

$$\begin{cases} 1, \quad \text{agar } x = \frac{\pi}{2} + 2\pi k, k \in Z \text{ bo'lsa;} \\ 0, \text{agar } 2\pi k \le x < \pi + 2\pi k, k \in Z \ va \ x \ne \frac{\pi}{2} + 2\pi k, k \in Z \text{ bo'lsa;} \\ -1, \quad \text{agar } \pi + 2\pi k \le x < 2\pi k, k \in Z \ va \ x \ne \frac{\pi}{2} + 2\pi k, k \in Z \text{ bo'lsa} \end{cases}$$

ekanligini e'tiborga olsak quyidagi grafikni hosil qilamiz (6-shakl).



6-shakl.

89. *a*) $[x^2] = 2 \Rightarrow 2 \le x^2 < 3 \Rightarrow \sqrt{2} \le |x| < \sqrt{3}$. Avvalo $\sqrt{2} \le |x|$ dan $x \le -\sqrt{2}, x \ge \sqrt{2}$ va $|x| < \sqrt{3}$ dan $-\sqrt{3} < x < \sqrt{3}$ ga ega bo‘lamiz. Bulardan

$-\sqrt{3} < x \le -\sqrt{2}$ va $\sqrt{2} \le x < \sqrt{3}$.

b) $[3x^2 - x] = x + 1$ dan $x + 1$ butun son bo‘lishi kerak. Buning uchun x butun bo‘lishi kerak. x butun son bo‘lsa, $3x^2 - x$ ham butun son bo‘ladi. U holda $3x^2 - x = x + 1$ tenglamaga ega bo‘lamiz. Bundan $3x^2 - 2x - 1 = 0$. Bu tenglamani yechib $x_{1,2} = \frac{1 \pm \sqrt{1+3}}{3} = \frac{1 \pm 2}{3}$ ni ya'ni $x_1 = 1$, $x_2 = -\frac{1}{3}$ larga ega bo‘lamiz. Bu yerda $x_2 = -\frac{1}{3}$ kasr son bo‘lgani uchun tenglamani qanoatlantirmaydi. Javob $x = 1$.

c) $[x] = \frac{3}{4}x \Rightarrow \frac{3}{4}x \le x < \frac{3}{4}x + 1$ va $\frac{3}{4}x$ butun son bo‘lishi kerak. Bulardan $3x \le 4x < 3x + 4 \Rightarrow 0 \le x < 4$ $x = 0, \frac{4}{3}, \frac{8}{3}$. Bundan $x = 0, \frac{4}{3}, \frac{8}{3}$ ekanligi kelib chiqadi. Demak 3ta yechimi bor.

d) $[x^2] = x \Rightarrow x \le x^2 < x + 1$ va x butun son bo‘lishi kerak ekanligi kelib chiqadi. Bulardan$0 \le x^2 - x < 1$. Bu qo‘sh tengsizlikni yechamiz.

A. $x^2 - x - 1 < 0$ tengsizlikni yechamiz. Uning o‘ng tomoni shoxlari yuqoriga qaragan parabola bo‘lgani uchun ham tengsizlikning yechimlari $x^2 - x - 1 = 0$ tenglamaning ikkala yechimlari orasidagi sonlardan iborat bo‘ladi. $x^2 - x - 1 = 0$ tenglamaning yechimlari

$$x_{1,2} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

dan iborat. Shuning uchun ham $x^2 - x - 1 < 0$ tengsizlikning yechimi $\left(\frac{1-\sqrt{5}}{2}; \frac{1+\sqrt{5}}{2}\right)$ oraliqdan iborat.

B. Endi $x^2 - x \ge 0$ tengsizlikni yechamiz. Uning o‘ng tomoni shoxlari yuqoriga qaragan parabola bo‘lgani uchun ham tengsizlikning yechimlari $]-\infty, x_1] \cup [x_2, +\infty[$ dan iborat bo‘ladi. $x^2 - x = 0$ tenglamaning yechimlari$x_1 = 0$ va $x_2 = 1$ lardan iborat. Shuning uchun ham $x^2 - x \ge 0$ tengsizlikning yechimi $]-\infty, 0] \cup [1, +\infty[$ dan iborat.

Endi qarab chiqilgan A va B hollarni birlashtirib $0 \le x^2 - x < 1$ qo‘sh tengsizlikning yechimini topamiz. U holda $x \in \left]\frac{1-\sqrt{5}}{2}, 0\right] \cup$

$\left[1,\frac{1+\sqrt{5}}{2}\right[$ va x butun son bo'lishi kerak. Demak, qaralayotgan tengsizlikning butun sonlardagi yechimlari $x = 0, 1$ dan iborat.

90. $[12{,}4m] = 87 \Rightarrow 87 \le 12{,}4m < 88 \Rightarrow \frac{870}{124} \le m < \frac{880}{124} \Rightarrow \frac{435}{62} \le m < \frac{440}{62} \Rightarrow\ 7\frac{1}{62} \le m < 7\frac{3}{31} \Rightarrow m \notin N.$

91. Agar x butun son bo'lsa, u holda $[-x] = -[x]$. Agar x kasr son bo'lsa,$[-x] = y$ deb olsak, $y < -x < y + 1$ bajarilishi kerak. Bundan $-y - 1 < x < -y$ yoki $[x] = -y - 1 = -[-x] - 1$. Shunday qilib

$$[-x] = \begin{cases} -[x]\text{ga; agar } x \text{ butun son bo'lsa;} \\ -[x] - 1 \text{ ga; agar } x \text{ kasr son bo'lsa.} \end{cases}$$

92. $x_i = [x_i] + \propto_i\quad 0 \le \propto_i < 1$ deb olsak

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} [x_i] + \sum_{i=1}^{n} \propto_i$$

bo'ladi. Bundan

$$\left[\sum_{i=1}^{n} x_i\right] = \sum_{i=1}^{n} [x_i] + \left[\sum_{i=0}^{n} \propto_i\right].$$

Bu yerda

$$\left[\sum_{i=0}^{n} \propto_i\right] \ge 0$$

bo'lgani uchun

$$\left[\sum_{i=1}^{n} x_i\right] \ge \sum_{i=1}^{n} [x_i] \qquad (*)$$

bajariladi.

93. 12-masalada $x_1 = x_2 = \dots = x_n = x$ deb olamiz. U holda (*) munosabat $[nx] \ge n[x]$ ko'rinishni oladi.

94. $[1, N]$ kesimda m soniga karrali sonlarning soni $\left[\frac{N}{m}\right]$ ga teng. Shuning uchun ham 10^6 va 10^7sonlari orasidagi 786 ga karrali natural sonlarning soni

$$\left[\frac{10^7}{786}\right] - \left[\frac{10^6}{786}\right] = \left[\frac{10000000}{786}\right] - \left[\frac{1000000}{786}\right] = 12722 - 1272 = 11450.$$

95. 1000 dan kichik natural sonlarning soni 999 ta ularning orasida 5 ga karralilari soni $\left[\frac{999}{5}\right]$ ga, 7 ga karralilari soni $\left[\frac{999}{7}\right]$ ga teng. Bu sonlar orasida 5 va7ga karralilari ham bor. Shuning uchun ham 1000 dan kichik 5ga ham 7 ga ham bo'linmaydigan natural sonlar soni $999-\left[\frac{999}{5}\right]-\left[\frac{999}{7}\right]+\left[\frac{\left[\frac{999}{5}\right]}{7}\right]=999-199-142+28=686$ ga teng.

96. $36=2^2\cdot 3^2$ bo'lgani uchun n soni 36 bilan o'zaro tub bo'lishi uchun $(n,2)=(n,3)=1$ bo'lishi kerak. Shuning uchun ham 36 soni bilan o'zaro tub 100 dan katta bo'lmagan natural sonlarning soni $100-\left[\frac{100}{2}\right]-\left[\frac{100}{3}\right]+\left[\frac{100}{6}\right]=100-50-33+16=116-83=33.$

97. Agar ko'paytmada 2 va 5 birgalikda ko'paytuvchi sifatida necha marta qatnashsa ko'paytma shuncha nol bilan tugaydi. Albatta 2017! da 2 soni 5 ga qaraganda ko'proq ko'paytuvchi sifatida qatnashadi. Shuning uchun ham masalani yechish uchun 5 ning 2017! da nechanchi daraja bilan qatnashishini aniqlash kifoya.

$$\alpha=\left[\frac{2017}{5}\right]+\left[\frac{2017}{5^2}\right]+\left[\frac{2017}{5^3}\right]+\left[\frac{2017}{5^4}\right]=403+80+16+3$$
$$=502.$$

Demak, 2017! ko'paytma 502 ta nol bilan tugaydi.

98. $N!$ ning tub ko'paytuvchilarga yoyilmasida p tub soni

$$\alpha=\left[\frac{N}{p}\right]+\left[\frac{N}{p^2}\right]+\cdots+\left[\frac{N}{p^k}\right],\quad p^k\le N$$

daraja bilan qatnashadi $p^n=N$ deb olsak,

$$\alpha=\left[\frac{p^n}{p}\right]+\left[\frac{p^n}{p^2}\right]+\cdots+\left[\frac{p^n}{p^n}\right]=p^{n-1}+p^{n-2}+\cdots+p+1=\frac{(1-p^n)}{1-p}$$
$$=\frac{p^n-1}{p-1}$$

hosil boladi.

99. $6=3\cdot 2$ bo'lgani uchun 100! ko'paytmada 6 ning qaysi daraja bilan qatnashishini aniqlash uchun 3 ning qaysi daraja bilan qatnashishini aniqlash kifoya.

$$\alpha=\left[\frac{100}{3}\right]+\left[\frac{100}{9}\right]+\left[\frac{100}{27}\right]+\left[\frac{100}{81}\right]=33+11+3+1=48.$$

Demak,100! ko'paytmada 6 soni 48-daraja bilan qatnashadi.

100. Ma'lumki, $n!$ sonining kanonik yoyilmasi $n! = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdots p_k^{\alpha_k}$ ko'rinishida bo'lib, bu yerda p_i lar tub sonlar, α_i lar esa p_i tub sonining $n!$ sonida qanday daraja qatnashishini bildiradi va

$$\alpha = \left[\frac{N}{p}\right] + \left[\frac{N}{p^2}\right] + \cdots + \left[\frac{N}{p^n}\right]$$

ko'rinishda topiladi. Demak ,

$$\alpha_1 = \left[\frac{11}{2}\right] + \left[\frac{11}{2^2}\right] + \left[\frac{11}{2^3}\right] = 5 + 2 + 1 = 8;$$

$$\alpha_2 = \left[\frac{11}{3}\right] + \left[\frac{11}{3^2}\right] = 3 + 1 = 4;$$

$$\alpha_3 = \left[\frac{11}{5}\right] = 2; \quad \alpha_4 = \left[\frac{11}{7}\right] = 1; \quad \alpha_5 = \left[\frac{11}{11}\right] = 1$$

bo'lgani uchun $11! = 2^8 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11$.

101. Avvalo berilgan N sonining ko'rinishini $N = \frac{1000!}{100! \cdot 7^\alpha}$ shaklda yozib olamiz va bu yerda N soni butun son bo'lishligi uchun $1000!$ ning kanonik yoyilmasi tarkibida 7 tub soni qanday daraja k bilan qatnashishini aniqlashimiz kerak: (N ning suratida)

$$k = \left[\frac{1000}{7}\right] + \left[\frac{1000}{7^2}\right] + \left[\frac{1000}{7^3}\right] = 142 + 20 + 2 = 164;$$

(N ning maxrajida)

$$l = \left[\frac{100}{7}\right] + \left[\frac{100}{7^2}\right] + \alpha = 14 + 2 + \alpha = 16 + \alpha.$$

Bularga asosan $N = \frac{7^{164} \cdot Q}{7^{16+\alpha}} = 7^{148-\alpha} \cdot Q$. Bu yerda Q natural son va $(Q, 7) = 1$. Bundan $148 - \alpha \geq 0 \Rightarrow 0 \leq \alpha \leq 148$. Demak, α ning eng katta qiymati 148 ga teng.

102. Ma'lumki, $(2m)!! = m! \cdot 2^m$. Bundan, agar $p = 2$ bo'lsa, u holda $2^k \leq m < 2^{k+1}$ bo'lgani uchun, izlangan daraja ko'rsatkich $m + \sum_{i=1}^{k} \left[\frac{m}{2^i}\right]$ ga teng bo'ladi. Agar $p > 2$ bo'lsa , u holda izlangan daraja ko'rsatkich $\sum_{i=1}^{s} \left[\frac{m}{p^i}\right]$ bu yerda $p^s \leq m < p^{s+1}$.

103. Berilgan tenglama avvalo ko'rinishida $[x]=1+2\left[\frac{x}{2}\right]$ yozib olamiz, , agar bu tenglamaning chap tomonini y belgilasak u holda quyidagiga ega bo'lamiz:

$$\begin{cases} y=[x] \\ y=1+2\left[\frac{x}{2}\right]. \end{cases}$$

Bundan esa

$$\begin{cases} y=[x] \\ \frac{y-1}{2}=1+2\left[\frac{x}{2}\right] \end{cases}$$

sistemani hosil qilamiz. $\frac{y-1}{2}$ ning butun qiymatlarini m belgilab

$$\begin{cases} 2m+1=[x] \\ m=\left[\frac{x}{2}\right] \end{cases} \text{ yoki } \begin{cases} 2m+1\le x<2m+2 \\ 2m\le x<2m+2 \end{cases} \text{ ni topamiz.}$$

Bu yerdan $2m+1\le x<2m+2,\quad m=0,\pm1,\ \pm2,...$ ni hosil qilamiz.

104. $y=ax^2+bx+c$ funksiya va demak $y=\left[ax^2+bx+c\right]$ funksiya $a>0$ bo'lganda quyidan va $a<0$da yuqorida chegaralangan . Ikkala holda ham $y=\left[ax^2+bx+c\right]=\left[a\left(\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4añ}{4a^2}\right)\right]$ funksiyaning qiymatlarining aniq chegarasi $\left[-\frac{b^2-4ac}{4a}\right]$ sondan iborat bo'ladi. Shuning учун $a>0$ bo'lganda berilgan tenglama $\left[-\frac{b^2-4ac}{4a}\right]\le d$ bo'lganda va faqat shu holda yechimga ega, agarda $a<0$ bo'lsa u holda $\left[-\frac{b^2-4ac}{4a}\right]\le d$ bo'lsa yechim mavjud bo'ladi.

105. Har bir $x=k\ (a\le k\le b)$ butun abssissaga egri chiziqli trapetsiya ichidagi va chegarasidagi $[f(x)]+1$ ta butun ordinata mos keladi. Shuning uchun ham izlanayotgan nuqtalar soni

$$\sum_{k=a}^{b}([f(k)]+1)$$

ga teng.

106. Buning uchun avvalo 1-chorakdagi shu aylana ichidagi butun nuqtalar sonini aniqlaymiz. Aylana tenglamasini y ga nisbatan yechib 1-chorakga mos qismi $y=\sqrt{6{,}5^2-k^2}$ni olib 25-misolni tatbiq etamiz. U holda $\sum_{k=0}^{6}([\sqrt{6{,}5^2-k^2}]+1)=7+7+7+6+6+5+3=41$ hosil bo'ladi. Demak, izlnayotgan nuqtalar soni $N=4\cdot 41-4\cdot 7=164-28=136$ ta.

107. n dan katta bo'lmagan va $p_1,p_2,\dots,p_k$ tub sonlarning har biri bilan o'zaro tub bo'lgan sonlarning soni $B(n;p_1;p_2;\dots;p_k)=[n]-\left[\frac{n}{p_1}\right]-\left[\frac{n}{p_2}\right]-\dots-\left[\frac{n}{p_k}\right]+\left[\frac{n}{p_1p_2}\right]+\dots+\left[\frac{n}{p_{k1}p_k}\right]-\left[\frac{n}{p_1p_2p_3}\right]-\dots-\left[\frac{n}{p_{k-2}p_{k-1}p_k}\right]+\dots+(-1)^k\left[\frac{n}{p_1p_2\cdots p_k}\right]$ formula bilan topiladi. Shunga asosan $1575=3^2\cdot 5^2\cdot 7$ bo'lgani uchun $12317-\left[\frac{12317}{3}\right]-\left[\frac{12317}{5}\right]-\left[\frac{12317}{7}\right]+\left[\frac{12317}{15}\right]+\left[\frac{12317}{21}\right]+\left[\frac{12317}{35}\right]-\left[\frac{12317}{105}\right]=12317-4105-2463-1759+821+586+351-117=5631.$

II.3-§.

108.1). Bu yerda $375=3\cdot 5^3$ bo'lgani uchun (1) va (2)-formulalardan

$$\tau(375)=\tau(3\cdot 5^3)=(1+1)(3+1)=8;$$
$$\sigma(375)=\frac{3^2-1}{3-1}\cdot\frac{5^4-1}{5-1}=4\cdot\frac{624}{4}=624$$

larga ega bo'lamiz.

2).$720=2^4\cdot 3^2\cdot 5$ bo'lgani uchun(1) va (2)- formulalardan

$$\tau(720)=5\cdot 3\cdot 2=30;$$
$$\sigma(720)=\frac{2^5-1}{2-1}\cdot\frac{3^3-1}{3-1}\cdot\frac{5^2-1}{5-1}=31\cdot 13\cdot 6=31\cdot 78=2418$$

lar kelib chiqadi.

3). Bu yerda $957=3\cdot 11\cdot 29$ bo'lgani uchun (1) va (2)-formulalardan

$$\tau(957)=(1+1)(1+1)(1+1)=8;$$

$$\sigma(957)=\frac{3^2-1}{3-1}\cdot\frac{11^2-1}{11-1}\cdot\frac{29^2-1}{29-1}=4\cdot12\cdot30=48\cdot30=1440$$

lar kelib chiqadi.

4).$988=2^2\cdot13\cdot19$ bo'lgani uchun(1) va (2)- formulalardan

$$\tau(988)=3\cdot2\cdot2=12;$$

$$\sigma(988)=\frac{2^3-1}{2-1}\cdot\frac{13^3-1}{13-1}\cdot\frac{19^2-1}{19-1}=7\cdot14\cdot20=1960.$$

5).$990=2\cdot3^2\cdot5\cdot11;\quad \tau(990)=2\cdot3\cdot2\cdot2=24,$

$$\sigma(990)=\frac{2^2-1}{2-1}\cdot\frac{3^3-1}{3-1}\cdot\frac{5^2-1}{5-1}=3\cdot13\cdot6\cdot12=2808.$$

6).$1200=2^4\cdot3\cdot5^2;\ \tau(1200)=5\cdot2\cdot3=30,$

$$\sigma(1200)=\frac{2^5-1}{2-1}\cdot\frac{3^2-1}{3-1}\cdot\frac{5^3-1}{5-1}=31\cdot4\cdot31=3844.$$

7). $1440=2^5\cdot3^2\cdot5;\ \tau(1440)=6\cdot3\cdot2=36,$

$$\sigma(1440)=\frac{2^6-1}{2-1}\cdot\frac{3^3-1}{3-1}\cdot\frac{5^2-1}{5-1}=63\cdot13\cdot6=4914.$$

8). $1500=2^2\cdot3\cdot5^3;\ \tau(1500)=3\cdot2\cdot4=24,$

$$\sigma(1500)=\frac{2^3-1}{2-1}\cdot\frac{3^2-1}{3-1}\cdot\frac{5^4-1}{5-1}=7\cdot4\cdot156=4368.$$

9). $1890=2\cdot3^3\cdot5\cdot7;\tau(1890)=2\cdot4\cdot2\cdot2=32,$

$$\sigma(1890)=\frac{2^2-1}{2-1}\cdot\frac{3^4-1}{3-1}\cdot\frac{5^2-1}{5-1}\cdot\frac{7^2-1}{7-1}=3\cdot40\cdot6\cdot8=5760.$$

10). $4320=2^5\cdot3^3\cdot5;\ \tau(4320)=6\cdot4\cdot2=48,$

$$\sigma(4320)=\frac{2^6-1}{2-1}\cdot\frac{3^4-1}{3-1}\cdot\frac{5^2-1}{5-1}=63\cdot40\cdot6=15120.$$

109. 1). $360=2^3\cdot3^2\cdot5,\ d=2^\alpha\cdot3^\beta\cdot5^\gamma,\ 0\le\alpha\le3,\ 0\le\beta\le2, 0\le\gamma\le1$. Shuning uchun ham

$(1+2+4+8)(1+3+9)(1+5)=(1+2+4+8)(1+3+9+5+15+45)=1+3+9+5+15+45+2+6+18+10+30+90+4+12+36+20+60+180+8+24+72+40+120+360;$

Bo'luvchilar:

$1,2,3,4,5,6,8,9,10,12,15,18,20,24,30,36,40,45,60,72,90,$
$120,180,360$.

Ularning jami soni $24\ ta$.

2).$720 = 2^4 \cdot 3^2 \cdot 5, \quad (1+2+4+8+16)(1+3+9)(1+5)$
$= (1+2+4+8+16)(1+3+9+5+15+45)$
$= 1+2+4+8+16+3+6+12+24+48+9+18$
$+ 36+72+144+5+10+20+40+80+15+30$
$+ 60+120+240+360+720.$

Bo'luchilar $1,2,3,4,5,6,8,9,10,12,15,16,18,20,24,30,$
$36,40,45,48,60,72,80,90,120,144,180,240,360,720. \textit{Jami}: 30ta.$

3).$954 = 2 \cdot 3^2 \cdot 53, \quad (1+2)(1+3+9)(1+53)$
$= (1+2)(1+3+9+53+159+447)$
$= 1+3+9+53+159+447+2+6+18+106+318$
$+ 954.$

Bo'luchilar:
$1,2,3,6,9,18,53,106,159,318,477,954.$ *Jami*: 12 *ta* .

4).$988 = 2^2 \cdot 13 \cdot 19, \quad (1+2+4)(1+13)(1+19)$
$= (1+2+4)(1+13+19+247)$
$= 1+13+19+247+2+26+38+494+4+52+76$
$+ 988$

Bo'luvchilar: $1,2,4,13,19,26,38,52,76,247,494,988.$
Jami: 12 *ta*.

5).$600 = 2^3 \cdot 3 \cdot 5^2, \quad (1+2+4+8)(1+3)(1+5+25) =$
$(1+3+5+15+25+75)(1+2+4+8) = 1+3+5+15+$
$25+75+2+6+10+30+50+150+4+12+20+60+100+$
$300+8+24+40+120+200+600$

Bo'luvchilar:
$1,2,3,4,5,6,8,10,12,15,20,24,25,30,40,50,60,75,100,$
$120,150,200,300,600.$ *Jami*: 24 *ta*.

110. $\tau(x) = 6, \ \sigma(x) = 28, \ x = p_1{}^{\alpha} \cdot p_2{}^{\beta} \quad \alpha \geq 1, \ \beta \geq 1$ bo'lgani uchun $\tau(x) = (\alpha+1)(\beta+1) = 6,$ bundan $\alpha = 1, \beta = 2$ va $x = p_1 \cdot p_2{}^2$.

Bu holda

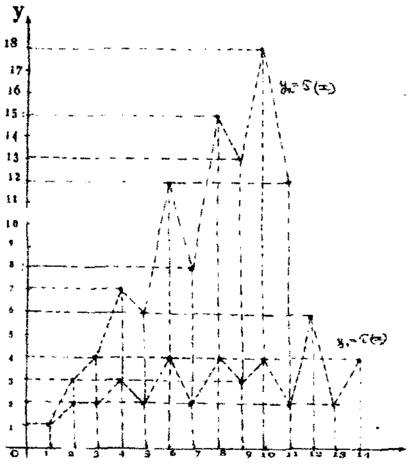
$\sigma(x) = (p_1+1)\frac{p_2{}^3-1}{p_2-1} = (p_1+1)(p_2{}^2+p_2+1) = (p_1+1)(p_2(p_2+1)+1) = 28,$ bu yerda $p_2(p_2+1)$ juft son bo'lgani uchun $p_2(p_2+1)+1$ toq son, shuning uchun ham $p_1+1=4, \ p_2(p_2+1)+1=7, \ p_1=3, \ p_2(p_2+1)=6, \ p_2=2$ demak, $x = p_1p_2{}^2 = 3 \cdot 4 = 12.$

111. $N = p^{\alpha} \cdot q^{\beta}, \quad N^2 = p^{2\alpha} \cdot q^{2\beta}, \quad N^3 = p^{3\alpha} \cdot q^{3\beta}$. Bulardan $\tau(N^2) = (2\alpha + 1)(2\beta + 1) = 15 = 3 \cdot 5$, bundan $\alpha = 1, \ \beta = 2$ (yoki $\alpha = 2; \ \beta = 1$). $\tau(N^3) = (3\alpha + 1)(3\beta + 1) = 4 \cdot 7 = 28$ ta.

112. $\tau(x)$ va $\sigma(x)$ larning grafigini sxematik tasvirlang. Buning uchun berilgan funksiyalarning qiymatlari jadvalini tuzib olamiz:

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\tau(x)$	1	2	2	3	2	4	2	4	3	4	2	6	3	4	4	5
$\sigma(x)$	1	3	4	7	6	12	8	15	13	18	12	28	14	24	24	31

Bu qiymatlarni Dekart koordinatalar sistemasida belgilab quyidagi grafiklarga



7-shakl.

ega bo'lamiz (7-shakl).

113. $p_2 - p_1 = 2, \ p_1 = p_2 - 2, \ \sigma(p_1) = \frac{p_1^{\,2}-1}{p_1-1} = p_1 + 1 = 1 + (p_2 - 2) = p_2 - 1 = \varphi(p_2)$.

114. $m = 2^{\alpha}$ bo'lsin. U holda $\sigma(m) = \frac{2^{\alpha+1}-1}{2-1}$ bo'lgani uchun tenglama $2^{\alpha+1} - 1 = 2 \cdot 2^{\alpha} - 1 = 2^{\alpha+1} - 1$, ya'ni $m = 2^{\alpha}$ $(\alpha =$

$0,1,2,3,\ldots$) ko'rinishdagi sonlar berilgan tenglamaning yechimi, $\alpha = 0,1,2,3,\ldots$ qiymatlar bersak m ning cheksiz ko'p natural qiymatlari hosil bo'ladi.

115. 1). Agar p tub soni m yoki n ning kanonik yoyilmasiga biror α daraja ko'rsatkichi bilan kirsa, u holda $\tau(mn)$ da ham, shuningdek $\tau(m)\tau(n)$ da ham $(\alpha+1)$ ko'paytuvchi qatnashadi. Agarda m va n larning kanonik yoyilmasida mos ravishda p^{α}va p^{β} lar qatnashsa, u holda mn ning kanonik yoyilmasida $p^{\alpha+\beta}$ishtirok etadi. Bu holda $\tau(mn)$da $\alpha+\beta+1$ ko'paytuvchi qatnashadi. $\alpha+\beta+1<(\alpha+1)(\beta+1)$bo'lgani uchun $\tau(m)\tau(n)>\tau(mn)$ bo'ladi, ya'ni agar $(m,n)>1$ bo'lsa, $\tau(m)\tau(n)>\tau(mn)$ bo'lar ekan.

2). Agar p tub soni m yoki n ning kanonik yoyilmasiga biror α daraja ko'rsatkichi bilan kirsa, u holda $\sigma(mn)$ da ham, shuningdek $\sigma(m)\sigma(n)$ da ham $\frac{p^{\alpha+1}-1}{p-1}$ ko'paytuvchi qatnashadi.

Agar m va n larning kanonik yoyilmasiga mos ravishda p^{α} va p^{β} lar tegishli bo'lsa, u holda mn ning kanonik yoyilmasida $p^{\alpha+\beta}$ qatnashadi. Bu holda $\sigma(mn)$ ning tarkibida qatnashuvchi $\frac{p^{\alpha+\beta+1}-1}{p-1}$ ko'paytuvchiga, $\sigma(m)\cdot\sigma(n)$ ning tarkibidagi

$$\frac{p^{\alpha+1}-1}{p-1}\cdot\frac{p^{\beta+1}-1}{p-1}=\frac{p^{\alpha+\beta+2}-p^{\alpha+1}-p^{\beta+1}+1}{(p-1)^2}$$

ko'paytma mos keladi. Bu yerda

$$\frac{p^{\alpha+\beta+2}-p^{\alpha+1}-p^{\beta+1}+1}{(p-1)}-\left(p^{\alpha+\beta+1}-1\right)$$
$$=\frac{p^{\alpha+\beta+2}-p^{\alpha+1}-p^{\beta+1}+1-p^{\alpha+\beta+2}+p^{\alpha+\beta+1}-1}{(p-1)}=$$
$$\frac{p(1-p^{\alpha})-p^{\beta}+p^{\alpha+\beta}}{p-1}=\frac{p(p^{\alpha}-1)(p^{\beta}-1)}{p-1}>0$$

Ya'ni

$$\frac{p^{\alpha+1}-1}{p-1}\cdot\frac{p^{\beta+1}-1}{p-1}>\frac{p^{\alpha+\beta+1}-1}{p-1}.$$

Demak, agar $(m,n)>1$bo'lsa, u holda $\sigma(m)\sigma(n)>\sigma(mn)$ bo'ladi.

116. m ning barcha natural bo'luvchilari $d_1, d_2, \dots, d_{\tau(m)}$ bo'lsin, u holda biz

$$\delta(m) = \prod_{i=1}^{\tau(m)} d_i$$

uchun formula chiqarishimiz kerak. Bunda $\frac{m}{d_1}, \frac{m}{d_2}, \dots \frac{m}{d_{\tau(m)}}$ lar ham m ning barcha bo'luvchilari bo'lgani uchun

$$\delta(m) = \prod_{i=1}^{\tau(m)} \frac{m}{d_i} = m^{\tau(m)} \prod_{i=1}^{\tau(m)} \frac{1}{d_i} = m^{\tau(m)} \cdot \frac{1}{\prod_{i=1}^{\tau(m)} d_i} = \frac{m^{\tau(m)}}{\delta(m)}.$$

Bunda $\delta^2(m) = m^{\tau(m)}$, yoki $\delta(m) = \sqrt{m^{\tau(m)}}$. Xususiy holda $\delta(10) = \sqrt{10^{\tau(10)}} = \sqrt{10^4} = 10^2 = 100.$

117. Masalaning shartiga asosan $m = \sqrt{m^{\tau(m)}}$, bundan $\tau(m) = 2$, ya`ni m natural soni faqat 2ta bo'luvchiga ega bo'lishi kerak, demak, u tub son bo'lishi kerak. Shunday qilib, o'zining barcha natural bo'luvchilari ko'paytmasiga teng bo'lgan sonlar natural sonlar to'plami tub sonlar to'plami bilan ustma-ust tushadi.

118. n ning kanonik yoyilmasi $n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \dots p_k^{\alpha_k}$ bo'lsin. U holda $\sigma_k(n) = (1 + p_1^k + p_1^{2k} + \dots + p_1^{\alpha_1 k})(1 + p_2^k + p_2^{2k} + \dots + p_2^{\alpha_2 k}) \dots (1 + p_s^k + p_s^{2k} + \dots + p_s^{\alpha_s k}) = \frac{p_1^{k(\alpha_1+1)}-1}{p_1^k-1} \cdot \frac{p_2^{k(\alpha_2+1)}-1}{p_2^k-1} \dots \frac{p_s^{k(\alpha_s+1)}-1}{p_s^k-1}$, ya'ni

$$\sigma_k(n) = \prod_{i=1}^{s} \frac{p^{k(\alpha_i+1)} - 1}{p_i^k - 1}.$$

Tushunarliki, $\sigma_0(n) = \tau(n)$, $\sigma_1(n) = \sigma(n)$.

119. 1).$\sigma_2(12) = \sigma_2(2^2 \cdot 3) = \frac{2^{2\cdot3}-1}{2^2-1} \cdot \frac{3^{2\cdot2}-1}{3^2-1} = \frac{63}{3} \cdot \frac{80}{8} = 210.$

2).$\sigma_2(18) = \sigma_2(2 \cdot 3^2) = \frac{2^{2\cdot2}-1}{2^2-1} \cdot \frac{3^{2\cdot3}-1}{3^2-1} = \frac{15\cdot728}{3\cdot8} = 5 \cdot 91 = 455.$

3).$\sigma_3(36) = \sigma_3(2^2 \cdot 3^2) = \frac{2^{3\cdot3}}{2^3-1} \cdot \frac{3^{3\cdot3}-1}{3^2-1} = \frac{511}{7} \cdot 91 = 73 \cdot 91 =$ 6643.

4).$\sigma_2(16) = \sigma_2(2^4) = \frac{2^{2\cdot5}-1}{2^2-1} = \frac{1023}{3} = 341.$

5). $\sigma_3(8) = \sigma_3(2^3) = \frac{2^{3\cdot 4}-1}{2^3-1} = \frac{4095}{7} = 585.$

120. 1). $\sigma(28) = \sigma(2^2 \cdot 7) = \frac{2^3-1}{2-1} \cdot \frac{4^2-1}{7-1} = 7 \cdot \frac{48}{6} = 7 \cdot 8 = 56 = 2 \cdot 28.$

Ya'ni $n = 28$ da $\sigma(n) = 2n$ tenglik o'rinli. Shuning uchin ham $n = 28$ – mukammal son.

2). $\sigma(469) = \sigma(2^4 \cdot 31) = \frac{2^5-1}{2-1} \cdot \frac{31^2-1}{31-1} = 31 \cdot 32 = 992 = 2 \cdot 496.$

3). $\sigma(8128) = \sigma(2^6 \cdot 127) = \frac{2^7-1}{2-1} \cdot \frac{127^2-1}{127-1} = 127 \cdot 128 = 16256 = 2 \cdot 8128.$

121. $\sigma(N) = \sigma(p^n) = \frac{p^{n+1}-1}{p-1} = p^n + (p^{n-1} + p^{n-2} + \dots + p + 1) = p^n + \frac{(1-p^n)}{1-p} = p^n + \frac{p^n-1}{p-1} < 2p^n = 2 \cdot N,$ ya`ni $\sigma(n) < 2N.$

122. $\sigma(N) = \sigma\left(p^\alpha \cdot q^\beta\right) = \frac{p^{\alpha+1}-1}{p-1} \cdot \frac{q^{\beta+1}-1}{q-1} < \frac{p^{\sigma+1}}{p-1} \cdot \frac{q^{\beta+1}}{q-1} = N \cdot \frac{p}{p-1} \cdot \frac{q}{q-1}$

Shart bo'yicha $p \geq 3, q \geq 5.$ Shuning uchin ham $\sigma(N) < \frac{3}{2} \cdot \frac{5}{4} N = \frac{15}{8} N < 2N.$

Demak $\sigma(N) < 2N.$

123. 1). Shartga ko'ra $\delta(n) = 5832,$ 9-masalada istalgan formulaga asosan $\delta(n) = \sqrt{n^{\tau(n)}} = 5832 = 2^3 \cdot 3^6.$

Demak $n = 2^\alpha \cdot 3^\beta$ ko'rinishda bo'lishi kerak. Bulardan $\sqrt{(2^\alpha \cdot 3^\beta)^{\tau(2^\alpha \cdot 3^\beta)}} = (2^\alpha \cdot 3^\beta)^{\frac{(\alpha+1)(\beta+1)}{2}} = 2^3 \cdot 3^6,$ ya`ni

$$2^{\frac{\alpha(\alpha+1)(\beta+1)}{2}} = 2^3, 3^{\frac{\beta(\alpha+1)(\beta+1)}{2}} = 3^6$$

ga ega bo'lamiz. Bularga asosan $\alpha(\alpha + 1)(\beta + 1) = 6,$ $\beta(\alpha + 1)(\beta + 1) = 12$ bundan

$$\begin{cases} \alpha(\alpha + 1)(\beta + 1) = 1 \cdot 3 \cdot 3 \\ (\alpha + 1)\beta(\beta + 1) = 2 \cdot 2 \cdot 3 \end{cases} \Rightarrow \alpha = 1, \ \beta = 2 \text{ ekanligi kelib}$$

chiqadi va $n = 2 \cdot 3^2 = 18$ hosil bo'ladi.

2). Shartga ko'ra $\sqrt{n^{\tau(n)}} = 3^{30} \cdot 5^{40}$, n ni $n = 3^\alpha \cdot 5^\beta$ ko'rinishda izlaymiz. U holda

$$(3^\alpha \cdot 5^\beta)^{\frac{(\alpha+1)(\beta+1)}{2}} = 3^{30} \cdot 5^{40}$$

ga ega bo'lamiz. Bundan $\alpha(\alpha + 1)(\beta + 1) = 60;$ $(\alpha + 1)\beta(\beta + 1) = 80.$

Buni quyidagicha yozib olish mumkin:

$$\begin{cases}\alpha(\alpha+1)(\beta+1)=3\cdot4\cdot5\\ \beta(\alpha+1)(\beta+1)=4\cdot4\cdot5\end{cases}\Rightarrow \alpha=3,\qquad \beta=4.$$

Demak $n=3^3\cdot5^4=27\cdot625=16875.$

124. N sonining barcha bo'luvchilarini o'sib borish tartibida joylashtirib chiqamiz: $1, d_1, d_2, \dots, \frac{N}{d_2}, \frac{N}{d_1}, \frac{N}{1}$. Bularning soni $(\alpha_1+1)(\alpha_2+1)\dots(\alpha_k+1)$ ta. Bularni 2 tadan olib $1\cdot\frac{N}{1}, d_1\cdot\frac{N}{d_1}, d_2\cdot\frac{N}{d_2}, \dots$ N ning barcha 2 ta ko'paytuvchi ko'rinishida ifodalanishlariga ega bo'lamiz. Ularning son $\frac{(\alpha_1+1)(\alpha_2+1)\dots(\alpha_k+1)}{2}$ ga teng, agar N to'liq kvadrat bo'lmasa va $\frac{(\alpha_1+1)(\alpha_2+1)\dots(\alpha_k+1)+1}{2}$ ga teng, agar N to'liq kvadrat bo'lsa. Bularni birlashtirsak N ni 2 ta ko'paytuvchi ko'rinishda ifodalashlar soni $\left[\frac{1+(\alpha_1+1)(\alpha_2+1)\dots(\alpha_k+1)}{2}\right]$ ga teng degan xulosaga kelamiz.

125. Bizda $N=2^\alpha\cdot3^\beta\cdot7^\gamma$. Bundan

$$\tau(N)=(\alpha+1)(\beta+1)(\gamma+1);$$
$$\tau(5N)=(\alpha+1)(\beta+2)(\gamma+1)=(\alpha+1)(\beta+1)(\gamma+1)+8;$$
$$\tau(7N)=(\alpha+1)(\beta+1)(\gamma+2)=(\alpha+1)(\beta+1)(\gamma+1)+12;$$
$$\tau(8N)=(\alpha+4)(\beta+1)(\gamma+1)=(\alpha+1)(\beta+1)(\gamma+1)+18.$$

Bulardan $(\alpha+1)(\gamma+1)(\beta+2-\beta-1)=8$, ya`ni

$$\begin{cases}(\alpha+1)(\gamma+1)=8\\ (\alpha+1)(\beta+1)=12\\ (\beta+1)(\gamma+1)=6\end{cases}\quad \text{ga ega bo'lamiz.}$$

$$(\alpha+1)(\beta+1)(\gamma+1)=\sqrt{8\cdot12\cdot6}=\sqrt{16\cdot36}=4\cdot6=4\cdot3\cdot2.$$

Bulardan $(\alpha+1)=4,\ \ \alpha=3; (\beta+1)=3,\ \ \beta=2; (\gamma+1)=2,\ \gamma=1$

va $N=2^3\cdot5^2\cdot7=1400.$

126. Masalaning sharti bo'yicha $N=2^x\cdot3^y\cdot5^z$ va $\frac{N}{2}=2^{x-1}\cdot3^y\cdot5^z$, $\frac{N}{3}=2^x\cdot3^{y-1}\cdot5^z$, $\frac{N}{5}=2^x\cdot3^y\cdot5^{z-y}$. Bulardan

$$\begin{cases} \tau\left(\frac{N}{2}\right) = \tau(N) - 30 \\ \tau\left(\frac{N}{3}\right) = \tau(N) - 35 \\ \tau\left(\frac{N}{5}\right) = \tau(N) - 42 \end{cases}$$

$$\Rightarrow \begin{cases} x(y+1)(z+1) = (x+1)(y+1)(z+1) - 30 \\ y(x+1)(z+1) = (x+1)(y+1)(z+1) - 35 \\ z(x+1)(y+1) = (x+1)(y+1)(z+1) - 42. \end{cases}$$

Oxirgi sistemani quyidagicha yozib olish mumkin:

$$\begin{cases} (y+1)(z+1) = 30 \\ (x+1)(z+1) = 35 \\ (x+1)(y+1) = 42. \end{cases}$$

Buni tanlash usuli bilan yechamiz:$(x+1)^2(y+1)^2(z+1)^2 = 2^2 \cdot 3^2 \cdot 5^2 \cdot 7^2 \Rightarrow (x+1)(y+1)(z+1) = 2 \cdot 3 \cdot 5 \cdot 7$,bu yerda $(x+1)(y+1) = 42$ bo'lishi kerak, shuning uchun $x+1=7$ $y+1=6$, u holda $(x+1)(z+1) = 35$ dan $(z+1) = 5$ kelib chiqadi va bu yechimlar $(y+1)(z+1) = 30$ tenglamani qanoatlantiradi. Shunday qilib $x=6$, $y=5$, $z=4$va$N = 2^6 \cdot 3^5 \cdot 5^4 = 64 \cdot 243 \cdot 625 = 9720000$.

127. $2^{\alpha+1} - 1$ tub son bo'lsin, u holda $N = 2^\alpha(2^{\alpha+1} - 1)$ ning mukammal son ekanligini ko'rsatamiz. $N = 2^{\alpha+1} - 1 = p$ deb olsak

$\sigma(N) = \sigma(2^\alpha \cdot p) = \frac{2^{\alpha+1}-1}{2-1} \cdot \frac{p^2-1}{p-1} = (2^{\alpha+1} - 1)(p+1) = (2^{\alpha+1} - 1)(2^{\alpha+1}) = 2N$, ya`ni N –mukammal son.

128. Buni isbotlash uchun har qanday juft mukammal sonning $2^\alpha(2^{\alpha+1} - 1)$ ko'rinishida ifodalanishini ko'rsatish yetarli. Bunda $2^{\alpha+1} - 1$ tub son. Faraz qilaylik $N = 2^\alpha \cdot q, (q; 2) = 1$ juft son mukammal son bo'lsin, ya`ni u uchun

$\sigma(N) = 2N$ tenglik bajarilsin. Bundan $\sigma(2^\alpha q) = 2^{\alpha+1} q$ yoki

$$\frac{2^{\alpha+1} - 1}{2-1}\sigma(q) = 2^{\alpha+1} q.$$

Bu yerdan $\sigma(q) = \frac{2^{\alpha+1}}{2^{\alpha+1}-1} q$ va q soni $2^{\alpha+1} - 1$ ga bo'linishi kerak. U holda $q = (2^{\alpha+1} - 1)k$ va $\sigma(q) = 2^{\alpha+1} k$ bo'ladi. Bu yerdan k va $(2^{\alpha+1} - 1)k$ lar q ning bo'luvchilari bo'lib, ularning yig'indisi uchun

$2^{\alpha+1}k = \sigma(q)$ bajariladi. U holda q ning boshqa bo'luvchilari yo'q bo'lishi kerak. Demak, $q = (2^{\alpha+1} - 1)k$ soni tub son ekan, ya'ni $k = 1$ va $2^{\alpha+1} - 1$ tub son.

129. $N = 2^{\alpha} \cdot p_1 p_2$ deb olsak, masalaning shartiga ko'ra $\sigma(N) = \sigma(2^{\alpha} p_1 p_2) = \frac{2^{\alpha+1}-1}{2-1} \cdot \frac{p_1{}^2-1}{p_1-1} \cdot \frac{p_2{}^2-1}{p_2-1} =$

$$(2^{\alpha+1} - 1)(p_1 + 1)(p_2 + 1) = 3N = 3 \cdot 2^{\alpha} p_1 p_2, \quad (1)$$

bu yerda $p_1 > p_2$ toq sonlar.

Agar $\alpha = 0$ bo'lsa $(p_1 + 1)(p_2 + 1) = 3p_1p_2$ yoki $p_1 + p_2 + 1 = 2p_1p_2$. Bu oxirgi tenglik o'rinli emas, chunki chap toq son o'ng tomoni esa juft son. Demak, $\alpha \neq 0$ bo'lsa. $\alpha = 1$ bo'lsin. U holda $3(p_1 + 1)(p_2 + 1) = 6p_1p_2$yoki $p_1 + p_2 + 1 = p_1p_2$, ya'ni $p_1 + 1 = p_2(p_1 - 1)$. Bunda $p_1 - 1$ juft son, ya'ni $p_1 - 1 = 2n$, u holda $2n + 2 = 2np_2$ bundan $n + 1 = np_2 \to n(p_2 - 1) = 1 \to n = 1$, $p_2 = 2$. Bunday bo'lishi uchun ham mumkin emas, chunki masalaning shartida p_2 – toq tub son. Demak, $\alpha \neq 1$. $\alpha = 2$

$\alpha = 2$ bo'lsin. Bu holda (1) dan $7(p_1 + 1)(p_2 + 1) = 12p_1p_2 \to 7p_1 + 7p_2 + 7 = 5p_1p_2 \to 7(p_1 + p_2 + 1) = 5p_1p_2$. Bundan $p_1 = 7$ (yoki $p_2 = 7$) va $8 + p_1 = 5p_2 \to p_2 = 2$ yoki $(p_1 = 2)$. Bunday bo'lishi ham mumkin emas.

Demak, $\alpha \neq 2, \alpha = 3$ bo'lsin. Bu holda (1) dan $15(p_1 + 1)(p_2 + 1) = 24p_1p_2 \to 5(p_1 + 1)(p_2 + 1) = 8p_1p_2$. Bundan $5(p_1 + p_2 + 1) = 3p_1p_2vap_1 = 5$ (yoki $p_2 = 5$) hamda $6 + p_2 = 3p_2 \to p_2 = 3$ $(p_1 = 3)$. Shunday qilib, berilgan masalaning shartini qanoatlantiruvchi eng kichik natural son $N = 2^3 \cdot 5 \cdot 3 = 120$ ekan.

130. Faraz qilaylik $N = p_1{}^{\alpha_1} p_2{}^{\alpha_2} \cdots p_k{}^{\alpha_k}$ bo'lsin. U holda

$$\tau(N) = (\alpha_1 + 1)(\alpha_2 + 1) \ldots (\alpha_k + 1)$$

a). Agar $\tau(N)$ toq son bo'lsa, $(1 + \alpha_i)$ $(i = 1,2, \ldots, k)$ ko'paytuvchilarning har biri toq son bo'lishi kerak, ya'ni$\alpha_i (i = 1,2, \ldots, k)$ lar juft bo'lishi kerak. Bu esa N butun sonning to'la kvadratiga teng degani.

b). Aksincha, agar N biror sonning kvadratiga teng bo'lsa, α_i $(i = 1,2, \ldots, k)$ lar juft sonlar $\alpha_i + 1$ lar esa toq natural sonlar bo'lishi kerak. U holda $\tau(N) = \prod_{i=1}^{k}(1 + \alpha_i)$ ham toq son bo'ladi.

II.4-§.

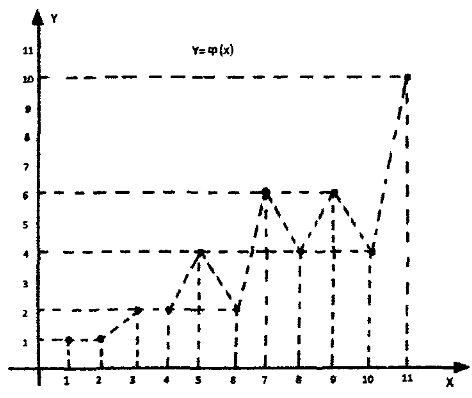
131. Eyler funksiyasi$y = \varphi(x)$ ning qiymatlari jadvalini tuzamiz.

x	1	2	3	4	5	6	7	8	9	10	11
$\varphi(x)$	1	1	2	2	4	2	6	4	6	4	10

Bu quymatlarni (nuqtalarni) Dekart koordinatalar sistemasida belgilab chiqib uzlukli chiziq bilan belgilab chiqsak, $y = \varphi(x)$ funksiyaning o'zgarishini xarakterlovchi chiziqqa ega bo'lamiz.

132. 1).$\varphi(125) = \varphi(5^3) = 5^3 - 5^2 = 100$;

2). 1000 ni tub ko'paytuvchilarga ajratib $\varphi(x)$ ning multiplikativligidan foydalanamiz.$\varphi(1000) = \varphi(2^3 \cdot 5^3) = \varphi(2^3) \cdot \varphi(5^3) = (2^3 - 2^2)(5^3 - 5^2) = 4 \cdot 100 = 400$;



8-shakl.

3).$\varphi(180) = \varphi(18 \cdot 10) = \varphi(2^2 \cdot 3^2 \cdot 5) = \varphi(2^2) \cdot \varphi(3^2) \cdot \varphi(5) = (2^2 - 2)\cdot(3^2 - 3)(5 - 1) = 2 \cdot 6 \cdot 4 = 48$;

4).$\varphi(360) = \varphi(2^3 \cdot 3^2 \cdot 5) = (2^3 - 2^2)(3^2 - 3)(5 - 1) = 4 \cdot 6 \cdot 4 = 64$;

5).$\varphi(1440) = \varphi(12^2 \cdot 10) = 4(2^5 \cdot 3^2 \cdot 5) = (2^5 - 2^4)(3^2 - 3)(5 - 1) = 16 \cdot 6 \cdot 4 = 384$;

6).$\varphi(1890) = \varphi(2)\varphi(3^3)\varphi(5)\varphi(7) = (2 - 1)(3^3 - 3^2) \cdot 4 \cdot 6 = 18 \cdot 24 = 432$;

7). $\varphi(11^3) = 11^3 - 11^2 = 121 \cdot 11 = 1331$;

8). $\varphi(23^2) = 23^2 - 23 = 506$;

9).$\varphi(12 \cdot 17) = \varphi(12) \cdot \varphi(17) = \varphi(2^2 \cdot 3) \cdot 16 = 16 \cdot (2^2 - 2) \cdot 2 = 32 \cdot 2 = 64$;

10).$\varphi(24 \cdot 28 \cdot 45) = \varphi(2^3 \cdot 3 \cdot 2^2 \cdot 7 \cdot 3^2 \cdot 5) = \varphi(2^5 \cdot 3^3 \cdot 5 \cdot 7)$=$(2^5 - 2^4)(3^3 - 3^2) \cdot 4 \cdot 6 = 24 \cdot 16 \cdot 18 = 6912$.

133. $\frac{a}{m}$; $a \leq m$; $(a; m) = 1$, ta'rifiga ko'ra, bunday kasrlar soni $\varphi(m)$ ta.

134. Berilgan oraliqda jami 120 ta natural son bor. Shulardan 120 bilan o'zaro tublari $\varphi(120) == \varphi(2^3 \cdot 3 \cdot 5) = (2^3 - 2^2) \cdot 2 \cdot 4 =$ 32 ta. Shuning uchun ham izlanayotgan natural sonlarning soni $120 - 32 = 88$ ta.

135.a). $\varphi(2^\alpha) = 2^\alpha - 2^{\alpha-1} = 2^{\alpha-1}(2-1) = 2^{\alpha-1}$.

b). $\varphi(p^\alpha) = p^\alpha - p^{\alpha-1} = p^{\alpha-1}(p-1) = p^{\alpha-1}\varphi(p)$.

c). $\varphi(m^\alpha) = m^{\alpha-1}\varphi(m)$ni isbotlash uchun m ning kanonik yoyilmasi $m = p_1^{\gamma_1} p_2^{\gamma_2} \dots p_k^{\gamma_k}$ ni qaraymiz. Bundan $m^\alpha = p_1^{\alpha\cdot\gamma_1} p_2^{\alpha\cdot\gamma_2} \dots p_k^{\alpha\cdot\gamma_k}$ va

$$\varphi(m^\alpha) = m^\alpha\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\cdots\left(1-\frac{1}{p_k}\right) = m^{\alpha-1}\cdot m\cdot\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\cdots\left(1-\frac{1}{p_k}\right) =$$
$$= m^{\alpha-1}\cdot\varphi(m).$$

136. Agar $(m, 2) = 1$ bo'lsa, Eyler funksiyasi multiplikativ bo'lgani uchun $\varphi(2m) = \varphi(2)\varphi(m) = \varphi(m)$.

Agar $(m, 2) > 1$ bo'lsa,$(m, 2) = 2$ bo'ladi. Bu holda $m = 2^\alpha \cdot m_1$, $(m_1; 2) = 1$deb yozib olamiz va $\varphi(2m) = \varphi(2^{\alpha+1} \cdot m_1) = \varphi(2^{\alpha+1})\varphi(m_1) = 2^\alpha\varphi(m_1) = 2\varphi(2^\alpha) \cdot \varphi(m_1) = 2\varphi(2^\alpha \cdot m_1) = 2\varphi(m)$.

137.a). $\varphi(4n + 2) = \varphi\big(2(2n + 1)\big) = \varphi(2)\varphi(2n + 1) = \varphi(2n + 1)$.

b).Agar $(n, 2) = 1$ bo'lsa, u holda $(n, 4) = 1$ bo'ladi. Shuning uchun ham

$$\varphi(4n) = \varphi(4)\varphi(n) = 2\varphi(n).$$

Agarda $n = 2^{\alpha} \cdot k,\ (k; 2) = 1$ bo'lsa, u holda $\varphi(4n) = \varphi(2^{\alpha+2} \cdot k) = \varphi(2^{\alpha+2}) \cdot \varphi(k) = 2^{\alpha+1} \cdot \varphi(k) = 2\varphi(2^{\alpha+1}) \cdot \varphi(k) = 2\varphi(2^{\alpha+1} \cdot k) = 2\varphi(2n)$.

138. *a).* $\varphi(5^x) = 100 \to 5^x - 5^{x-1} = 100 \to 5^{x-1} \cdot 4 = 100 \to 5^{x-1} = 5^2 \to x = 3$.

b). $\varphi(7^x) = 294 \Rightarrow 7^{x-1} \cdot 6 = 294 \Longrightarrow 7^{x-1} = 49 \Longrightarrow 7^{x-1} = 7^2 \Rightarrow x - 1 = 2 \Rightarrow x = 3$.

c). $\varphi(p^x) = p^{x-1} \Rightarrow p^{x-1}(p-1) = p^{x-1}$. Bu tenglama $p > 2$ bo'lsa yechimga ega emas. $p = 2$ da ixtiyoriy natural son x tenglamaning yechimi bo'ladi.

d). $\varphi(3^x \cdot 5^y) = 600 \Rightarrow \varphi(3^x) \cdot \varphi(5^y) = 600 \Rightarrow 3^{x-1} \cdot 2 \cdot 5^{y-1} \cdot 4 = 600 \Rightarrow 3^{x-1} \cdot 5^{y-1} = 75 \Rightarrow 3^{x-1} \cdot 5^{y-1} = 3 \cdot 5^2 \Rightarrow x - 1 = 1;\ y - 1 = 2 \Rightarrow x = 2;\ y = 3$.

139. $m = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}$ bo'lsin u holda

$$\varphi(m) = p_1^{\varphi_1 - 1}(p_1 - 1) p_2^{\varphi_2 - 1}(p_2 - 1) \dots p_k^{\varphi_k - 1}(p_k - 1)$$

bo'ladi. Bu yerda har bir toq p_k ko'paytuvehiga juft $p_i - 1$ ko'paytuvchi mos keladi va $\varphi(m)$ juft son bo'ladi. Agarda $m = 2^{\alpha} > 3$ ko'rinishida bo'lsa, $\varphi(m) = \varphi(2^{\alpha}) = 2^{\alpha-1}$juft son bo'ladi.

140. $x = m$ soni $\varphi(x) = a$ ning ildizi bo'lsa u holda $\varphi(m) = a$ bajariladi. Bu holda $\varphi(2m) = \varphi(m) = a$; chunki shartga ko'ra $(2; m) = 1$. Bu yerdan $x = 2m$ soni ham berilgan tenglamaning ildizi ekanligi kelib chiqadi.

141. m ning ham n ning ham bo'luvchisi bo'lgan p tub soniga $\varphi(mn)$ da bitta $(1 - \frac{1}{p}) < 1$ ko'paytuvchi mos keladi. $\varphi(m)\ \varphi(n)$da esa ikkita shunday ko'paytuvchi $\left(1 - \frac{1}{p}\right)^2$ mos keladi. $\left(1 - \frac{1}{p}\right)^2 < 1 - \frac{1}{p}$ bo'lgani uchun $(m; n) > 1$ bo'lsa, $\varphi(m)\varphi(n) < \varphi(mn)$ bo'ladi. Xususiy holda $\varphi^2(m) \le \varphi(m^2)$, bu yerda tenglik faqat $m = 1$ da bajariladi.

142. $q_1, q_2, \dots\dots\dots, q_t$ lar faqat m ning kanonik yoyilmasiga kiruvchi tub sonlar,$p_1, p_2, \dots, p_k$ lar m va n larning ikkalasining ham kanonik yoyilmasiga kiruvchi tub sonlar,$l_1, l_2, \dots, l_s$ lar faqat n ning kanonik yoyilmasiga kiruvchi tub sonlar bo'lsin. U holda

$$\varphi(m \cdot n) = mn \prod_{i=1}^{t}\left(1 - \frac{1}{q_i}\right) \prod_{i=1}^{k}\left(1 - \frac{1}{p_i}\right) \prod_{i=1}^{s}\left(1 - \frac{1}{l_i}\right) =$$

$$\left\{m\prod_{i=1}^{t}\left(1-\frac{1}{q_i}\right)\prod_{i=1}^{k}\left(1-\frac{1}{p_i}\right)\right\}$$

$$\cdot\left\{n\prod_{i=1}^{k}\left(1-\frac{1}{p_i}\right)\prod_{i=1}^{s}\left(1-\frac{1}{l_i}\right)\right\}\frac{d}{d\prod_{i=1}^{k}\left(1-\frac{1}{p_i}\right)}$$

$$=\varphi(m)\varphi(n)\cdot\frac{d}{\varphi(d)}.$$

Izoh: Agar $\frac{d}{\varphi(d)}\geq 1$ ekanligini inobatga olsak, isbotlangan tenglikdan 11-misoldagi munosabat to‘g‘ridan to‘g‘ri kelib chiqadi.

143. $[m;n]=\frac{mn}{(m;n)}\rightarrow \mu\cdot\delta=mn$ bo‘lgani uchun $\varphi(mn)=$ $\varphi(\mu\delta)=\varphi(\mu)\varphi(\delta)\cdot\frac{d}{\varphi(d)}=\varphi(\mu)\cdot\varphi(\delta)$. $d=(\mu;\delta)=$ $\left(\frac{mn}{(m;n)};mn\delta\right)=1$

144. Yig‘indini bevosita $\varphi(p^{\alpha})=p^{\alpha}-p^{\alpha-1}$ formuladan foydalanib hisoblaymiz: $\varphi(1)+\varphi(p)+\varphi(p^2)+\cdots....+\varphi(p^{\alpha})=$

$$=1+p-1+p^2-p+\cdots+p^{\alpha}-p^{\alpha-1}=p^{\alpha}.$$

145. Agar a natural sonning kanonik yoyilmasi $a=p_1^{\alpha_1}p_2^{\alpha_2}\dots p_k^{\alpha_k}$ va θ multiplikativ funksiya bo‘lsa, u holda

$$\sum_{d\backslash a}\theta(d)=\left(1+\theta(p_1)+\theta(p_1^2)+\cdots+\theta\left(p_1^{\alpha_1}\right)\right)\times$$
$$\times\left(1+\theta(p_2)+\theta(p_2^2)+\cdots+\theta\left(p_2^{\alpha_2}\right)\right)\times\dots$$
$$\times\left(1+\theta(p_k)+\theta(p_k^2)+\cdots+\theta\left(p_k^{\alpha_k}\right)\right)\qquad(1)$$

ayniyat o‘rinli. Haqiqatan ham (1) ning o‘ng tomonidagi qavs ichidagi ifodalarni ko‘paytirib qavslarni ochsak va $\theta(a)$ ning multiplikativligidan foydalanib quyidagiga ega bo‘lamiz:

$$\prod_{i=1}^{k}\left(1+\theta(p_i)+\theta(p_i^2)+\cdots+\theta(p_i^{\alpha_1})\right)$$
$$=1+\theta(p_1)\ +\theta(p_2)+\cdots+\theta(p_k)+\cdots$$
$$+\,\theta(p_1^{\alpha_1})\theta(p_2^{\alpha_2})\ldots\theta(p_k^{\alpha_k})$$
$$=\sum_{\beta_1}^{\alpha_1}\sum_{\beta_2}^{\alpha_2}\ldots\sum_{\beta_k}^{\alpha_k}\theta(p_1^{\beta_1}p_2^{\beta_2}\ldots p_k^{\beta_k})=\sum_{d/a}\theta(d).$$

Bu yerda biz $a=p_1^{\alpha_1}p_2^{\alpha_2}\ldots p_k^{\alpha_k}$ sonining ixtiyoriy bo‘luvchisi d ni $d=p_1^{\alpha_1}p_2^{\alpha_2}\ldots p_k^{\alpha_k}$, $0\le\beta_i\le\alpha_i$, $i=1,2,\ldots,k$ ko‘rinishidagi ifodalash mumkinligidan foydalandik. Endi (1) da $\theta(d)=\varphi(d)$ deb olamiz. U holda (1) dan

$$\sum_{d/a}\varphi(d)=\prod_{i=1}^{k}\left(1+\varphi(p_i)+\varphi(p_i^2)+\cdots+\varphi(p_i^{\alpha_i})\right)$$
$$=\prod_{i=1}^{k}\left(1+p_i-1+p_i^2-p_i+\cdots+p_i^{\alpha_i}-p_i^{\alpha_i-1}\right)=\prod_{i=1}^{k}p^{\alpha_i}$$
$$=a.$$

146. Avvalo, agar $(a;m)=1$ bo‘lsa, u holda $(a;m-a)=1$ ekanligini ko‘rsatamiz. $(a;m-a)=d>1$ bo‘lsin deb faraz etaylik. U holda $a=da_1, m-a=d\cdot t$ deb yoza olamiz. Bu yerdan $m=a+dt=d(a_1+t)$ ga, ya’ni $(a;m)=d>1$ga ega bo‘lamiz. Bu esa $(a;m)=1$ ga qarama-qarshidir.

Endi m dan kichik va m bilan o‘zaro tub sonlarni o‘sib borish tartibida yozib chiqamiz:

$$1,\quad a_1,\quad a_2,\quad \ldots, m-a_2,\quad m-a_1,\quad m-1. \qquad (2)$$

Bularning soni $\varphi(m)$ ta. Bu yerda har bir a_i ga bitta $m-a_i$ soni mos keladi. Ularning yig‘indisi $a_i+(m-a_i)=m$ ga teng. Bunday juftliklar soni $\frac{1}{2}\varphi(m)$ ta. Shunday qilib (2) dagi sonlar yig‘indisini S deb belgilasak $S=\frac{1}{2}m\varphi(m)$ ga ega bo‘lamiz.

147. 16 – masalada isbotlangan formuladan foydalanib

$$S_1=\frac{1}{2}p\varphi(p)=\frac{1}{2}p(p-1);\quad S_2=\varphi(p^2)=p^2-p=p(p-1)$$

$$\frac{S_2}{S_1}=\frac{p(p-1)}{\frac{1}{2}p(p-1)}=2$$

topamiz.

148. 1). $\varphi(x)=p-1,\ x=p^{\alpha}\cdot y,\ (p;y)=1$ deb olamiz. $\varphi(x)=\varphi(p^{\alpha}\cdot y)=p^{\alpha-1}(p-1)\varphi(y)=p-1$ yoki $p^{\alpha-1}\cdot\varphi(y)=1$ hosil bo'ladi. Bundan $\alpha=1,\ \ \varphi(y)=1$ yoki $y=1$ va $y=2$. $\alpha=1$ va $y=1$ da $x=p=2$ tenglama bitta yechimi, $p>2$ bo'lsa tenglama 2 ta p va $2p$ yechimga ega bo'ladi.

2). $\varphi(x)=14\ \Rightarrow\ \varphi(x)=2\cdot 7dan\varphi(x)\ \vdots\ 7$ ya'ni x ning yoyilmasida 7 qatnashishi kerak u holda $\varphi(x)\ \vdots\ 6$, lekin $\varphi(x)$ ifoda 6 bo'linmaydi. Demak, tenglama yechimga ega emas.

3). $\varphi(x)=8=2^3\Rightarrow\ \ \varphi(x)\ \vdots\ 2,\ \varphi(x)\ \vdots\ 4,\ \varphi(x)\ \vdots\ 8.$

a) $x=2^{\alpha}\cdot 3^{\beta}\cdot 5^{\gamma}$ bo'lsin u holda $\varphi(x)=(2^{\alpha}-2^{\alpha-1})(3^{\beta}-3^{\beta-1})(5^{\gamma}-5^{\gamma-1})=8\Rightarrow 2^{\alpha-1}\cdot 3^{\beta-1}\cdot 5^{\gamma-1}\cdot 2\cdot 4=8\Rightarrow 2^{\alpha-1}\cdot 3^{\beta-1}\cdot 5^{\gamma-1}=1\Rightarrow\alpha=1;\ \beta=1;\ \gamma=1$ va $x=30,\ \varphi(30)=4\cdot 2=8;$

b) $x=2^{\alpha}\cdot 3^{\beta}\ \Rightarrow\ \ \varphi(x)=2^{\alpha-1}\cdot 3^{\beta-1}\cdot 2=8\ \Rightarrow 2^{\alpha-1}\cdot 3^{\beta-1}=4$
$\Rightarrow\alpha=3,\ \beta=1\Rightarrow\ \ x=8\cdot 3=24.$

c). $x=2^{\alpha}\cdot 5^{\gamma}\Rightarrow\varphi(x)=2^{\alpha-1}\cdot 5^{\gamma-1}\cdot 4=8\Rightarrow 2^{\alpha-1}\cdot 5^{\gamma-1}=2\ \Rightarrow\ \gamma=1;\ \alpha=2\Rightarrow x=4\cdot 5=20.$

d). $x=3^{\beta}\cdot 5^{\alpha}\Rightarrow\ \ \varphi(x)=3^{\beta-1}\cdot 5^{\gamma-1}\cdot 8=8\Rightarrow 3^{\beta-1}\cdot 5^{\gamma-1}=1$
$\Rightarrow\ \beta=1,\qquad\gamma=1\Rightarrow x=15.$

e). $x=2^{\alpha};\ \ \varphi(x)=2^{\alpha-1}=8=2^3\Rightarrow\ \alpha=4\ \Rightarrow x=16.$

Demak, **javob** $x=15;16;20;24;30.$

4). $\varphi(x)=12=2^2\cdot 3.$ Mumkin bo'lgan hollarni qarab chiqamiz.

a) $x=2^2\cdot 3^{\beta}\cdot 7^{\gamma}\Rightarrow\varphi(x)=2^{\alpha-1}\cdot 2\cdot 3^{\beta-1}\cdot 6\cdot 7^{\gamma-1}=12\ \Rightarrow 2^{\alpha-1}\cdot 3^{\beta-1}\cdot 7^{\gamma-1}=1\Rightarrow\alpha=1,\beta=1,\gamma=1\ \Rightarrow\ x=42.$

b) $x=2^{\alpha}\cdot 3^{\beta}\Rightarrow\varphi(x)=2^{\alpha-1}\cdot 2\cdot 3^{\beta-1}=12\ \Rightarrow\ 2^{\alpha}\cdot 3^{\beta-1}=2^2\cdot 3\Rightarrow\alpha=2;\ \beta=2\ \Rightarrow\ \ x=36.$

c)$x=2^{\alpha}\cdot 7^{\gamma}\Rightarrow\ \ \varphi(x)=2^{\alpha-1}\cdot 6\cdot 7^{\gamma-1}=12\Rightarrow 2^{\alpha-1}\cdot 7^{\gamma-1}=2;\ \gamma=1;\ \alpha=2\Rightarrow x=28.$

d) $x=3^{\beta}\cdot 7^{\gamma}\ \ \Rightarrow\varphi(x)=3^{\beta-1}\cdot 2\cdot 6\cdot 7^{\alpha-1}=12\Rightarrow 3^{\beta-1}\cdot 7^{\gamma-1}=1\Rightarrow\ \ \beta=1,\gamma=1\ \Rightarrow x=21.$

e) $x=2^{\alpha}\cdot 13^{\delta}\Rightarrow\varphi(x)=2^{\alpha-1}\cdot 13^{\delta-1}\cdot 12=12\Rightarrow 2^{\alpha-1}\cdot 13^{\delta-1}=1\Rightarrow\delta=1,\ \alpha=1\ \Rightarrow x=26.$

g) $x=13^{\delta}\ \Rightarrow\ \varphi(x)=13^{\delta-1}\cdot 12=12\Rightarrow 13^{\delta-1}=1\Rightarrow\delta=1\Rightarrow x=13.$

Javob: $x=\ 5;13;21;26;28;36;42.$

149. 1). $\varphi(x)=2^{\alpha};\ \ x=2^{k}\cdot 3^{l}\cdot 5^{m}$

a) $x=2^{k}\Rightarrow\varphi(x)=2^{k-1}=2^{\alpha}\ \Rightarrow k-1=\alpha\Rightarrow\ \ k=\alpha+1\Rightarrow x=2^{\alpha+1}.$

b) $x=2^{k}\cdot 5^{m}\Rightarrow\varphi(x)=2^{k-1}\cdot 5^{m-1}\cdot 4=2^{\alpha}\Rightarrow\ \ m-1=0; m=1,$
$k+1=\alpha,\qquad k=\alpha-1\Rightarrow x=2^{\alpha-1}\cdot 5.$

c) $x=2^{k}\cdot 3^{l}\Rightarrow\ \varphi(x)=2^{k-1}\cdot 3^{l-1}\cdot 2=2^{\alpha}\Rightarrow\ k=\alpha,\ l=1\Rightarrow x=2^{\alpha}\cdot 3.$

d) $x=3^{l}\cdot 5^{m}\ \Rightarrow\varphi(x)=3^{l-1}\cdot 2\cdot 5^{m-1}\cdot 4=2^{\alpha}\Rightarrow\alpha=3\,;l=1\,;\ m=1\Rightarrow\ x=15\,.$

e) $x=2^{k}\cdot 3^{l}\cdot 5^{m}\Rightarrow\varphi(x)=2^{k-1}\cdot 3^{l-1}\cdot 2\cdot 5^{m-1}\cdot 4=2^{\alpha}\Rightarrow$
$2^{k+2}\cdot 3^{l-1}\cdot 5^{m-1}=2^{\alpha}\Rightarrow k=\alpha-2\,;\ l=1\,;\ m=1\Rightarrow\ x$
$=2^{\alpha-2}\cdot 15.$

Javob: $x=2^{\alpha+1};\ 2^{\alpha-1}\cdot 5;\ \ 2^{\alpha}\cdot 3;\ 15;\ \ 2^{\alpha-2}\cdot 15.$

2). $\varphi(p^{x})=6\cdot p^{x-2}\ \Rightarrow\ p^{x-1}(p-1)=6p^{x-2}\ \Rightarrow\ p(p-1)=6\Rightarrow\ p=3$ ixtiyoriy x qanoatlantiradi $p\neq 3$ da yechimi yoq.

150. $\varphi(m)=3600$, bunda $m=3^{\alpha}\cdot 5^{\beta}\cdot 7^{\gamma}$. $3600=2^{4}\cdot 3^{2}\cdot 5^{2}\Rightarrow\ \varphi(m)=3^{\alpha-1}\cdot 2\cdot 5^{\beta-1}\cdot 4\cdot 7^{\gamma-1}\cdot 6=2^{4}\cdot 3^{2}\cdot 5^{2}\ \Rightarrow 3^{\alpha-1}\cdot 5^{\beta-1}\cdot 7^{\gamma-1}=3\cdot 5^{2}\Rightarrow\alpha-1=1\,;\ \alpha=2\,;\ \beta-1=2\,;\ \beta=3\,;\ \gamma=1\ \Rightarrow\ m=3^{2}\cdot 5^{3}\cdot 7=7875.$

151. $\varphi(x)=120,\ \ x=p_1\cdot p_2$ va $p_1-p_2=2\Rightarrow\varphi(x)=(p_1-1)(p_2-1)=120\,;\ p_1=p_2+2\ \Rightarrow(p_2+1)(p_2-1)=120\ \Rightarrow p_2=11\,;\ p_1=13\,;\ \ x=143.$

152. Masalaning shartiga ko‘ra: $\varphi(m)=11424\,;\ m=p_1^2\cdot p_2^2$. Bulardan va $11421=2^{5}\cdot 3\cdot 7\cdot 17$ ekanligidan $\varphi(p_1^2\cdot p_2^2)=(p_1^2-p_1)(p_2^2-p_2)=p_1(p_1-1)p_2(p_2-1)=2^{5}\cdot 3\cdot 7\cdot 17=16\cdot 17\cdot 7\cdot 6$ hosil bo‘ladi. Bundan esa $p_1=17\,;\ p_2=7;\ \ m=(p_1\cdot p_2)^2=119^2=14161$ ni hosil qilamiz.

153. *a)*. $\varphi(x)=\varphi(px)$da agar $p=2$ bo'lsa $\varphi(x)=\varphi(2)\varphi(x)=\varphi(x)$, x ning barcha toq qiymatlari qanoatlantiradi, chunki bu holda

$(2;x)=1\,; p\geq 3$ bo'lsa $x=p_1^{\alpha_1}\cdot p_2^{\alpha_2}\ldots.p_k^{\alpha_k}$desak, $\varphi(x)=(p_1^{\alpha_1}-p_1^{\alpha_1-1})(p_2^{\alpha_2}-p_2^{\alpha_2-1})\ldots(p_k^{\alpha_k}-p_k^{\alpha_k-1})$.

Agar $(p;x)=1$ bo'lsa, $\varphi(px)=\varphi(x)(p-1)\neq\varphi(x)$. Agarda $(p;x)=p;\ (p=p_i)$ bo'lsa, $px=p_1^{\alpha_1}\cdot p_2^{\alpha_2}\ldots p_i^{\alpha_i+1}\cdots p_k^{\alpha_k}$ va $\varphi(px)=(p_1^{\alpha_1}-p_1^{\alpha_1-1})(p_2^{\alpha_2}-p_2^{\alpha_2-1})\ldots(p_i^{\alpha_i+1}-p_i^{\alpha_i})\cdots(p_k^{\alpha_k}-p_k^{\alpha_k-1})=p_i\cdot\varphi(x)\neq\varphi(x)$ Demak, $p=2$ da berilgan tenglamani x ning barcha toq qiymatlari qanoatlantiradi; $p\geq 3$ bo'lsa tenglama yechimga ega emas.

b). $\varphi(px)=p\varphi(x)$. 1). Agar $(x;p)=1$ bo'lsa, $\varphi(px)=\varphi(p)\varphi(x)=(p-1)\varphi(x)\Rightarrow\varphi(x)(p-1)=p\varphi(x)\ \Rightarrow\ \varphi(x)=0$. Demak yechimi yo'q.

2). $x=p_1^{\alpha_1}\cdot p_2^{\alpha_2}\ldots.p_k^{\alpha_k}$ bo'lsa, $(p;x)=p\,;\ (p=p_i)\,;\ \varphi(px)=p_i\cdot\varphi(x)=p\varphi(x)$. Demak, bu holda berilgan tenglamani x ning p ga karra natural qiymatlari qanoatlantiradi.

c).$\varphi(p_1\cdot x)=\varphi(p_2\cdot x);\ p_1\neq p_2\,;\ x=q_1^{\alpha_1}\cdot q_2^{\alpha_2}\ldots.q_k^{\alpha_k}$;

1) $p_1\neq q_i$; yani$(x;p)=1$ bo'lsa $\varphi(p_1\,x)=(p_1-1)\,\varphi(x)$, agarda $(x;p_1)=p_1$bo'lsa, $\varphi(p_1x)=p_1\varphi(x)$.

2) $p_2\neq q_i$yani $(x;p_2)=1\Rightarrow\varphi(p_2x)=(p_2-1)\varphi(x)$; agarda $(x;\,p_2)=p_2\,;\ \varphi(p_2x)=p_2\varphi(x)$.

Bulardan quyidagi tenglamalarni hosil qilamiz:

1)$(p_1-1)\varphi(x)=(p_2-1)\varphi(x)$; 3) $(p_2-1)\varphi(x)=p_1\varphi(x)$;

2)$(p_1-1)\varphi(x)=p_2\varphi(x)$; 4) $p_2\varphi(x)=p_1\varphi(x)$.

1)dan $(p_1-1-p_2+1)\varphi(x)=0\Rightarrow(p_1-p_2)\varphi(x)=0\ \Rightarrow\ p_1-p_2\neq 0$.

Demak $\varphi(x)=0$ bo'lishi kerak bu holda tenglama yechimga ega emas.

2) dan $(p_1-p_2-1)\varphi(x)=0\,;\ p_1=p_2+1\,;\ p_1=3\,;\ p_2=2$ da tenglamani x ning berilgan shartilarini qanoatlantiruvchi, yani $(x;3)=1$ va $(x;2)=2$ (x ning 2 ga bo'linib 3 ga bo'linmaydigan qiymatlari) tenglamani qanoatlantiradi.

3) dan $((p_1-p_2-1)\varphi(x)=0$. Bundan yuqoridagi singari $p_1=2\,;\ p_2=3$ da bajariladi. Yani x ning 3 gabo'linib 2 bilano'zaro tub qiymatlarining berilgan tenglamani qanoatlantirishi kelib chiqadi.

4) dan $(p_2-p_1)\varphi(x)=0\,;\ p_1\neq p_2$ bo'lgani uchun bu holda tenglama yechimga ega emas.

154. *a)*.$\varphi(x) = \frac{x}{2} \Rightarrow \frac{x}{2}$ – butun son bo'lishi kerak. Shuning uchun ham $x = 2^{\alpha} \cdot q,\ (q;2) = 1$ deb yozish mumkin. Bu holda $\varphi(x) = 2^{\alpha-1} \cdot \varphi(q) = 2^{\alpha-1} \cdot q \Rightarrow \varphi(q) = q \Rightarrow q = 1.$ Bundan $x = 2^{\alpha}$ tenglamaning yechimi ($\alpha \geq 1$) bo'ladi.

b).$\varphi(x) = \frac{x}{3} \Rightarrow x = 3^{\beta} \cdot q \Rightarrow \varphi(x) = 3^{\beta-1} \cdot 2 \cdot \varphi(q) = \frac{3^{\beta} \cdot q}{3} \Rightarrow \varphi(q) = \frac{q}{2} \Rightarrow q = 2^{\alpha} \Rightarrow x = 2^{\alpha} \cdot 3^{\beta}.$

c).$\varphi(x) = \frac{x}{4} \Rightarrow x = 2^{\alpha} \cdot q\,;\ \alpha \geq 2\ ;\ (2^{\alpha}; q) = 1 \Rightarrow \varphi(x) = 2^{\alpha-1} \cdot \varphi(q) = \frac{2^{\alpha} \cdot q}{4} = 2^{\alpha-2} \cdot q$. Bundan $\varphi(q) = \frac{q}{2}$ ni hosil qilamiz. Bundan esa a) ga asosan $q = 2^{k}$ kelib chiqadi, lekin bizda $(2^{\alpha}; q) = 1$ bo'lishi kerak edi, bu qarama-qarshilikdan berilgan tenglamani yechimga ega emas degan xulosa kelib chiqadi.

155. $\varphi(p^{x}) = a \to p^{x-1}(p-1) = a \Rightarrow$

$$(x-1)\ln p = \ln\frac{a}{p-1} \Rightarrow x = 1 + \frac{\ln\frac{a}{p-1}}{\ln p},$$

bundan a birga teng yoki juft son .

156. $p_i(i = 1,2 \ldots, k)$ barcha tub sonlar bo'lsin. U holda $a = p_1 p_2 \ldots p_k$ soni uchun

$$\varphi(a) = (p_1 - 1)(p_2 - 1) \ldots (p_k - 1). \qquad (*)$$

Ikkinchi tomondan esa har bir $\leq a$ natural son $p_1, p_2, \ldots, p_k$ tub sonlarning birortasiga bo'linadi va a bilan o'zaro tub emas. Shuning uchun ham $\varphi(a) = 1$. Shunday qilib (*) ga asosan $(p_1 - 1)(p_2 - 1) \ldots (p_k - 1) = 1$ hosil bo'ladi. Bunday bo'lishi mumkin emas. Bu qarama-qarshilik tub sonlar soni chekli k ta bo'lsin deganimizdan kelib chiqdi. Demak, tup sonlar soni cheksiz ko'p.

157. $\frac{a}{b}\,; (a;b) = 1\,;\ 0 < a < b$ musbat, to'g'ri, qisqarmas kasr berilgan bo'lsin. Maxraji b ga teng musbat, to'g'ri, qisqarmas kasrlar soni $\varphi(b)$ ta. Shuning uchun ham izlanayotgan son $\varphi(2) + \varphi(3) + \cdots + \varphi(n)$ ga teng bo'ladi.

158. $x \leq 300$ va $(x; 300) = 20$ bajarilishi kerak. Bundan $\left(\frac{x}{20}; 15\right) = 1.$

$y = \frac{x}{20}$ deb olsak $(y; 15) = 1$ va $y \leq 15$ bo'lishi kerak, bunday y lar soni $\varphi(15) = 8$ ta. Bular $y = 1, 2, 4, 7, 8, 11, 13, 14$ va bunga mos x lar $x = 20, 40, 80, 140, 160, 220, 260, 280$ lardan iborat.

III. 1-§.

159. Barcha butun sonlarni 1 ga bo'lsak 0 qoldiq qoladi, ya'ni barcha butun sonlar 1 moduli bo'yicha o'zaro taqqoslanuvchi.

160. 8 moduli bo'yicha taqqoslanuvchi sonlar $8q + r$, $0 \leq r < 8$; masalan $r = 1$ da 9,17 lar 8 moduli bo'yicha o'zaro taqqoslanadi, chunki $9 = 8 \cdot 1 + 1$ va $17 = 8 \cdot 2 + 1$.

161. a) $1 \equiv -5(mod6), 1 \equiv 6 - 5(mod6), 1 \equiv 1(mod6);$

b) $546 \equiv 0(mod13), 546 \equiv 13 \cdot 42 + 0, 0 \equiv 0(mod13);$

c) $2^3 \equiv 1(mod4), 8 \equiv 1(mod4),\ 0 \equiv 1(mod4)$?

d) $3m \equiv -1(modm), 0 \equiv m - 1(modm)$?

Demak, $a), b)$ taqqoslamalar o'rinli, $c), d)$ lar o'rinli emas.

162. $a \equiv b(modm)$ taqqoslamaning o'rinli ekanligini ko'rsatish uchun a va b larni m ga bo'lganda bir xil qoldiq qolishini, yoki $(a - b) \vdots m$ ni ko'rsatish yetarli.

a) $121 \equiv 13145(mod2)$, chunki $121 \equiv 2 \cdot 60 + 1$ va $13145 \equiv 2 \cdot 6572 + 1$.

Berilgan sonlarni 2 ga bo'lsak bir xil qoldiq qoladi. Shuning uchun ham ular 2 moduli bo'yicha taqqoslanuvchi.

b) $121347 \equiv 92817(mod10)$, bu yerda $121347 = 12134 \cdot 10 + 7$, $92817 = 9281 \cdot 10 + 7$. Demak ta'rifga ko'ra taqqoslama o'rinli.

c) $31 \equiv -9(mod10), 31 - (-9) \equiv 40 \vdots 10$. Demak, taqqoslama o'rinli.

d) $(m - 1)^2 \equiv 1(modm)$, bu yerda $(m - 1)^2 - 1 = m^2 - 2m = m(m - 2) \vdots m$. Demak, taqqoslama o'rinli.

e) $2m + 1 \equiv (m + 1)^2(modm)$, chunki $2m + 1 - (m + 1)^2 = 2m + 1 - m^2 - 2m - 1 = -m^2 \vdots m$. Demak, berilgan taqqoslama o'rinli.

163. *a)* $5^{1812} = (5^2)^{906} = (25 \cdot 1 + 0)^{906} \equiv 0(mod25)$. Shuningdek

$1964 = 1950 + 14 = 78 \cdot 25 + 14 \equiv 14(mod25)$, demak, bu sonlar 25 moduli bo'yicha teng qoldiqli emas, ya'ni $5^{1812} \not\equiv 1964(mod25)$.

b) agar $a \equiv b(mod m)$bo'lsa, $(a,m) = (b,m)$ bo'lishi kerak. Bizning misolimizda $(7^{103};87) = 1$; $(3;87) = 3$. Demak, $7^{103} \not\equiv 3(mod 87)$.

c) $4^{1965} \equiv 25(mod 10)$ da$(4^{1965};10) = 2$va$(25,10) = 5$, $5 \neq 2$ bo'lgani uchun $4^{1965} \not\equiv 25(mod\ 10)$.

d) $30 \cdot 17 \equiv 81 \cdot 19(mod 6) da\ 30 \cdot 17 \equiv 0(mod 6), 81 \cdot 19 \not\equiv (mod 6)$ demak, taqqoslama o'rinli emas.

e) $(2n+1)(2m+1) \equiv 2k(mod 6)$. Bu yerdan tenglikka o'tsak,

$(2n+1)(2m+1) = 2k + 6t = 2(k+3t)$. Bu tenglikning o'ng tomoni 2 ga bo'linadi, chap tomoni esa 2 ga bo'linmaydi. Shuning uchun ham taqqoslama o'rinli emas.

164. a butun soni va $m > 0$ butun soni berilgan bo'lsin. U holda qoldiqli bo'lish haqida teoremaga asosan $a = m \cdot q + r$, $0 \leq r < m$ deb yoza olamiz. Bundan $a - r = mq$, ya'ni$(a - r) \vdots m$. U holda ta'rifga asosan $a \equiv r(mod m)$.

165. $x \equiv 2(mod 10)$ ni tenglik ko'rinishida yozsak $x = 2 + 10t$, $t \in Z, x = 2, 12, 22, -8, -18$.

166. $a) x \equiv 0(mod 3), x = 3t, t \in Z$; b) $x \equiv 1(mod 2), x = 1 + 2t$, $t \in Z$.

167. $a) 20 \equiv 8(mod m) \Rightarrow \left.\begin{matrix} 20 = mq + r \\ 8 = mq_1 + r \end{matrix}\right\} \Rightarrow 12 = m(q - q_1) \Rightarrow$ $m = 1,2,3,4,6,12$.

b)$3p + 1 \equiv p + 1(mod m) \Rightarrow 3p + 1 - p - 1 \equiv 0(mod m) \Rightarrow 2p \equiv 0(mod m) \Rightarrow 2p \vdots m$. $2p$ ning bo'luvchilari $m = 1, 2, p, 2p$.

168. $13 \equiv 5(mod m) \rightarrow 13 - 5 \equiv 0(mod m) \rightarrow 8 \equiv 0(mod m) \Rightarrow m = 1,2,4,8$.

169. Ta'rifga ko'ra 10 modul bo'yicha taqqoslanuvchi butun sonlarni 10 ga bo'lganda bir xil qoldiq qolishi kerak, ya'ni ular $a = 10 \cdot q + r, 0 \leq r < 10$ shartni qanoatlantirishi kerak. Misol uchun $r = 1$ deb olsak barcha 10 ga bo'lganda $1, 11, 101, 1001, \ldots$ larga ega bo'lamiz.

170. Berilgan taqqoslamalardan qaysilari o'rinli ekanligini aniqlash uchun m modul bo'yicha taqqoslanuvchi sonlarning ayirmasi shu modulga qoldiqsiz bo'linishini tekshirib ko'rish kifoya.

a)da $1 - (-11) = 1 + 11 = 12$ va 12 soni 6 ga qoldiqsiz bo'linadi. Demak, berilgan taqqoslama o'rinli.

b)da $3n - n^2 = n(3-n)$va$n(3-n)$ soni n ga qoldiqsiz bo'liadi. Demak, berilgan taqqoslama o'rinli.

c)da $2^6 - 1 = 63 = 7 \cdot 9$ va $7 \cdot 9$ soni 7 ga qoldiqsiz bo'linadi. Demak, berilgan taqqoslama o'rinli.

d)da $3m - 1 = 2m + (m-1)$ va $2m + (m-1)$ soni $m > 1$ga qoldiqsiz bo'linmaydi. Demak, berilgan taqqoslama o'rinli emas.

Shunday qilib berilgan taqqoslamalardan a), b), c) lar o'rinli, d) esa o'rinli emas.

171. Berilgan taqqoslamani parametrik tenglik qilib yozsak $x = 7 + 5t$, bunda t ixtiyoriy butun son. Bundan $x = 2 + 5 + 5t = 2 + 5(t+1) = 2 + 5t_1$, t_1 - ixtiyoriy butun son. Demak x 5ga bo'lganda 2 qoldiq qoluvchi sonlardan $\cdots, -13, -8, -3, 2, 7, 12, 17, \cdots$ iborat bo'lar ekan.

172. Faraz etaylik $\left.\begin{matrix} x \equiv \alpha \\ y \equiv \beta \\ z \equiv \gamma \end{matrix}\right\}(modm)$ bo'lsin. U holda $\left.\begin{matrix} ax^3 \equiv a\alpha^3 \\ bx^2y \equiv b\alpha^2\beta \\ cxyzc = c\alpha\beta\gamma \\ dz \equiv d\gamma \end{matrix}\right\}(modm)$ bajariladi. Bundan $F(x,y,z) \equiv F(\alpha,\beta,\gamma)(modm)$ kelib chiqadi.

173. $3^n \equiv -1(modm)$ ni $3^4 = 81 \equiv 1(mod10)$ taqqoslamaga hadlab ko'paytirsak $3^{n+4} \equiv -1(mod10)$ hosil bo'ladi.

174. $2^{5n} - 1 = (2^5)^n - 1 = (31+1)^n - 1 \equiv (1^n - 1)(mod31) \equiv 0(mod31)$demak$(2^{5n} - 1) \vdots 31$.

175. $x = 3n + 1$bo'lsa $1 + 3^x + 9^x = 1 + 3^{3n+1} + 9^{3n+1} = 1 + 3 \cdot 3^{3n} + 9 \cdot 9^{3n} = 1 + 3 \cdot (3^3)^n + 9 \cdot (9^3)^n = 1 + 3(26+1)^n + 9(128+1)^n = 1 + 3(13 \cdot 2 + 1)^n + 9(13 \cdot 56 + 1)^n \equiv 1 + 3 \cdot 1^n + 9 \cdot 1^n(mod13) \equiv 13(mod13) \equiv 0(mod13)$. Demak, $1 + 3^x + 9^x$ soni $x = 3n + 1$ $(n = 0,1,2,\dots)$ bo'lganda 13 ga bo'linadi.

176. $(a+b)^p$ ni Nyuton binomi formulasidan foydalanib yoyib, keyin p moduli bo'yicha taqqoslamaga o'tamiz. $(a+b)^p = a^p + pa^{p-1}b + \frac{p(p-1)}{21}a^{p-2}b^2 + \cdots + ab^{p-1} + b^p \equiv a^p + b^p(modp)$, ya'ni $(a+b)^p \equiv a^p + b^p(modp)$.

177. Masalaning sharti bo'yicha $a \equiv b(modp^n)$. Buni tenglik qilib yozsak $a = b + p^n \cdot t$, $(t = 0, \pm1, \pm2 \dots)$. Bu tenglikni ikkala tomonini

p-darajaga ko'taramiz, u holda $a^p = (b + p^n t)^p = b^p + p^{n+1}q$, $(q = 0, \pm 1, \pm 2, \dots)$. Oxirgi tenglik esa $a^p \equiv b(mod p^{n+1})$ taqqoslamaga teng kuchli.

178. Agar $(x; m) = 1$ bo'lsa $ax \equiv bx(mod m)$ taqqoslamaning ikkala tomonini x ga qisqartirish mumkin, ya'ni $a \equiv b(mod m)$, bundan $a \equiv b\left(mod \frac{m}{(x,m)}\right)$ taqqoslama o'rinli ekanligi kelib chiqadi. Agar $(x, m) = d > 1$ bo'lsa $x = dx_1$ va $m = dm_1$, $(m_1; x_1) = 1$ deb yoza olamiz. Bulardan foydalanib $ax \equiv bx(mod m)$ taqqoslamani $adx_1 \equiv bdx_1(mod m_1 d)$ deb yoza olamiz. Berilgan taqqoslamaning ikkala tomonini va modulini ularning umumiy bo'luvchisiga qisqartirish mumkin. Shuning uchun ham oxirgi taqqoslamani $ax_1 \equiv bx_1(mod m_1)$ ko'rinishda yozish mumkin. Bundan, $(x_1; m_1) = 1$ bo'lgan uchun, $a \equiv b(mod m_1)$ ga, ya'ni $a \equiv b(mod \frac{m}{d})$ ga ega bo'lamiz. Bunda $d = (m, x)$ bo'lgani uchun $a \equiv b\left(mod \frac{m}{(x,m)}\right)$ ni hosil qilamiz.

179. Bunda $\overline{a_4a_3a_2a_1a_0} \equiv 0(mod 33)$ taqqoslamani $a_4 10^4 + \overline{a_3a_2} \cdot 10^2 + \overline{a_1a_0} \equiv 0(mod 33)$ ko'rinishda yozib olamiz va undan $9999a_4 + 99\overline{a_3a_2} \equiv 0(mod 33)$ ayniy taqqoslamani hadlab ayiramiz. U holda isbotlanishi talab etilgan taqqoslama $a_4 10^4 + \overline{a_3a_2} + \overline{a_1a_0} \equiv 0(mod 33)$ hosil bo'ladi.

180. 1). Berilgan taqqoslamalarni $p - 1 \equiv -1(mod p)$, $p - 2 \equiv -2(mod p), \dots, p - n \equiv -n(mod p)$ ko'rinishida yozib olib hadlab ko'paytiramiz. U holda $(p-1)(p-2)\dots(p-n) \equiv (-1)^n n!\,(mod p)$ hosil bo'ladi. Bunda $(n!, p) = 1$ bo'lgani uchun oxirgi taqqoslamaning ikkala tomonini $n!$ ga bo'lib $\frac{(p-1)(p-2)\dots(p-n)}{n!} \equiv (-1)^n(mod p)$ ni hosil qilamiz. Buning chap tomoni C_{p-1}^n ga teng. Shuning uchun ham $C_{p-1}^n \equiv (-1)^n(mod p)$ bajariladi.

2) 22.1-misoldagi singari $p - 2 \equiv -2(mod p), \dots, p - n \equiv -n(mod p)$, $p - (n+1) \equiv -(n+1)(mod p)$ lardan $(p-2)(p-3)\dots(p-n)\big(p-(n+1)\big) \equiv (-1)^n(n+1)!\,(mod p)$ ni, bundan esa $\frac{(p-2)(p-3)\dots(p-n)(p-(n+1))}{n!} \equiv (-1)^n(n+1)(mod p)$ ni hosil qilamiz. Shuning uchun ham $C_{p-2}^{n+1} \equiv (-1)^n(n+1)(mod p)$.

181. 1). $9^{10} = (10-1)^{10} = 100t + 1 \equiv 1(mod 100)$ bo'gani uchun $9^{10q+r} \equiv 9^r(mod 100)$ bo'ladi. $9^9 = (9^2)^4 \cdot 9 = 81^4 \cdot 9 \equiv$

$9(mod10)$ dan $9^9 = 9 + 10t_1$; u holda $9^{9^9} \equiv 9^{9+10t_1}(mod100) \equiv 9^9(mod100) \equiv (9^3)^3 \equiv 729^3(mod100) \equiv 29^3(mod100) \equiv 24389(mod100) \equiv 89(mod100)$. Demak, izlanayotgan oxirgi ikkita raqam 8 va 9.

2) $7^4 = 2401 \equiv 1(mod100)dan7^{100} = (7^4)^{25} \equiv 1(mod100)$. Bu yerdan $7^{9^{9^9}} \equiv 7^{100q+89}(mod100)$ (1) – misolga qarang) $7^{100q+89} \equiv (7^{100})^q \cdot 7^{89}(mod100) \equiv 7^{89}(mod100) \equiv 7^{88} \cdot 7(mod100) \equiv (7^4)^{22}7(mod100) \equiv 7(mod100)$. Demak, izlanayotgan oxirgi 2ta raqam 0 va 7.

182. $p > 2$ – toq tub son bo'lgani uchun $p + 2$ ham toq son bo'ladi, ya'ni $p \equiv p + 2 \equiv 1(mod2)$ (1) bajariladi. Bundan $p^{p+2} + (p+2)^p \equiv (2k+1)^{p+2} + (2q+1)^p \equiv 2(mod2) \equiv 0(mod2)$. Shuningdek tushunarliki, $p \equiv -1(mod\,p+1)$ va $p + 2 \equiv 1(mod\,p+1)$ bajariladi. Oxirgi 2 ta taqqoslamadan $p^{p+2} + (p+2)^p \equiv (-1)^{p+2} + 1^p(mod\,p + 1) \equiv -1 + 1(mod\,p+1) \equiv 0(mod\,p+1)$ (2). (1) va (2) dan$p^{p+2} + (p+2)^p \equiv 0(mod\,2p+2)$ taqqoslama kelib chiqadi.

183. Qaralayotgan sonlarni juft-jufti bilan birlashtirib (noldan tashqarilarini) $\pm\frac{p-x}{2}$, $(x = 1,2,\ldots p-2)$ ko'rinishda yozish mumkin. Endi agarda bu sonlar ichida $p > 2$ moduli bo'yicha o'zaro taqqoslanuvchilari bor desak $\pm\frac{p-x}{2} \equiv 0(modp)$ yoki $\frac{p-x_1}{2} \equiv \pm\frac{p-x_2}{2}(modp)$ larning birortasi bajarilishi kerak. Bulardan $x \equiv p(modp)$va$x_1 \equiv \pm x_2(modp)$ larga ega bo'lamiz. Birinchi holda $x = 0$ (chunki $x < p$), ikkinchi holda esa $x_1 = x_2$ yoki $x_1 = -x_2$ga ega bo'lamiz. Bu esa qaralayotgan sonlar orasida o'zaro taqqoslanuvchilari yo'q ekanligini bildiradi.

184. Berilgan $i \equiv i - m(modm)$ taqqoslamadan $i = 1,2,\ldots,m$ da $1 \equiv 1 - m, 2 \equiv 2 - m, \ldots, m - 2 \equiv (m-2) - m \equiv 2, m - 1 \equiv m - 1 - m \equiv -1,\ m \equiv -m(modm)$ larga ega bo'lamiz. Bularning barchasini n-darajaga ko'tarib keyin hadlab qo'shsak:

$$1^n + 2^n + \cdots + m^n \equiv (-1)^n + (-2)^n + \cdots + (-m)^n(modm) \qquad (1)$$

hosil bo'ladi. Bundan agar $n = 2k + 1$toq son bo'lsa (shart bo'yicha m va n lar toq sonlar), $1^n + 2^n + \cdots + m^n \equiv -(1^n + 2^n + \cdots + m^n)(modm)$, yoki

$$2\sum_{i=1}^{m} i^n \equiv 0(mod m), \text{ya'ni} \sum_{i=1}^{m} i^n \equiv 0(mod m)$$

kelib chiqadi.

185. Taqqoslamaning o'rinli ekanligini matematik induksiya metodidan foydalanib isbotlaymiz. $n = 1$ da berilgan $2^{3^n} \equiv -1(mod 3^{n+1})$ taqqoslama $2^3 \equiv -1(mod 9)$ ko'rnishni oladi. Bu taqqoslama $2^3 \equiv 8(mod 9)$ ayniy taqqoslamaga teng kuchli. Demak, $n = 1$ da taqqoslama o'rinli. Endi faraz etaylik berilgan taqqoslama $n = k$ uchun$2^{3^k} \equiv -1(mod 3^{k+1})$ o'rinli bo'lsin va biz $n = k + 1$ uchun uning, ya'ni $2^{3^{k+1}} \equiv -1(mod 3^{k+2})$ ning o'rinli ekanligini ko'rsatamiz.

$$2^{3^{k+1}} + 1 = \left(2^{3^k}\right)^3 + 1^3 = \left(2^{3^k} + 1\right)\left(2^{3^{k}.2} - 2^{3^k} + 1\right)$$

bu yerda induktivlik farazimizga ko'ra $2^{3^k} + 1 \equiv 0(mod 3^{k+1})$ va $2 \equiv (-1)(mod 3)$ bo'lgani uchun $2^{2\cdot 3^k} - 2^{3^k} + 1 \equiv 0(mod 3)$bo'ladi. Bulardan $2^{3^k} + 1 \equiv 0(mod 3^{k+1})$ ning bajarilishi kelib chiqadi. Demak, matematik induksiya metodiga ko'ra berilgan taqqoslama ixtiyoriy natural n soni uchun o'rinli.

186. Masalaning shartiga ko'ra$2^{3^n} + 1 \equiv 0(mod 3^{n+1})$ bajariladi. U holda $2^{3^n} + 1 \equiv 0(mod 3^n)$ taqqoslama albatta bajariladi.Agar bundan $m = 3^n, (n = 1,2,3,\dots)$deb olsak, $2^m + 1 \equiv 0(mod m)$ taqqoslama kelib chiqadi. Bu yerda $m = 3^n, (n = 1,2,3,\dots)$ bo'lgani uchun $2^m + 1 \equiv 0(mod m)$ taqqoslama natural sonlarda cheksiz ko'p yechimga ega bo'ladi.

187. Taqqoslamaning o'rinli ekanligini n bo'yicha matematik induksiya metodini qo'llab isbotlaymiz. $n = 1$ da berilgan$(m-1)^{m^n} \equiv -1(mod m^{n+1})$ taqqoslama $(m-1)^m \equiv -1(mod m^2)$ ko'rinishni oladi. Bundan $(m-1)^m + 1 \equiv 0(mod m^2)$, yoki $(m > 1 -$ toq son$)(m - 1 + 1)((m-1)^{m-1} - (m-1)^{m-2} + \dots + 1) \equiv 0(mod m^2)$. Bu taqqoslamaning ikkala tomoni va moduli m ga bo'lib

$(m-1)^{m-1} - (m-1)^{m-2} + \dots + 1 \equiv 0(mod m)$ga ega bo'lamiz. Bundan$(-1)^{m-1} - (-1)^{m-2} + \dots + 1 \equiv 0(mod m)$. Yoki $\underbrace{1 + 1 + 1 + 1 + \dots + 1}_{mta} \equiv 0(mod m) \to m \equiv 0(mod m)$. Shunday qilib berilgan taqqoslama $n = 1$ da o'rinli ekan. Endi faraz etaylik $n = k$

uchun berilgan taqqoslama, ya'ni $(m-1)^{m^k} \equiv -1(mod m^{k+1})$ o'rinli bo'lsin. Biz berilgan taqqoslamaning $n=k+1$ bo'lganda, ya'ni $(m-1)^{m^{k+1}} \equiv -1(mod m^{k+2})$ taqqoslamaning o'rinli ekanligini isbotlaymiz. Bu yerda m –toq son va

$$(m-1)^{m^{k+1}}+1=\left[(m-1)^{m^k}\right]^m+1=\left[(m-1)^{m^k}+1\right]\Big((m-1)^{(m-1)m^k}-(m-1)^{(m-2)m^k}+\cdots+1\Big) \equiv 0(mod m^{k+2}).$$

Oxirgi taqqoslamaning o'ng tomonidagi birinchi ko'paytuvchi uchun induktivlik farazimizga asosan $(m-1)^{m^k}+1 \equiv 0(mod m^{k+1})$ bajariladi. Ikkinchi ko'paytuvchi uchun esa $(m-1)^{(m-1)m^k}-(m-1)^{(m-2)m^k}+\cdots+1 \equiv \underbrace{1+1+\cdots+1}_{mta} \equiv m(mod m) \equiv 0(mod m)$ bajariladi. Keyingi 2 ta taqqoslamadan $(m-1)^{m^{k+1}} \equiv -1(mod m^{k+2})$ kelib chiqadi. Shunday qilib matematik induksiya prinspiga asosan berilgan taqqoslama ixtiyoriy n natural soni uchun o'rinli.

188. Masalaning shartiga ko'ra $(m-1)^{m^n} \equiv -1(mod m^{n+1})$ taqqoslama o'rinli. Bundan $m=5$ da $4^{5^n} \equiv -1(mod 5^{n+1})$, ya'ni $4^{5^n}+1 \equiv 0(mod 5^{n+1})$. Bu holda $4^{5^n}+1 \equiv 0(mod 5^n)$ taqqoslama albatta bajarilishi kerak. Endi agar biz $5^n \equiv x$ $(n=1,2,3,\dots)$ deb olsak $2^{2x}+1 \equiv 0(mod x)$ taqqoslamaga ega bo'lamiz. Bu yerda $x=5^n$ $(n=1,2,3,\dots)$ bo'lgani uchun oxirgi taqqoslama natural sonlarda cheksiz ko'p yechimga ega.

189.1). Bu yerda $2^{4n+1} \equiv 2\cdot(2^4)^n(mod 5)$, ya'ni $2^{4n+1}=2+5t, t\in N$ bo'lgani uchun $N=3^{2^{4n+1}}+2=3^{2+5t}+2=9\cdot(3^5)^t+2=9(243)^t+2 \equiv 9(11\cdot 22+1)^t+2 \equiv 9+2(mod 11) \equiv 0(mod 11)$, ya'ni $N>11$ va $N\vdots 11$. Demak u murakkab son.

2).Bu yerda $3^{4n+1}=3\cdot(81)^n=3\cdot(8\cdot 10+1)^n \equiv 3(mod 10)$, ya'ni $3^{4n+1}=3+10k,\ k\in N$. Shuning uchun ham $M=2^{3^{4n+1}}+3=2^{3+10k}+3=2^3\cdot(2^5)^{2k}+3=8(32)^{2k}+3 \equiv 8(-1)^{2k}+3(mod 11) \equiv 0(mod 11)$. Bu yerdan $M>11$ bo'lgani uchun $M\vdots 11$ va u murakkab son degan xulosa kelib chiqadi.

190.1).$2^x+7^y=19^z$ tenglamani qaraymiz. $19 \equiv 1(mod 3)$ bo'lganidan $19^z \equiv 1(mod 3)$. Lekin $2^x \equiv (-1)^x(mod 3)$ va $7^y \equiv 1(mod 3)$ bo'lgani uchun $2^x+7^y \equiv (-1)^x+1(mod 3)$. Bu yerdan, agar x juft son bo'lsa, $2^x + 7^y \equiv 2(mod3)$; agarda x –toq son bo'lsa, $2^x + 7^y \equiv 0(mod3)$ larga ega bo'lamiz. Shunday qilib $2^x + 7^y \not\equiv 19^z(mod3)$. Bundan $2^x + 7^y = 19^z$ tenglama x, y, z natural sonlarda yechimga ega emas degan xulosaga kelamiz.

2).Endi $2^x + 5^y = 19^z$ tenglamani qaraymiz. Bu holda 1)-misolga asosan $2^x + 5^y = (-1)^x + (-1)^y(mod3)$. Agar bu yerda x va y larning ikkalasi ham toq son bo'lsa, $2^x + 5^y \equiv -2 \equiv 1(mod3)$ bo'ladi hamda $2^x + 5^y = 19^z(mod3)$ kelib chiqadi. Agar $2^x + 5^y = 19^z$ tenglama x, y, z larning biror natural qiymatlarida o'rinli bo'lsa, $2^x + 5^y$ va 19^z lar ixtiyoriy modul bo'yicha ham taqqoslanuvchi bo'lishi kerak $x = 2n + 1, y = 2n + 1$ bo'lsin. $2^x + 5^y = 19^z(mod5)$ taqqoslamani qaraymiz. $2^{2n+1} + 5^{2n+1} = 2 \cdot 4^n + 5^{2n+1} \equiv 2(-1)^n(mod5)$, qaralayotgan tenglamaning ikkinchi tomoni $19^z \equiv (-1)^z(mod5)$ bo'lgani uchun $2^{2n+1} + 5^{2n+1} \not\equiv 19^z(mod5)$. Demak, $2^x + 5^y = 19^z$ tenglama x, y, z- natural sonlarda yechimga ega emas.

Izoh: Bu tenglamalarning yechimga ega emasligini taqqoslamalardan foydalanmasdan turib ham isbotlash mumkin. Masalan birinchi tenglamadan $2^x = 19^z - 7^y = (19^z - 1) - (7^y - 1) = 18(19^{z-1} + 19^{z-2} + \dots + 1) - 6(7^{y-1} + 7^{y-2} + \dots + 1) = 3[6(19^{z-1} + 19^{z-2} + \dots + 1) - 2(7^{y-1} + 7^{y-2} + \dots + 1)]$. Bu yerdan ko'rinadiki $(19^x - 7^y):3$. Lekin 2^x soni 3 ga bo'linmaydi. Demak, $2^x \neq 19^z - 7^y$, ya'ni $2^x - 7^y \neq 19^z$.

191. Masala shartiga ko'ra $11a + 2b \equiv 0(mod\ 19)$ bo'lib, bu yerda taqqoslamalarning xossasiga ko'ra $30a + 2b \equiv 0(mod\ 19)$ => $15a + b \equiv 0(mod\ 19)$ => $b \equiv 4a(mod\ 19)$ ekanligini hosil qilamiz. Bunday holda $18a + 5b \equiv 18a + 20a \equiv 38a \equiv 0(mod\ 19)$ bo'lib, bundan esa $18a + 5b \equiv 0(mod\ 19)$ ekanligi kelib chiqadi. Bu esa $\frac{18a+5b}{19}$ ning ham butun son ekanligini isbotlaydi.

192. Berilgan taqqoslamada $n^2 - 1 = (n-1)(n+1)$ bo'lib n toq son bo'lgani uchun $(n-1)$va $(n+1)$ lar ketma-ket keluvchi juft sonlar bo'ladi. Shuning uchun ham $n - 1$ soni 2ga bo'linsa, $n + 1$ soni 4ga bo'linadi. U holda ularning ko'paytmasi 8 ga bo'linadi. Shu tasdiqni taqqoslamalar tilida $n^2 - 1 \equiv 0(mod8)$ ko'rinishda yoziladi.

193. Bu yerda $11\cdot 31-1=340=5\cdot 68$ va $2^5\equiv -1(mod11)$ bo'lgani uchun $2^{11\cdot 31-1}=(2^5)^{68}\equiv(-1)^{68}\equiv 1(mod11)$. Shuningdek $2^5\equiv 1(mod31)$ bo'lgani uchun $2^{11\cdot 31-1}=(2^5)^{68}\equiv 1^{68}\equiv 1(mod31)$.

Agar taqqoslama bir necha modul bo'yicha o'rinli bo'lsa, u shu modullarning eng kichik umumiy karralisi bo'yicha ham o'rinli bo'ladi (8-xossa). Shuning uchun ham $2^{11\cdot 31-1}\equiv 1(mod11\cdot 31)$. Bu oxirgi taqqoslamaning ikkala tomonini ayniy taqqoslama $2\equiv 2(mod11\cdot 31)$ga ko'paytirsak isbotlanishi talab etilgan taqqoslama kelib chiqadi.

194. Bu yerda $1,2,3,\dots,\frac{p-1}{2},\frac{p+1}{2},\dots p-2,p-1$ sonlarini qarab ulardan quyidagi $\frac{p-1}{2}$ ta taqqoslamalarni tuzamiz:

$$1\equiv-(p-1)(modp),\ 2\equiv-(p-2)(modp),\dots,\frac{p-1}{2}\equiv -\frac{p+1}{2}(modp).$$

Bu taqqoslamalarning har birini $2k+1$ darajaga ko'tarib qo'shamiz. U holda

$$1^{2k+1}+2^{2k+1}+3^{2k+1}+\dots+\left(\frac{p-1}{2}\right)^{2k+1}\equiv-(p-1)^{2k+1}-$$

$(p-2)^{2k+1}-(p-3)^{2k+1}-\dots-\left(\frac{p+1}{2}\right)^{2k+1}$ $(modp)$ hosil bo'ladi.

Bundan

$$1^{2k+1}+2^{2k+1}+3^{2k+1}+\dots+\left(\frac{p-1}{2}\right)^{2k+1}+\left(\frac{p+1}{2}\right)^{2k+1}+\dots$$
$$+(p-3)^{2k+1}+(p-2)^{2k+1}+(p-1)^{2k+1}\equiv 0(modp).$$

III.2-§.

195. $m=10$ moduli bo'yicha barcha sinflarni $x=10\cdot q+r$, $0\le r<10$ ko'rinishda yozish mumkin. Bu tenglamani taqqoslama ko'rinishida yozsak $x\equiv r(mod10)$, bunda $r=0,1,2,\dots,9$. Buni $x\equiv 0,1,2,\dots,9(mod10)$ ko'rinishida yozsak bo'ladi.

196. 1). $m=9$ bo'lsa, m moduli bo'yicha chegirmalarning to'la sistemalari: $1,2,3,4,\dots,9$ 9 moduli bo'yicha eng kichik musbat chegirmalarining to'la sistemasi. $-9,-8,-7,\dots,-2,-1$ 9 moduli bo'yicha eng katta manfiy chegirmalarining to'la sistemasi; $0;\pm1;\ \pm2;\ \pm3;\ \pm4-$ 9 moduli bo'yicha absolyut qiymati jihatidan eng kichik chegirmalarining to'la sistemasi.

Endi $m = 9$ modul bo'yicha chegirmalarning keltirilgan sistemalarini yozamiz. Ular mos ravishda quyidagicha bo'ladi (buning uchun yuqorida yozilgan to'la sistemadagi chegirmalardan 9 bilan o'zaro tublarini ajratib olish kifoya):

1,2,4,5,7,8; $-1,-2,-4,-5,-7,-8$; ±1; ±2; ±4.

2). $m = 8$ – moduli bo'yicha chegirmalarning izlanayotgan to'la sistemalari:

1,2,3,4, ... ,8; $-8,-7,-6,-5,\dots,-2,-1$; ±1; ±2; ±3; ±4.

$m = 8$ – moduli bo'yicha chegirmalarning izlanayotgan keltirilgan sistemalari:

$1,3,5,7$; $-1,-3,-5,-7$; ±1; ±3.

3). $p = 13$ – moduli bo'yicha chegirmalarning izlanayotgan to'la sistemalari: $1,2,3,4,\dots,13$; $-13,-12,-11,\dots,-2,-1$;

0, ±1, ±2, ±3, ±4, ±5, ±6.

$p = 13$ - moduli bo'yicha chegirmalarning izlanayotgan keltirilgan sistemalari: $1,2,3,4,\dots,12$; $-12,-11,\dots,-2,-1$; ±1, ±2, ±3, ±4, ±5, ±6.

4). $m = 12$ – moduli bo'yicha chegirmalarning izlanayotgan to'la sistemalari:

$1,2,3,4,\dots,12$; $-12,-11,-10,\dots,-2,-1$; ±1, ±2, ±3, ±4, ±5, ±6.

$m = 12$ – moduli bo'yicha chegirmalarning izlanayotgan keltirilgan sistemalar $1,5,7,11$; $-1,-5,-7,-11$; ±1; ±5.

5). $p = 7$-moduli bo'yicha chegirmalarning izlanayotgan to'la sistemalari: $1,2,3,4,5,6,7$; $-7,-6,-5,-4,-3,-2,-1$;

$0,\pm1,\pm2,\pm3$.

$p = 7$ – moduli bo'yicha chegirmalarning izlanayotgan keltirilgan sistemalari: $1,2,3,4,5,6$; $-7 - 6,-5,-4,-3,-2,-1$; $\pm1,\pm2,\pm3$.

6). $m = 10$ – moduli bo'yicha chegirmalarning izlanayotgan to'la sistemalari: $1,2,3,4,\dots,10$; $-10,-9,-8,\dots,-2,-1$; ±1, ±2, ±3, ±4, ±5,

$m = 10$ – moduli bo'yicha chegirmalarning izlanayotgan keltirilgan sistemalari: $1,3,7,9$; $-9,-7,-3,-1$; ±1, ±3.

197. $x = 10q + r, 0 \le r < 10$ dan $x = 10q$, $x = 10q + 1$, $x = 10q + 2$, $x = 10q + 3$, $x = 10q + 4$, $x = 10q + 5$, $x = 10q + 6, x = 10q + 7, x = 10q + 8$, $x = 10q + 9$.

198. *a)* $(10, x) = 1$ va $x \leq 10$ bo'lishi kerak. Ularning soni $\varphi(10) = 4$ ta va ular $x = 10q + 1, x = 10q + 3, x = 10q + 7,$ $x = 10q + 9,$ bularni taqqoslama ko'rinishida yozsak.

$x \equiv 1(mod10), x \equiv 3(mod10), x \equiv 7(mod10),$
$x \equiv 9(mod10),$ yoki qisqacha yozsak $x \equiv 1, 3, 7, 9(mod10)$.

b) $(10, x) = 2$ va $x \leq 10$ bo'lishi kerak, 3 – misoldan $x = 10q + 2,$ $x = 10q + 4,$ $x = 10q + 6,$ $x = 10q + 8,$ yoki bulardan $x \equiv 2, 4, 6, 8(mod10)$.

c) $(10, x) = 5$ va $x \leq 10$ bo'lishi kerak, ya'ni 3-misoldan $x = 10q + 5,$ ya`ni $x \equiv 5(mod10)$.

d) $(10, x) = 10$ va $x \leq 10$ bo'lishi kerak, 3-misoldan $x = 10q,$ ya`ni $x \equiv 0(mod10)$.

199. Buni isbotlash uchun quyidagi 2 ta holatni e'tiborga olish kifoya. Birinchidan *md* modul bo'yicha sinflar soni, *m* modul bo'yicha sinflar sonidan *d* marta ko'p. Ikkinchidan *m* modul bo'yicha taqqoslanmaydigan sonlar *md* modul bo'yicha ham taqqoslanmaydi.

200. Masalan:
$1, 2, 3, 4, 5, 6, 7, 8, 9, 10; -10, -9, -8, -7, -6, -5, -4, -3, -2, -1;$
$\pm 1, \pm 2, \pm 3, \pm 4, \pm 5,$ umumiy holda $x = 10q + r, 0 \leq r < 10$va $q \in Z$.

201. $\frac{Z}{10Z} = \{C_0, C_1, C_2, \ldots, C_9\}$ to'plamlarni qarasak va bu to'plamda qo'shish hamda ko'paytirish amallarini (2) va (3) tengliklar yordamida aniqlash bu to'plam shu amallarga nisbatan yopiq ekanligini jadvallardan ko'rish qiyin emas.

+	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
C_1	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_0
C_2	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_0	C_1
C_3	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_0	C_1	C_2
C_4	C_4	C_5	C_6	C_7	C_8	C_9	C_0	C_1	C_2	C_3
C_5	C_5	C_6	C_7	C_8	C_9	C_0	C_1	C_2	C_3	C_4
C_6	C_6	C_7	C_8	C_9	C_0	C_1	C_2	C_3	C_4	C_5
C_7	C_7	C_8	C_9	C_0	C_1	C_2	C_3	C_4	C_5	C_6
C_8	C_8	C_9	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7
C_9	C_9	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8

*	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
C_0	C_0	C_0	C_0	C_0	C_0	C_0	C_0	C_0	C_0	C_0
C_1	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
C_2	C_0	C_2	C_4	C_6	C_8	C_0	C_2	C_4	C_6	C_8
C_3	C_0	C_3	C_6	C_9	C_2	C_5	C_8	C_1	C_4	C_7
C_4	C_0	C_4	C_8	C_2	C_6	C_0	C_4	C_8	C_2	C_6
C_5	C_0	C_5	C_0	C_5	C_0	C_5	C_0	C_5	C_0	C_5
C_6	C_0	C_6	C_2	C_8	C_4	C_0	C_6	C_2	C_8	C_4
C_7	C_0	C_7	C_4	C_1`	C_8	C_5	C_2	C_9	C_6	C_3
C_8	C_0	C_8	C_6	C_4	C_2	C_0	C_8	C_6	C_4	C_2
C_9	C_0	C_9	C_8	C_7	C_6	C_5	C_4	C_3	C_2	C_1

$\langle \frac{z}{10\,z}; +; \cdot \rangle$ ning halqa bo‘lishi uchun additiv Abel gruppasi, multipikativ yarim guruh va distributuvlik sharti $(C_i + C_s)C_j = C_iC_j + C_sC_j$ bajarilishi kerak.

Endi shu shartlarning bajarilishini tekshiramiz.

I. AdditivAbel gruppasi: a) $\forall C_i, C_e, C_s \in \frac{Z}{10Z}$ elementlar uchun $(C_i + C_e) + C_s = C_i + (C_e + C_s)$ - assotsiativlik sharti bajarilishi kerak. Bu yerda $C_i = (10q + i)$,$C_e = (10q + e)$, $C_s = (10q + s)$ bo‘lgani uchun $(C_i + C_e) + C_s = C_{i+e} + C_s = C_{i+e+s}$ (yoki $C_{i+e-m+s-m} = C_{i+e+s-m}$). Shuningdek $C_i + (C_e + C_s) = C_i + C_{e+s} = C_{i+e+s}$ (yoki $C_{i+e+s-2m}$). Bu tengliklarning o‘ng tomonlari teng, demak, chap tamonlari ham teng bo‘lishi kerak. Bundan assotsiativlik shartining bajarilishi kelib chiqadi.

b) $\forall C_i \in \frac{Z}{10Z}$ uchun $\exists\, C_0 \in \frac{Z}{10Z}$ bo‘lib $C_i + C_0 = C_0 + C_i = C_i$ bajariladi, ya`ni qaralayotgan to‘plamda nol element mavjud.

c) $\forall C_i \in \frac{Z}{10Z}$ uchun $\exists\, C_{10-i} \in \frac{Z}{10Z}$ bo‘lib $C_i + C_{10-i} = C_{10-i} = C_{10} = C_0$ bajariladi, ya`ni qaralayotgan to‘plamda $\forall\, C_i$ ga qarama-qarshi element C_{10-i} mavjud.

d) $\forall C_i \in \frac{Z}{10Z}$ uchun $C_i + C_j = C_j + C_i = C_{i+j}$ (yoki C_{i+j-m}) bajariladi.

Shunday qilib qaralayotgan to‘plam qo‘shishga nisbatan additiv Abel gruppasi bo‘lar ekan.

II. $<\frac{Z}{10Z};\cdot>$ ning multiplikativ yarim gurppa bo'lishini tekshiramiz: $\forall C_i, C_j, C_e \in \frac{Z}{10Z}$ uchun $C_i(C_j \cdot C_e) = (C_i \cdot C_j)C_e$ ning bajarilishini ko'rsatish yetarli tenglikning chap tomoni $C_i(C_j \cdot C_e) = C_i \cdot C_{je} = C_{ije} = C_r$, bunda $ije = 10q + r$. O'ng tomoni $(C_i \cdot C_j)C_e = C_{ij} \cdot C_e = C_{ije} = C_r$, Bulardan isbotlanishi kerak bo'lgan tenglik kelib chiqadi.

III. Distributivlik sharti $\forall C_i, C_j, C_e \in \frac{Z}{mZ}$ lar uchun $(C_i + C_j)C_e = C_iC_e + C_jC_e$ tenglikning bajarilishini tekshiramiz. Bu tenglik chap tomoni (soddalik uchun $i + j + l < m$ deb qaraymiz; $i + j + l > m$ holi ham shunga o'xshash qaraladi). $(C_i + C_j)C_e = C_{i+j} \cdot C_e = C_{(i+j)e}$. O'ng tomoni $C_i \cdot C_e + C_jC_e = C_{ie} + C_{je} = C_{ij+je} = C_{(i+j)e}$ demak, bu tenglikning chap tomonlari teng, o'ng tomonlari ham teng bo'lishi kerak. Bundan ega isbotlanish talab etilgan tenglik kelib chiqadi. Shunday qilib $\langle\frac{Z}{10Z}; +; *\rangle$ sistema halqa bo'lar ekan.

202. m moduli bo'yicha chegirmalarning to'la sistemasida m ta chegirma bo'lib ular shu modul bo'yicha o'zaro taqqoslanmaydigan bo'lishi kerak. Bizga 5 ta son $20, -4, 22, 18, -1$, berilgan. Demak, $m = 5$ deb olib, berilgan sonlarning 5 moduli bo'yicha o'zaro taqqoslanuvchi emas ekanligini ko'rsatamiz. Buning uchun berilgan sonlarni manfiy bo'lmagan eng kichik chermalar ko'rinishiga keltirib olamiz. U holda $0, 1, 2, 3, 4$ larga ega bo'lamiz. Bular $m = 5$ moduli bo'yicha o'zaro taqqoslanmaydi. J: $m = 5$.

203. Berilgan $20, 31, -8, -5, 25, 14, 8, -1, 13$ va 6 sonlarning soni 10 bo'lib ularni eng kichik musbat chegirmalar ko'rinishida yozsak: $0, 1, 2, 5, 5, 4, 8, 9, 3, 6$ hosil bo'ladi. Bunda -5 va 25 lar $m = 10$ moduli boyicha o'zaro taqqoslanuvchi, ya`ni ular bitta sinifga tegishli. Shuning uchun ham berilgan sonlar $m = 10$ moduli bo'yicha chegirmalarning to'la sistemasini tashkil etmaydi.

204. Istalgan m ta ketma-ket kelgan $x + b$, $x = 0, 1, 2, \dots, m-1$ sonlarni qaraymiz. Bu yerda $(m, 1) = 1$ va 1-teoremani ($a = 1$ deb) qo'llasak $x + b, x = 0, 1, 2, \dots, m-1$ sonlarni m moduli bo'yicha chegirmalarning to'la sistemasini hosil qiladi degan xulosaga kelamiz.

205. Berilgan sonlarning soni m ta bo'lib ular m moduli bo'yicha o'zaro taqqoslanmaydi. Agar $-\frac{m-i}{2} \equiv \frac{m-j}{2} (mod m)$ desak ($1 \le i, j < m$

$-\frac{m-i}{2}-\frac{m-j}{2}\equiv 0(modm)\Rightarrow\frac{-2m+j+i}{2}\equiv 0(modm)$ yoki $\frac{j+i}{2}\equiv 0(modm)\Rightarrow j\equiv -i(mod)\Rightarrow j=-i+mt.$ U holda $\frac{-m+i}{2}\equiv\frac{m+i-mt}{2}(mod)\Rightarrow\frac{i}{2}\equiv\frac{i}{2}(modm),$ ya`ni $-\frac{m-i}{2}$ va $\frac{m-j}{2}$ chegirmalar bitta sinfdan olingan. Demak, $-\frac{m-i}{2}\not\equiv\frac{m-j}{2}(modm)$ va berilgan sonlar m moduli bo'yicha chegirmalarining to'la sistemasini tashkil etadi.

206. $(10,3)=1$ bo'lgani uchun 1- teoremaga ko'ra agar x o'zgaruvchi $m=10$ moduli bo'yicha chegirmalarning to'la sistemasini qabul qilsa,$3x-1$ ham shu sistemani qabul qiladi, ya`ni

x	0	1	2	3	4	5	6	7	8	9
$3x-1$	9	2	5	8	1	4	7	0	3	6

Bu yerda $3x-1$ning qiymatlarini 10 moduli bo'yicha manfiy bo'lmagan eng kichik chegirma ko'rinishida yozdik.

207. 4 modul chegirmalarning to'la sistemasida 4 ta 4 moduli bo'yicha o'zaro taqqoslanmaydigan chegirma bo'lishi kerak.Bizga ma'lumki, agar $(a,m)=1$ bo'lib x o'zgaruvchi m moduli bo'yicha chegirmalarning keltirilgan sistemasini qabul qilsa $ax+b$ ham shu sistemami qabul qiladi. Bizning misolimizda $a=5$, $b=0$, $m=4$ va $(5,4)=1$. Shuning uchun ham$x$ ga $x=0$, 1, 2, 3 qiymatlar bersak $5x=$ 0, 5, 10,15 lar hosil bo'ladi. Bularni manfiy bo'lmagan eng kichik chegirmalar ko'rinishida yozib olsak 0, 1, 2, 3 izlanayotgan sistema hosil bo'ladi.

208. ax_i+b $(i=1,2,\dots,m)$ ko'rinishidagi sonlar m moduli bo'yicha chegirmalarning to'la sistemasini tashkil qilsa, ularning soni m ta bo'lib m moduli bo'yicha o'zaro taqqoslanmasligi kerak.

U holda $x_i(i=1,2,\dots,m)$ lar qiymatlari ham m ta bo'lib ular ham m moduli bo'yicha o'zaro taqqoslanmaydigan bo'ladilar. Haqiqatan ham, agar $x_l\equiv x_r(modm)$ desak, $(a,m)=1$ sonini tanlab olib taqqoslamaning ikkala tomonini a ga ko'paytiramiz, u holda $ax_l\equiv ax_r(modm)$ bo'ladi. Bu taqqoslamaga $b\equiv b(modm)$ ayniy taqqoslamani hadlab qo'shsak $ax_l+b\equiv ax_r+b(modm)$ hosil bo'ladi. Masalaning shartiga ko'ra bunday bo'lishi mumkin emas. Bu qarama-qarshilik $x_l\equiv x_r(modm)$ deganimizdan kelib chiqdi va demak, $x_l\not\equiv$

$x_r(modm)$. Shuning uchun ham qaralayotgan sonlar $x_i(i = 1,2,\dots,m)m$ moduli bo'yicha chegirmalarning to'la sistemasini tashkil etadi.

209. $f(x_i) = a_n x_i^n + a_{n-1}x_i^{n-1} + \dots + a_1 x_i + a_0, (i = 1,2,\dots,m)$, $(a_i, m) = 1$ ko'rinishidagi sonlar m moduli bo'yicha chegirmalarning to'la sistemasini tashkil qilsa, demak, ularning soni m ta va $f(x_i) \not\equiv f(x_j)(\text{mod}m)$ bajariladi. Bu holda $x_i(i = 1,2,\dots,m)$ larning soni ham m ta bo'ladi va ular m moduli bo'yicha o'zaro taqqoslanmaydigan bo'ladi. Haqiqatan ham, agar $x_s \equiv x_k(modm)$ desak, $x_s^2 \equiv x_k^2(modm)$, $\dots, x_s^{n-1} \equiv x_k^{n-1}(modm), x_s^n \equiv x_k^n(modm)$, $a_0 \equiv a_0(modm)$lar bajariladi. Bu taqqoslamalarning ikkala tomonini mos ravishda $a_1, a_2, \dots, a_{n-1}, a_n$ larga ko'paytirib keyin qo'shsak $f(x_s) \equiv f(x_k)(modm)$ ga ega bo'lamiz. Lekin masalaning shartiga ko'ra $f(x_s) \not\equiv f(x_k)(modm)$. Bu qarama-qarshilik $x_i(i = 1,2,\dots,m)$ lar ichida o'zaro taqqoslanuvchilar yo'q ekanligini bildiradi va demak ular m moduli bo'yicha chegirmalarning to'la sistemasini tashkil qiladi. Aksincha tasdiq ham shunga o'xshash isbotlanadi.

210. m moduli bo'yicha chegirmalar keltirilgan sistemasida $\varphi(m)$ta chegirma bo'lib ularning har biri m moduli bilan o'zaro tub bo'lishi kerak. Masalada $m = 6$, $\varphi(6) = \varphi(2)\cdot\varphi(3) = (2-1)(3-1) = 2$. $x \le 6$ va $(x;\ 6) = 1$ shartlarni qanoatlantiruvchi sonlarni yozib olish kifoya: $1,5$; $-5,5$; $-5,-1$; $7,\ 11$; $13,17$.

211. Qulaylik uchun berilgan chegirmalarni eng kichik musbat chegirmalar ko'rinishida yo'zib olamiz. U holda $7,1,11,3,5$ va $\varphi(12) = \varphi(2^2\cdot 3) = \varphi(2^2)\cdot\varphi(3) = (2^2-2)(3-1) = 4$ bo'lgani uchun 12 modulli bo'yicha chegirmalarning keltirilgan sistemasida 4ta chegirma bo'lish kerak va ularning har biri 12 bilan o'zaro tub bo'lishi kerak. Bizda 5 ta chegirma bor, lekin$(3,12) = 3$. Shuning uchun ham berilgan sonlar sistemasi 12 moduli bo'yicha chegirmalarning keltirilgan sistemasini tashkil etmaydi.

212. p modul bo'yicha chegirmalarning to'la sistemasi sifatida $1,2,3,\dots,p-1$, p larni olish mumkun. Bularning ichidan p bilan o'zaro tublarini ajratib olsak: $1,2,3,\dots,p-1$ chegirmalarning keltirilgan sistemasi hosil bo'ladi. Bu sistemadagi chegirmalar soni $p-1$ ta.

213. Berilgan chegirmalar soni $\varphi(p) = p-1$ ta va ularning ham biri p bilan o'zaro tub, ya`ni $\left(\frac{p-i}{2};\ p\right) = 1$, bunda $p > 2$ tub son, $i =$

$2k+1$ toq son $\frac{p-i}{2}<p$ va demak $\frac{p-i}{2}$ soni p tub soniga bo‘linmaydi. Qaralayotgan chegirmalarning p moduli bo‘yicha har xil siniflarga tegishli ekanligi 212-masalada isbotlangan edi. Demak qaralayotgan sonlar sistemasi $p>2$ moduli bo‘yicha chegirmalarning keltirilgan sistemasini tashkil etadi.

214. Qaralayotgan sistemada $\varphi(7)=7-1=6$ ta son bor. Ularning har biri 7 bilan o‘zaro tub, chunki $(5;\ 7)=1$. Ular turli sinflarga tegishli, chunki $5^i\equiv 5^j(\mathrm{mod}7)$ $(0<j\le i\le 6)$ dan $5^{i-j}\equiv 1(\mathrm{mod}7)$, bundan $i=j$ kelib chiqadi. Demak, chegirmalarning keltirilgan sistemasining ta`rifiga asosan berilgan sonlar sistemasi 7 modul bo‘yicha chegirmalarning keltirilgan sistemasini tashkil etadi.

215. $ax_i, (i=1,2,\dots\varphi(m))$ sonlarni m moduli bo‘yicha chegirmalarning keltirilgan sistemasini tashkil etsa, ularning soni $\varphi(m)$ ta bo‘lib $(ax_i;m)=1$ va $ax_1\not\equiv ax_s\ (modm)$ bo‘lishi kerak. Bundan $(a;m)=1$ va$(x_i;m)=1$ kelib chiqadi. Bizda $x_i\ (i=1,2,\dots\varphi(m))$ larning soni $\varphi(m)$ ta va $(x_i;m)=1$ $x_s\not\equiv x_k\ (modm)$ ekanligini ko‘rsatamiz. Faraz etaylik $x_s\equiv x_k\ (modm)$ bo‘lsin, u holda bu taqqoslamaning ikkala tomoni $a, (a,m)=1$ soni ko‘paytiramiz. U holda $ax_s\equiv ax_k\ (modm)$ taqqoslamaga ega bo‘lamiz. Masalaning sharti bo‘yicha $ax_s\not\equiv ax_k(modm)$. Bu qarama-qarshilik $x_s\equiv x_k\ (modm)$ bo‘lsin degan farazimizdan kelib chiqdi. Demak $x_s\not\equiv x_k\ (modm)$ ekan. Shunday qilib, agar $ax_i\ (i=1,2,\dots\varphi(m))$ sonlari m modul bo‘yicha chegirmalarning keltirilgan sistemasini tashkil qilsa, $x_i\ (i=1,2,\dots\varphi(m))$ sonlari ham m moduli bo‘yicha chegirmalarning keltirilgan sistemasini tashkil qilar ekan.

216. x o‘zgaruvchining qiymatlari $x_1,x_2,\dots,x_{\varphi(m)}$ (bunda $(x_i,m)=1$ va $x_i\not\equiv x_j(modm)$)lar m modul bo‘yicha chegirmalarning keltirilgan sistemasini tashkil etgani uchun bu qiymatlarni $ax+b$ ga qo‘yib $\varphi(m)$ ta $ax_1+b,\ ax_2+b,\ \dots\ ,ax_{\varphi(m)}+b$ songa ega bo‘lamiz.

Endi ularning har xil sinflarga tegishli ekanligini va m modul bilan o‘zaro tub ekanligini ko‘rsatamiz. Agar $ax_i+b\equiv ax_j+b(modm)$ desak, bu taqqoslamalarning xossalariga ko‘ra $ax_i\equiv ax_j(modm)$ ga teng kuchli. Buning ikkala tomonini $a,(a,m)=1$soniga qisqartirsak $x_i\equiv x_j(modm)$ ga ega bo‘lamiz. Bu esa$x_i\not\equiv x_j(modm)$shartga ziddir. Demak, qaralayotgan sonlar m moduli bo‘yicha har xil sinflarga tegishli ekan. $(ax_i + b,\ m) = d > 1$ desak, $ax_i + b \equiv 0(mod d)$ va $m \equiv 0(mod d)$ ga ega bo'lamiz. $b = m \cdot b_1$ va $m \equiv d \cdot m_1$ bo'lgani uchun $b = d \cdot (m_1 \cdot b_1)$ bo'ladi, ya'ni b soni d ga bo'linadi. U holda $ax_i + b \equiv 0(mod d)$ dan $ax_i \equiv 0(mod d)$ ni hosil qilamiz. $(a, m) = 1$ dan $(a, d) = 1$ ekanligi kelib chiqadi. Shuning uchun $ax_i \equiv 0(mod d)$ dan $x_i \equiv 0(mod d)$ bajarilishi kerak degan xulosa kelib chiqadi. Bunday bo'lishi mumkin emas, chunki $(x_i, m) = 1$ va demak, $(x_i, d) = 1$. Bu yerdan $\varphi(m)$ta $ax_1 + b,\ ax_2 + b,\ \dots\ , ax_{\varphi(m)} + b$ larning har xil sinflarga tegishli ekanligi kelib chiqadi.

217. $(a; m) = d$ shart $\left(\frac{a}{d}; \frac{m}{d}\right) = 1$ ga teng kuchli. Shuning uchun ham a ning o'rniga $\frac{a}{d}$ va m ning o'rniga $\frac{m}{d}$ ni olib 1- teoremani qo'llaymiz. U holda 1-teoremadan – agar x o'zgaruvchi $\frac{m}{d}$ moduli bo'yicha chegirmalarning to'la sistemasini qabul qilsa, $\frac{a}{d}x + b$ ham $\frac{m}{d}$ moduli bo'yicha chegirmalarning to'la sistemasini qabul qiladi degan tasdiq kelib chiqadi.

218. $(a; m) = d$ shartdan $\left(\frac{a}{d}; \frac{m}{d}\right) = 1$ shart kelib chiqadi. Shuning uchun ham a ni $\frac{a}{d}$ bilan, m ni $\frac{m}{d}$ bilan almashtirib 2 – teoremani qo'llaymiz. U holda 2- teoremadan – "agar x o'zgaruvchi $\frac{m}{d}$ moduli bo'yicha chegirmalarning keltirilgan sistemasini qabul qilsa, u holda ax ham $\frac{m}{d}$ moduli bo'yicha chegirmalarning keltirilgan sistemasini qabul qiladi" – degan tasdiqqa ega bo'lamiz.

219. m=9 moduli bo'yicha chegirmalarning to'la sistemasida 9 ta son bo'lib ular o'zaro taqqoslanmaydigan bo'lishi kerak. Shuning uchun ham:

$1, 2, 3, 4, 5, 6, 7, 8, 9$–lar m=9 moduli bo'yicha musbat eng kichik chegirmalarning to'la sistemasi;

$0, 1, 2, 3, 4, 5, 6, 7, 8$– lar m=9 moduli bo'yicha manfiy bo'lmagan eng kichik chegirmalarning to'la sistemasi;

$0, \pm1, \pm2, \pm3, \pm4$– lar m=9 moduli bo'yicha absolyut qiymati jihatidan eng kichik chegirmalarning to'la sistemasi bo'ldi.

Endi m=9 moduli bo'yicha chegirmalarning keltirilgan sistemalarini 3 xil (musbat, manfiy bo'lmagan, absolyut qyimati jihatidan eng kichik chegirmalar) ko'rinishda yozish uchun to'la sistemalardagi chegirmalarning m bilan o'zaro tublarini ajratib olish kifoya, ya'ni ularning har birida $\varphi(9)=6$ ta chegirma bo'ladi. Shuning uchun ham:

$1,2,4,5,7,8$ - lar $m=9$ moduli bo'yicha musbat eng kichik chegirmalarning keltirilgan sistemasi;

$1,2,4,5,7,8$ – lar $m=9$ moduli bo'yicha manfiy bo'lmagan eng kichik chegirmalarning keltirilgan sistemasi;

$\pm1,\pm2,\pm4$– lar $m=9$ moduli boyicha absolyut qiymati jihatidan eng kichik chegirmalarning keltirilgan sistemasi bo'ladi.

Shuni ham ta'kidlash kerakki, bu misolda $m=9$ moduli bo'yicha musbat eng kichik chegirmalarning va manfiy bo'lmagan eng kichik chegirmalarning keltirilgan sistemalari bir xil bo'lar ekan.

III.3-§.

220. a) $(a,7)=1$ bo'lganligi uchun Ferma teoremasiga ko'ra $a^6\equiv 1(mod7)$ bajariladi. Bundan $a^{12}\equiv 1(mod7)$, ya'ni $(a^{12}-1)\,\vdots\,7$.

b) $(a,65)=1$ dan $(a;5\cdot13)=(a;5)=(a;13)=1$ kelib chiqadi. Demak, Ferma teoremasiga asosan $a^{12}\equiv 1(mod13)$ va $a^4\equiv 1(mod5)$. Oxirgi taqqoslamaning ikkala tomonini kubga ko'tarsak $a^{12}\equiv 1(mod5)$ hosil bo'ladi. $a^{12}\equiv 1(mod5)$ va $a^{12}\equiv 1(mod13)$ hamda $(5;13)=1$ dan $a^{12}\equiv 1(mod65)$ kelib chiqadi. $(b;65)=1$ bo'lganligi uchun yuqoridagidek mulohaza yuritib $b^{12}\equiv 1(mod65)$ ni hosil qilamiz. $a^{12}\equiv 1(mod65)$ va $b^{12}\equiv 1(mod65)$ taqqoslamalardan $a^{12}-b^{12}\equiv 0(mod65)$ ga ega bo'lamiz. Bu esa $a^{12}-b^{12}\,\vdots\,65$ ga teng kuchli.

221. Kanonik yoyilmasiga 2 va 5 sonlari kirmaydigan natural sonni x desak $(x,10)=1$ va $\varphi(10)=4$ bo'lgani uchun Eyler teoremasiga ko'ra $x^4\equiv 1(mod10)$. Shuning uchun ham $x^{12}\equiv(x^4)^3\equiv 1(mod10)$. Demak, kanonik yoyilmasiga 2 va 5 sonlari kirmaydigan natural sonning 12-darajasining birlik raqami 1ga teng ekan.

222. $a\not\equiv 0(modp)$ bo'lgani uchun $(a;p)=1$ deb yozish mumkin. U holda, Ferma teoremasiga ko'ra $a^{p-1}\equiv 1(modp)$ bajariladi. Bundan $a^{p-1}-1\equiv 0(modp)$. Bu taqqoslamaning chap tomoniga p ni qo'shsak (taqqoslamaning istalgan tomoniga yoki ikkala tomoniga modulga karalli bo'lgan sonni qo'shish va ayirish mumkin) $a^{p-1}+p-1\equiv 0(modp)$ hosil bo'ladi. Bundan $(a^{p-1}+p-1)\,\vdots\,p$, ya'ni $a^{p-1}+p-1$ soni murakkab son.

223. Ferma teoremasiga asosan $2^{11-1} \equiv 1(mod11)$, $2^{30} \equiv 1(mod31)$ yoki$2^{11} \equiv 2(mod11)$, $2^{31} \equiv 2(mod31)$. Birinchi taqqoslamadan $(2^{11})^{31} \equiv 2^{31}(mod11) \equiv 2 \cdot (2^6)^5(mod11) \equiv 2 \cdot (-2)^5(mod11) \equiv 2 \cdot (-32)(mod11) \equiv 2(mod11)$. Shunga o'xshash $(2^{31})^{11} \equiv 2^{11}(mod31) \equiv 2 \cdot (2^5)^2(mod31) \equiv 2(mod11)$. Shunday qilib, $2^{11\cdot 31} \equiv 2(mod11)$ va $2^{31\cdot 11} \equiv 2(mod31)$ hamda $(11;31) = 1$ bo'lgani uchun $2^{11\cdot 31} \equiv 2(mod11 \cdot 31)$.

224. Ferma teoremasiga ko'ra $2^{12} \equiv 1(mod\ 13)$. Shuning uchun $2^{24} \equiv 1(mod\ 13)$. Bundan tashqari $2^6 \equiv 64 \equiv -1(mod\ 13)$ ekanligidan $2^{30} \equiv -1 \equiv 12(mod\ 13)$ bo'ladi. Demak, izlangan qoldiq 12 ga teng.

225. $3^{16} \equiv 1(mod17)$ bo'lganligi uchun $3^{59} \equiv 3^{11} \cdot (3^{16})^3 \equiv 3^{11}(mod17) \equiv (3^3)^3 \cdot 3^2(mod17) \equiv 10^3 \cdot 9(mod17) \equiv 1000 \cdot 9(mod17) \equiv 14 \cdot 9(mod17) \equiv 126(mod17) \equiv 7(mod17)$. Demak, 3^{59} ni 17 ga bo'lsak 7 qoldiq qoladi.

226. Ferma teoremasiga asosan $a^{p-1} \equiv 1(modp)$, $(a;p) = 1$. Bu taqqoslamaning ikkala tomonini n-darajaga ko'taramiz. U holda $a^{n(p-1)} \equiv 1(modp)$ hosil bo'ladi. Bundan va $a \equiv a(modp)$ dan $a^{n(p-1)+1} \equiv a(modp)$ kelib chiqadi. Keyingi taqqoslama $a \vdots p$ bo'lsa ham o'rinli. Shunday qilib ixtiyoriy a butun, n-natural va p tub sonlar uchun $a^{n(p-1)+1} \equiv a(modp)$ taqqoslama o'rinli.

227. 317 ni 15 ga bo'lgandagi qoldiq 2 ga teng bo'lgani uchun, ya'ni $317 \equiv 2(mod15)$ ekanligidan $317^{259} \equiv 2^{259}(mod\ 15)$ bo'lishini topamiz. Eyler teoremasiga ko'ra$2^{\varphi(15)} \equiv 1(mod15)$va $\varphi(15) = \varphi(3 \cdot 5) = \varphi(3) \cdot \varphi(5) = 2 \cdot 4 = 8$ bo'lgani uchun $2^8 \equiv 1(mod15)$. $259 = 32 \cdot 8 + 3$ ekanligidan $2^{259} = (2^8)^{32} \cdot 2^3 \equiv 8(mod15)$≡8(mod15) bo'ladi. Demak, 317^{259} sonini 15 ga bo'lgandagi qoldiq 8 ga teng ekan.

228. Bu yerda$(3,11) = 1$. Shuning uchun ham Eyler teoremasiga ko'ra $3^{\varphi(11)} \equiv 1(mod11)$. $\varphi(11) = 11 - 1 = 10$ bo'lganligi sababli $3^{10} \equiv 1(mod11)$ bo'ladi. Bundan

$$3^{80} = (3^{10})^8 \equiv 1^8 \equiv 1(mod11). \qquad (1)$$

Shunga o'xshash $(7,11) = 1$ va Eyler teoremasiga ko'ra $7^{\varphi(11)} \equiv 1(mod11)$ bo'lganligi sababli $7^{10} \equiv 1(mod11)$ bo'ladi. Bundan

$$7^{80} = (7^{10})^8 \equiv 1^8 \equiv 1(mod11). \qquad (2)$$

(1) va (2) taqqoslamalarni hadlab qo'shib

$$3^{80} + 7^{80} \equiv 2(mod 11)$$

ni hosil qilamiz. Demak, $3^{80} + 7^{80}$sonini 11 ga bo'lgandagi qoldiq 2 ga teng ekan.

229. Avvalo 3^{100} ni 7 ga bo'lgandagi qoldiqni topamiz. $(3;7) = 1$ bo'lganligi uchun Ferma teoremasidan $3^6 \equiv 1(mod\ 7)$ kelib chiqadi. Shuning uchun ham

$$3^{100} \equiv (3^6)^{16} \cdot 3^4(mod\ 7) \equiv 3^4(mod\ 7) \equiv 4(mod\ 7). \qquad (1)$$

Endi 4^{100} ni 7 ga bo'lgandagi qoldiqni aniqlaymiz. Bu yerda $(4;7) = 1$ va $4^6 \equiv 1(mod\ 7)$. Shuning uchun

$4^{100} \equiv (4^6)^{16} \cdot 4^4(mod7) \equiv 4^2 \cdot 4^2(mod7) \equiv 9 \cdot 9(mod7) \equiv 4(mod7)$ (2)

(1) va (2) taqqoslamalardan $4^{100} + 3^{100} \equiv 1(mod7)$ hosil bo'ladi. Demak, $4^{100} + 3^{100}$ ni 7 ga bo'lsak 1 qoldiq qoladi.

Izoh. $4^{100} + 3^{100} \equiv 3^{100} + (-3)^{100}(mod7) \equiv 2 \cdot 3^{100}(mod7)$ dan foydalanib ham shu natijani olish mumkin.

230. $197 = 35 \cdot 5 + 22$ bo'lganligi uchun $197^{157} \equiv (35 \cdot 5 + 22)^{157} \equiv 22^{157}(mod35)$. Bu yerda $(22;35) = 1$ va Eyler teoremasiga asosan $22^{\varphi(35)} \equiv 1(mod35)$ yoki $22^{24} \equiv 1(mod35)$. Bundan $22^{157} \equiv (22^{24})^6 \cdot 22^{13}(mod35)$

$\equiv 22^{13}(mod35) \equiv (22^2)^6 \cdot 22^1(mod35) \equiv (-6)^6 \cdot 22(mod35) \equiv ((-6)^2)^3 \cdot 22(mod35) \equiv 22(mod35)$. Shunday qilib, 197^{157} ni 35 ga bo'lgandagi qoldiq 22 chiqar ekan.

231. $2^{72} \equiv 1(mod73)$ va $2^{36} \equiv 1(mod37)$. Bulardan $2^{73} \equiv 2(mod73)$ va$2^{37} \equiv 2(mod37)$. Bu yerdagi birinchi taqqoslamaga asosan $(2^{73})^{37} \equiv 2^{37}(mod73) \equiv (2^6)^6 \cdot 2(mod73) \equiv (-9)^6 \cdot 2(mod73) \equiv ((-9)^2)^3 \cdot 2(mod73) \equiv 8^3 \cdot 2(mod73) \equiv 1024(mod73) \equiv 2(mod73)$, ya'ni

$$(2^{73})^{37} \equiv 2(mod73). \qquad (3)$$

Endi $2^{73} \equiv 2(mod37)$ taqqoslamadan $(2^{37})^{73} \equiv 2^{73}(mod37) \equiv (2^{36})^2 \cdot 2(mod37)) \equiv 2(mod37)$, ya'ni

$$(2^{37})^{73} \equiv 2(mod73). \qquad (4)$$

(3) va (4) taqqoslamalardan $(2^{37})^{73} \equiv 2(mod37 \cdot 73)$, yoki bundan $2^{n-1} \equiv 1(modn)$, bu yerda $n = 37 \cdot 73$.

232. $1^{30} \equiv 1(mod11), 2^{30} \equiv (2^{10})^3 \equiv 1(mod11), \ \dots\ , 10^{30} \equiv 1(mod11)$. Bu yerda $i = 1,2,3,\dots,10$ bo'lsa, $i^{10} \equiv 1(mod11)$

ekanligidan foydalandik. Bundan $1^{30}+2^{30}+\cdots+10^{30}\equiv 10(mod 11)\equiv -1(mod 11)$ kelib chiqadi.

233. *a)* $x^7\equiv x(mod 42)$ dan $x^7\equiv x(mod 2\cdot 3\cdot 7)$. Demak, biz $x^7\equiv x(mod 7)$, $x^7\equiv x(mod 3)$ va $x^7\equiv x(mod 2)$ taqqoslamalarning ixtiyoriy x butun soni uchun o'rinli ekanligini ko'rsatishimiz kerak. Birinchi taqqoslama Ferma teoremasidan bevosita kelib chiqadi. 2- va 3-larni bevosita chegirmalarning to'la sistemasini tekshirib ko'rish bilan ishonch hosil qilamiz. 2 moduli bo'yicha chegirmalarning to'la sistemasi 0 va 1 dan iborat va bularning ikkalasi ham $x^7\equiv x(mod 2)$ taqqoslamani qanoatlantiradi. 3 moduli bo'yicha chegirmalarning to'la sistemasi 0,1,2 dan iborat va bularning uchalasi ham $x^7\equiv x(mod 3)$ taqqoslamani qanoatlantiradi.

b) $x^{13}\equiv x(mod 2730)$ dan $x^{13}\equiv x(mod\ 2\cdot 3\cdot 5\cdot 7\cdot 13)$. Bu yerdan $x^{13}\equiv x(mod 13)$, (Ferma teoremasiga ko'ra); $x^{13}\equiv x(mod 2)$ (0,1 ni qo'yib tekshirsak); $x^3\equiv x(mod 3)$ dan $x^{13}\equiv (x^3)^4\cdot x\equiv x^5\equiv x^3\cdot x^2(mod 3)\equiv x^3(mod 3)\equiv x(mod 3)$. $x^{13}\equiv x(mod 5)$ va $x^{13}\equiv x(mod 7)$ lar ham shunga o'xshash isbotlanadi. Endi hosil bo'lgan $x^{13}\equiv x(mod 2), x^{13}\equiv x(mod 3), x^{13}\equiv x(mod 5), x^{13}\equiv x(mod 7)$ va $x^{13}\equiv x(mod 13)$ taqqoslamalarning ixtiyoriy x butun son uchun o'rinli ekanligidan $x^{13}\equiv x(mod\ 2\cdot 3\cdot 5\cdot 7\cdot 13)$ ning, yoki bundan $x^{13}\equiv x(mod 2730)$ ning o'rinli ekanligi kelib chiqadi.

234. p va q lar $(p;q)=1$ shartni qanoatlantiruvchi tub sonlar bo'lgani uchun $p^{q-1}\equiv 1(mod q)$ va $q^{p-1}\equiv 1(mod p)$. Bu taqqoslamalarni tenglik qilib yozsak $p^{q-1}-1=qt,\ q^{p-1}-1=pl, t, l\epsilon\ Z.$ Bulardan $(p^{q-1}-1)(q^{p-1}-1)=pqtl$ yoki $p^{q-1}\cdot q^{p-1}-p^{q-1}-q^{p-1}+1=pqtl.$ Endi taqqoslama qilib yozsak $q^{p-1}+p^{q-1}-p^{q-1}\cdot q^{p-1}-1\equiv 0(mod pq)$. Bundan $q^{p+1}+p^{q+1}\equiv 1(mod pq)$ kelib chiqadi.

235. 2^{100} sonining oxirgi ikkita raqamini topish uchun uni 100 ga bo'lishdan chiqqan qoldiqni topish kifoya. Bu yerda $100=25\cdot 4$ va Eyler teoremasiga ko'ra $2^{\varphi(25)}\equiv 1(mod 25)$, ya'ni $2^{20}\equiv 1(mod 25)$ hamda $2^{100}=2^{98}\cdot 2^2$ bo'lgani uchun $2^{98}\equiv 2^{80}\cdot 2^{18}(mod 25)\equiv 2^{18}(mod 25)\equiv (2^9)^2(mod 25)\equiv 144(mod 25)\equiv 19(mod 25)$. Buni tenglik qilib yozsak $2^{98}=19+25t$. Bu tenglikni ikkala tomonini 4 ga ko'paytirib taqqoslama ko'rinishida yozamiz. U holda $2^{100} = 76 + 100t$ yoki $2^{100} \equiv 76(mod\ 100)$. Demak, 2^{100} ning oxirgi raqami ikkita raqam 7 va 6.

236. Berilgan sonning oxirgi raqamini topish uchun uni 10 ga bo'lishdan chiqqan qoldiqni topish kifoya. (3,10) = 1va Eyler teoremasiga ko'ra $3^{\varphi(10)} \equiv 1(mod11)$. Bunda $\varphi(10) = \varphi(2 \cdot 5) = \varphi(2) \cdot \varphi(5) = (2-1) \cdot (5-1) = 4$ bo'lganligi sababli $3^4 \equiv 1(mod10)$ bo'ladi. Shuning uchun ham $3^{100} = (3^4)^{25} \equiv 1^{25} \equiv 1(mod10)$. Demak,$3^{100}$ sonining oxirgi raqami 1 ga teng bo'lar ekan.

237. 243^{402} sonining oxirgi uchta raqamini topish uchun uni 1000 ga bo'lishdan chiqqan qoldiqni topish kerak bo'ladi. $243 = 3^5$, $1000 = 10^3 = 5^3 \cdot 2^3$ bo'lgani uchun $(243; 1000) = 1$ va Eyler teoremasiga asosan $243^{\varphi(1000)} \equiv 1\ (mod1000)$ bajariladi. Bu yerda $\varphi(1000) = \varphi(2^3 \cdot 5^3) = \varphi(2^3) \cdot \varphi(5^3) = (2^3 - 2^2)(5^3 - 5^2) = 4 \cdot 100 = 400$ bo'lgani uchun $243^{400} \equiv 1(mod1000)$. Shuning uchun ham $243^{402} \equiv 243^{400} \cdot 243^2 \equiv 243^2(mod1000)$

$\equiv 59049(mod1000) \equiv 49(mod1000)$. Demak, uchta raqami 0,4,9.

238. Shartga asosan $(n; 6) = 1$. Bundan $(n; 2) = 1$ va $(n; 3) = 1$ bo'lgani uchun u toq son $n = 2k + 1$, u holda $n^2 - 1 = (n-1)(n+1) = (2k+1-1)(2k+1+1) = 4k(k+1)$ ifoda 8 ga bo'linadi , ya'ni $n^2 - 1 = 0(mod8)$ yoki bundan

$$n^2 \equiv 1(mod8). \qquad (5)$$

Ikkinchi tomondan $(n; 3) = 1$ bo'lgani uchun Ferma teoremasiga asosan

$$n^2 \equiv 1(mod3). \qquad (6)$$

Hamda $(8; 3) = 1$ bo'lgani uchun (5) va (6) dan $n^2 \equiv 1(mod24)$ kelib chiqadi.

239. Ferma teoremasiga ko'ra : $1^{p-1} \equiv 1(modp)$, $2^{p-1} \equiv 1(modp)$, ... , $(p-1)^{p-1} \equiv 1(modp)$. Bunda p- tub son . Bu taqqoslamaning har birini $k\epsilon N$ darajaga ko'tarib keyin hadlab qo'shamiz . U holda

$$1^{k(p-1)} + 2^{k(p-1)} + \dots + (p-1)^{k(p-1)} \equiv p - 1(modp)$$

hosil bo'ladi. Bundan

$$1^{k(p-1)} + 2^{k(p-1)} + \dots + (p-1)^{k(p-1)} + 1 \equiv 0(modp).$$

Buni qisqacha

$$\sum_{i=1}^{p-1} i^{k(p-1)} + 1 \equiv 0(modp)$$

ko'rinishda yozishimiz mumkin.

240. Ma'lumki, $a^p - a \equiv 0(modp)$. Shunga asosan $(a_1 + a_2 + \dots + a_n)^p \equiv a_1 + a_2 + \dots + a_n(modp)$. Bu yerda $a_1 \equiv a_1^p(modp)$, $a_2 \equiv a_2^p(modp)$,..., $a_n \equiv a_n^p(modp)$ ekanligini e'tiborga olsak : $(a_1 + a_2 + \dots + a_n)^p \equiv a_1^p + a_2^p + \dots + a_n^p(modp)$ ga, ya'ni isbotlanishi kerak bo'lgan taqqoslama $(\sum_{i=1}^{n} a_i)^p \equiv \sum_{i=1}^{n} a_i{}^p$ (modp) ga ega bo'lamiz.

241. Eyler teoremasiga asosan $(a;m) = 1$ bo'lsa , $a^{\varphi(m)} \equiv 1(modm)$ bo'ladi. Endi faraz etaylik x soni $a^x \equiv 1(modm)$ taqqoslamaning eng kichik yechimi bo'lib $\varphi(m) = x \cdot q + r$, $0 \le r < x$ bo'lsin u holda $a^{\varphi(m)} \equiv (a^x)^q \cdot a^r \equiv 1 \cdot a^r(modm) \equiv 1(modm)$, ya'ni $a^r \equiv 1(modm)$. Bu esa x soni

$a^x \equiv 1(modm)$ taqqoslamaning eng kichik yechimi deganimizga zid. Demak, $r = 0$ va $\varphi(m) = x \cdot q$, ya'ni x soni $\varphi(m)$ ning bo'luvchisi.

242. Ferma teoremasiga asosan

$$a_i^5 \equiv a_i(mod5) \text{ va } a_i^5 \equiv a_i(mod2)\,, a_i^5 \equiv a_i(mod3). \qquad (7)$$

Keyingi ikkita taqqoslamaning o'rinli ekanligini bevosita chegirmalarning to'la sistemasini qo'yib tekshirib ko'rish mumkin. Bulardan

$$a_i^5 \equiv a_i(mod30), \qquad i = 1,2,\dots,n.$$

Bu taqqoslamalarni hadlab qo'shsak

$$\sum_{i=1}^{n} a_i^5 \equiv \sum_{i=1}^{n} a_i(mod30),$$

ya'ni $M \equiv N(mod30)$. Bundan, agar N soni 30 bo'linsa, M ning ham 30 ga bo'linishi kelib chiqadi.

Izoh.(7) taqqoslamalar $a_i^5 \equiv a_i(mod6)$ ga teng kuchli bu taqqoslamani

$$a_i^5 - a_i \equiv a_i(a_i^4 - 1) \equiv a_i(a_i - 1)(a_i + 1)(a_i^2 + 1)(mod6)$$
$$\equiv (a_i - 1)a_i(a_i + 1)(a_i^2 + 1)(mod6).$$

Bunda $(a_i - 1)a_i(a_i + 1) \equiv 0(mod6)$ bo'lganligi uchun $a_i^5 - a_i \equiv 0(mod6)$ bajariladi.

243. Agar a soni 5 ga karrali bo'lsa, $a = 5k$ va $a^{100} \equiv (5k)^{100} \equiv 5^{100} \cdot k^{100} \equiv 0(mod125)$. Agarda $(a;5) = 1$ bo'lsa, u holda Eyler teoremasiga asosan $a^{\varphi(125)} \equiv 1(mod125)$. Bundan $a^{\varphi(125)} \equiv a^{\varphi(5^3)} \equiv a^{5^3-5^2} \equiv a^{100} \equiv 1(mod\ 125)$. Demak, agar a butun soni 5 ga karrali bo'lsa, a^{100} ni 125 ga bo'lishdan chiqqan qoldiq 0 ga teng, aks holda qoldiq 1 ga teng bo'lar ekan.

244. Masalaning shartiga ko'ra $(a;10) = 1$. Bu esa $(a;2) = 1$ va $(a;5) = 1$ larga teng kuchli. Agar $(a;5) = 1$ bo'lsa, 24-masalaga asosan

$$a^{100} \equiv 1(mod\ 125)\,. \qquad (8)$$

Ikkinchi tomondan esa Eyler teoremasiga asosan $a^{\varphi(8)} \equiv 1(mod8)$. Bundan $a^4 \equiv 1(mod8)$. Bu taqqoslamaning ikkala tomonini 25–darajaga ko'taramiz, u holda

$$a^{100} \equiv 1(mod\ 8) \qquad (9)$$

taqqoslama hosil bo'ladi. (8) va (9) dan $(8;125) = 1$ bo'lgani uchun $a^{100} \equiv 1(mod\ 1000)$ ni hosil qilamiz. Bu oxirgi taqqoslamaning ikkala tomonini n –darajaga ko'taramiz va keyin ikkala tomonini a ga ko'paytirsak

$$a^{100n+1} \equiv a(mod\ 1000)$$

ga ega bo'lamiz .

245. a soni 7 ga bo'linmasa, u holda $(a;7) = 1$ bo'ladi va $a^6 \equiv 1(mod\ 7)$ bo'ladi. Bu taqqoslamani avval m –darajaga keyin esa n –darajaga ko'taramiz. U holda $a^{6m} \equiv 1(mod\ 7)$ va $a^{6n} \equiv 1(mod\ 7)$ larga ega bo'lamiz. Bularni hadlab qo'shsak $a^{6m} + a^{6n} \equiv 2(mod\ 7)$ ni hosil qilamiz. Ya'ni agar a soni 7 ga bo'linmasa $a^{6m} + a^{6n}$ ni 7 ga bo'lsak 2 qldiq qolar ekan. Endi $a \vdots 7$ bo'lsin. U holda $a^{6m} \vdots 7$ va $a^{6n} \vdots 7$ bajariladi. Bundan $(a^{6m} + a^{6n}) \vdots 7$, ya'ni $a^{6m} + a^{6n} = 0(mod7)$.

246. Bu yerda $p \neq 5$ chunki, agarda $p = 5$ bo'lsa, $5^{25} + 1 \equiv 0(mod25)$ bo'lishi kerak.Lekin bu yerda ikkinchi qo'shiluvchi 25 ga bo'linmaydi. Berilgan taqqoslamani quyidagicha yozib olamiz:

$$5^{p^2} + 1 = (5^{p^2} - 5) + 6 = 5\left(5^{p^2-1} - 1\right) + 6 = 5[(5^{p-1})^{p+1} - 1] + 6 \equiv 0(modp^2).$$

Ferma teoremasiga asosan $5^{p-1} - 1 \equiv 0(modp)$. Bu yerda $(5^{p-1})^{p+1} - 1$ soni $5^{p-1} - 1$ ga karrali bo'lganligi uchun $[(5^{p-1})^{p+1} - 1]$ soni p ga bo'linadi. Demak, 6 ham p ga bo'linishi kerak. Bundan $p = 2$ yoki $p = 3$. Agar $p = 2$ bo'lsa, u holda $5^{2^2} + 1 = 5^4 + 1 \equiv 626 \not\equiv 0(mod2^2)$, agarda $p = 3$ bo'lsa, u holda $5^{3^2} + 1 =$

$5^9 + 1 \equiv 1953126 \equiv 0(mod 3^2)$. Shunday qilib izlanayotgan son $p = 3$ ekan.

247. Masalaning sharti bo'yicha p va $2p + 1$ lar tub sonlar. Shuning uchun ham Ferma teoremasiga ko'ra $(2p + 1)^2 \equiv 1(mod 3)$ va $p^2 \equiv 1(mod 3)$. Ikkinchi taqqoslamani 4 ga ko'paytirib $4p^2 \equiv 4(mod 3)$ birinchisidan ayiramiz, u holda $4p^2 + 4p + 1 - 4p^2 \equiv 1 - 4(mod 3)$ yoki $4p + 1 \equiv -3(mod 3)$. Bundan $4p + 1 \equiv 0(mod 3)$. Demak, $4p + 1$ soni 3 dan katta va 3 ga bo'linadi. Shuning uchun ham u murakkab son.

IV.1-§.

248. a). Bu holda 3 moduli bo'yicha chegirmalarning to'la sistemasi 0,1,2 dan iborat. Bu sonlarni berilgan taqqoslama qo'yib sinab ko'ramiz va $x_1 = 1, x_2 = 2$ larning uni qanoatlantirishiga ishonch hosil qilamiz. Demak, berilgan taqqoslamaning yechimlari $x \equiv 1(mod 3)$ va $x \equiv 2(mod 3)$ yoki buni $x = 1 + 3t, t\epsilon Z$ va $x = 2 + 3t, t\epsilon Z$ ko'rinishda yozishimiz mumkin.

b). 5 moduli bo'yicha chegirmalarning to'la sistemasi 0,1,2,3,4. Bu sonlarni berilgan taqqoslamaga qo'ysak ulardan $x_1 = 1$ va$x_2 = 2$ lar uni qanoatlantirishini ko'ramiz. Shuning uchun ham yechimlar$x \equiv 1(mod 5)$va $x \equiv 2(mod 5)$ lardan iborat. **Javob** $x = 1 + 5t, t\epsilon Z$ va $x = 2 + 5t, t\epsilon Z$.

c). 3 moduli bo'yicha chegirmalarning to'la sistemasi 0,1,2 lardan iborat. Bularning birortasi ham berilgan taqqoslamani qanoatlantirmaydi. Demak, taqqoslama yechimga ega emas.

d). 5 moduli bo'yicha chegirmalarning to'la sistemasi 0,1,2,3,4 lardan iborat. Bularni berilgan taqqoslamaga qo'yib sinab ko'rsak, $x_1 = 3$ uni qanoatlantiradi. Demak, javob $x \equiv 3(mod 5)$, ya'ni $x = 3 + 5t, t\epsilon Z$.

e). 7 moduli bo'yicha chegirmalarning to'la sistemasi 0,1,2,3,4,5,6 lardan iborat. Bularni taqqoslamaga bevosita olib borib qo'ysak, $x_1 = 1$ va$x_2 = 2$ lar uni qanoatlantiradi. **Javob:** $x = 1 + 7t, x = 2 + 7t, t\epsilon Z$.

f).15 moduli bo'yicha manfiy bo'lmagan eng kichik chegirmalarning to'la sistemasi $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14$ lardan iborat. Bularni berilgan taqqoslamaga qo'yib sinab ko'rib $x_1 = 11$ ning uni qanoatlantirishini topamiz. Demak, $x \equiv 11(mod 15)$, ya'ni $x = 11 + 15t, t\epsilon Z$ berilgan taqqoslamaning yechimi.

Izoh. Bu holda $x = 1,2,\ldots,14$ larning barchasi emas, balki $2x > 15$ shartni qanoatlantiruvchilari $x = 8,9,10,11,12,13,14$ni ham tekshirish kifoya bo'ladi.

249. 7 moduli bo'yicha chegirmalarning to'la sistemasini, tekshirish qulay bo'lsin uchun uni absolyut qiymati jihatidan eng kichik chegirmalar sistemasi ko'rinishda $0, \pm1, \pm2, \pm3$ yozib olamiz. Berilgan taqqoslamaga bu sonlarni qo'yib tekshirsak faqat 1 uni qanoatlantiradi, demaк $x \equiv 1(mod 7)$ berilgan taqqoslamaning yagona yechimi.

250. Bu yerda 3 moduli bo'yicha absolyut qiymati jihatidan eng kichik chegirmalarning to'la sistemasi $0, \pm1$ dan iborat, lekin bularning birortasi ham berilgan taqqoslamani qanoatlantirmaydi, ya'ni berilgan taqqoslama yechimga ega emas.

251.a). Avvalo koeffitsiyentlarini berilgan 15 moduli bo'yicha bo'yicha absolyut qiymati jihatidan eng kichik chegirmalar bilan almashtiramiz. Bunda $90 = 15 \cdot 6 + 0,\ 46 = 15 \cdot 3 + 1,\ 52 = 15 \cdot 3 + 7$ bo'lgani uchun berilgan taqqoslama $x^2 - 7x + 1 \equiv 0(mod 15)$ taqqoslamaga teng kuchli. Endi 15 moduli bo'yicha chegirmalrning to'la sistemasi $0, \pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm7$ larni qo'yib tekshirib ko'ramiz. U holda $x_1 = -4$ ning berilgan taqqoslamani qanoatlantiradi. Demak, berilgan taqqoslamaning yechimi $x = -4 + 15t, t\epsilon Z$.

b). Bunda $25 = 12 \cdot 2 + 1, 36 = 12 \cdot 3 + 0, 18 = 12 \cdot 1 + 6, 13 = 12 \cdot 1 + 1$bo'lgani uchun berilgan taqqoslama $3x^3 - 6x + 1 \equiv 0(mod 12)$ taqqoslamaga teng kuchli. Endi 12 moduli bo'yicha chegirmalrning to'la sistemasi $\pm1, \pm2, \pm3, \pm4, \pm5, \pm6$ larni qo'yib tekshirib ko'ramiz. Bularning birortasi ham berilgan taqqoslamani qanoatlantirmaydi.Taqqoslamaning yechimi yo'q.

Izoh. Buni quyidagicha izohlash ham mumkin.$3x^3 - 6x + 1 \equiv 0(mod 12)$ taqqoslama

$$\begin{cases} 3x^3 - 6x + 1 \equiv 0(mod 3) \\ 3x^3 - 6x + 1 \equiv 0(mod 4) \end{cases}$$

ga teng kuchli. Bu yerda birinchi taqqoslama $1 \equiv 0(mod 3)$ ziddiyatli taqqoslama bo'lgani uchun sistema va demak, berilgan taqqoslama ham yechimga ega emas.

c).$21x + 4 \equiv 7(mod 6) \rightarrow 3x - 2 \equiv 1(mod 6) \rightarrow 3x \equiv 3(mod 6) \rightarrow x \equiv 1(mod 2), x = 1 + 2t,$ tϵZ, $x \equiv 1,3,5(mod 6)$.

Yechimlar $x \equiv 1(mod6), x \equiv -2(mod6), x \equiv -1(mod6)$,ya'ni $x = 1 + 6t, x = -2 + 6t, x = -1 + 6t, t\epsilon Z$

d).$x^5 - 2x^3 + 13x - 1 \equiv 0(mod4) \rightarrow x^5 - 2x^3 + x - 1 \equiv 0(mod4)$. $x = \pm 1, \pm 2$ larni qo'yib tekshiramiz. U holda bularning birortasi ham bu taqqoslamani qanoatlantirmaydi va berilgan taqqoslama yechimga ega emas.

252. Bunda $12 = 3 \cdot 4 + 0$, $24 = 3 \cdot 8 + 0$ va $7 = 3 \cdot 2 + 1$ bo'lganligi uchun koeffitsiyentlarini absolyut qiymati jihatidan eng kichik chegirmalar bilan almashtirib $x^3 - x \equiv 0(mod3)$ ni hosil qilamiz. Ferma teoremasiga ko'ra p tub son bo'lganda $x^p - x \equiv 0(modp)$ bajariladi. Bizda p=3, ya'ni oxirgi taqqoslama va demak berilgan taqqoslama ham ayni taqqoslama. Shuning uchun ham noma'lum x ning barcha butun qiymatlari berilgan taqqoslamani qanoatlantiradi.

253. a).$x^3 - x + 6 \equiv 0(mod3)$. Bunda Ferma teoremasiga ko'ra $x^3 - x \equiv 0(mod3)$ va $6 \vdots 3$. Shuning uchun berilgan taqqoslama x ning ixtiyoriy butun qiymatida o'rinli.

b). $x(x^2 - 1) \equiv 0(mod6)$. Bu taqqoslamani $(x - 1)x(x + 1) \equiv 0(mod6)$ ko'rinishda yozib olish mumkin. Bu yerda chap tomondagi ifoda uchta ketma-ket sonlarning ko'paytmasi sifatida 6 ga bo'linadi, ya'ni berilgan taqqoslama x ning ixtiyoriy butun qiymatida o'rinli.

c). $20x^6 + x^5 - 10x^3 - x + 15 \equiv 0(mod5)$. Bunda koeffitsiyentlarni absolyut qiymat jihatdan eng kichik chegirmalar bilan almashtirib soddalashtiramiz.U holda $x^5 - x \equiv 0(mod5)$ ayniy taqqoslamaga ega bo'lamiz.

d).$x^{13} - 26x^{12} - x \equiv 0(mod13) \rightarrow x^{13} - x \equiv 0(mod13)$.

Bu taqqoslama x ning ixtiyoriy butun qiymatlarida bajariladigan ayniy taqqoslama.

254. a).Berilgan $5x \equiv 4(mod5)$taqqoslama $0 \equiv 4(mod5)$ ziddiyatli taqqoslamaga teng kuchli.Shuning uchun taqqoslama yechimga ega emas.

b). $x^2 - 2x + 3 \equiv 0(mod4)$. Bu yerda 4 moduli bo'yicha chegirmalarning to'la sistemasi $0, 1, 2, 3$ larni qo'yib tekshirib ko'ramiz.U holda ularning birortasi ham berilgan taqqoslamani qanoatlantirmaydi. Demak, taqqoslama yechimga ega emas.

c).$20x^5 + 5x^4 - 10x^3 - 6 \equiv 0(mod5)$ taqqoslama $-1 \equiv 0(mod5)$ ziddiyatli taqqoslamaga teng kuchli. Shuning uchun ham berilgan taqqoslama yechimga ega emas.

d). $x^{13} - 26x^{12} - x + 5 \equiv 0(mod13)$ taqqoslama $x^{13} - x + 5 \equiv 0(mod13)$ taqqoslamaga teng kuchli. Bu yerda $x^{13} - x \equiv 0(mod13)$ ayniy taqqoslama bo'lganligi uchun berilgan taqqoslama $5 \equiv 0(mod13)$ ziddiyatli taqqoslamaga teng kuchli bo'ladi. Shuning uchun ham berilgan taqqoslama yechimga ega emas.

255. *a)*.Bu yerda $y = x + a$ almashtirish olib berilgan taqqoslamaga qo'yamiz, u holda $(y + a)^n + a_1(y + a)^{n-1} + a_2(y + a)^{n-2} + \cdots + a_n \equiv 0(modm)$ ni hosil qilamiz. Oxirgi taqqoslamada y ning bir xil darajalari oldidagi koeffitsiyentlarni yig'sak $y^n + (a_1 + na)y^{n-1} + \cdots + (a^n + a_1 \cdot a^{n-1} + \cdots + a_n) \equiv 0(modm)$ hosil bo'ladi. a ixtiyoriy bo'lgani uchun uni $a_1 + na \equiv 0(modm)$ shart bajariladigan qilib tanlaymiz. U holda $y^n + b_2y^{n-2} + \cdots + b_n \equiv 0(modm)$ taqqoslamaga ega bo'lamiz.

b). $x^3 + 5x^2 + 6x - 8 \equiv 0(mod13)$. $a_1 = 5, m = 13, n = 3$a) qismdagiga asosan $a_1 + na \equiv 0(modm)$ dan $5 + 3a \equiv 0(mod13)$ ga ega bo'lamiz . Bundan $3a \equiv -5(mod13) \to -10a \equiv -5(mod13) \to 2a \equiv 1(mod13) \to 2a \equiv 14(mod13) \to a \equiv 7(mod13)$. Demak, $a = 7$ va biz $x = y + 7$ almashtirish bajaramiz u holda $(y + 7)^3 + 5(y + 7)^2 + 6(y + 7) - 8 = (y + 7)(y^2 + 14y + 49 + 5y + 35 + 6) - 8 = (y + 7)(y^2 + 19y + 90) - 8 = y^3 + 19y^2 + 90y + 7y^2 + 133y + 630 - 8 = y^3 + 26y^2 + 223y + 622 \equiv y^3 + 2y - 2 \equiv 0(mod13)$. Demak, izlanayotgan taqqoslama $y^3 + 2y - 2 \equiv 0(mod13)$ dan iborat.

256. Eyler teoremasiga ko'ra berilgan taqqoslamani $(x, 60) = 1$ shartni qanoatlantiruvchi barcha x lar qanoatlantiradi, ya'ni $\varphi(60) = \varphi(2^2 \cdot 3 \cdot 5) = \varphi(2^2) \cdot \varphi(3) \cdot \varphi(5) = (2^2 - 2) \cdot (3 - 1) \cdot (5 - 1) = 2 \cdot 2 \cdot 4 = 16$ ta yechimga ega. Bu yechimlar x ning $x \leq 60, (x, 60) = 1$ shartlarni qanoatlantiruvchi qiymatlari $x \equiv 1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59\ (mod60)$ dan iborat.

IV.2-§.

257. a). Bunda$(5,6) = 1$. Shuning uchun ham taqqoslama yagona yechimga ega. Bu yechimni tanlash usuli bilan topish uchun 6 moduli bo'yicha chegirmalarning to'la sistemasini biror ko'rinishda (masalan: $0, 1, 2, 3, 4, 5$) yozib olib bu sonlarni berilgan taqqoslamaga qo'yib tekshiramiz. $x_1=3$ berilgan taqqoslamani qanoatlantiradi. Shuning uchun berilgan taqqoslamaning yechimi $x \equiv 3(mod\ 6)$,ya'ni $x = 3 + 6t, t \in Z$.

b).$8x \equiv 3(mod\ 8)$taqqoslamada$(8,10) = 2$, lekin 3 soni 2ga bo'linmaydi. Shuning uchun ham taqqoslama yechimga ega emas.

c). $2x \equiv 6(mod\ 8)$ taqqoslamada $(2,8) = 2$ va 6 soni 2 ga bo'linadi. Shuning uchun ham berilgan taqqoslama 2 ta yechimga ega. Bu holda berilgan taqqoslama$x \equiv 3(mod\ 4)$ga teng kuchli. Demak, berilgan taqqoslamaning yechimlari $x \equiv 3,7(mod\ 8)$,ya'ni $x = 3 + 8t$ va $x = 7 + 8t, t \in Z$ lardan iborat bo'ladi.

d).$3x \equiv -6(mod\ 7)$ning o'ng tomoniga 7 ni (modulni) qo'shsak $3x \equiv 1(mod\ 7)$ taqqoslama hosil bo'ladi. Bunda $(3,7) = 1$ bo'lgani uchun u yagona yechimga ega. 7 moduli bo'yicha chegirmalarning to'la sistemasi 0,1,2,3,4,5,6 larni taqqoslamaga qo'yib tekshirib ko'rib $x \equiv 5(mod\ 7)$, ya'ni $x = 5 + 7t, t \in Z$ ning berilgan taqqoslamaning yechimi ekanligini topamiz.

e). $4x \equiv 3(mod 12)$da $(4,12) = 4$, lekin 3 soni 4 ga bo'linmagani uchun ham taqqoslama yechimga ega emas.

f).$6x \equiv 5(mod\ 9)$ da $(6,9) = 3$ va 5 soni 3 ga bo'linmaydi shuning uchun ham berilgan taqqoslama yechimga ega emas.

g). Bu yerda $(5,8) = 1$ va 8 –moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm1, \pm2, \pm3,$ 4 dan iborat. Bularni qo'yib tekshirib $x \equiv 3(mod 8)$ berilgan taqqoslamani yechimi ekanligini aniqlaymiz.

258. a).$5x \equiv 3(mod\ 7)$ taqqoslamada $(5,7) = 1$ bo'lgani uchun taqqoslama yagona yechimga ega. Bu yechimni taqqoslamalarning xossalaridan foydalanib topish uchun taqqoslamaning o'ng tomoniga modulni qo'shamiz.U holda $5x \equiv 10(mod\ 7)$ ni hosil qilamiz. Bu taqqoslamaning ikkala tomonini 5 ga qisqartiramiz.($(5;7) = 1$ bo'lgani uchun bu ishni amalga oshirish mumkin). U holda $x \equiv 2(mod\ 7)$ ya'ni $x = 2 + 7t, t \in Z$ berilgan taqqoslamaning yechimiga ega bo'lamiz.

b).$8x \equiv 3(mod 11)$ taqqoslamada $(8;11) = 1$. Demak, taqqoslama yagona yechimga ega. Bu taqqoslamaning o'ng tomonidan 11 ni ayirsak $8x \equiv -8(mod\ 11)$ taqqoslama hosil bo'ladi. Oxirgi taqqoslamaning ikkala tomonini 8 ga (chunki (8;11)=1) qisqartirsak $x \equiv -1(mod 11)$ taqqoslamaga ega bo'lamiz. Demak, berilgan taqqoslamaning yechimi $x = -1 + 11t, t \in Z$.

c). $4x \equiv 6(mod8)$ taqqoslamada $(4,8) = 4$ va 6 soni 4 ga bo'linmaydi, shuning uchun ham berilgan taqqoslama yechimga ega emas.

d).$4x \equiv 25(mod13)$ da $(4;13) = 1$ bo'lgani uchun taqqoslama yagona yechimga ega. Bu taqqoslamaning o'ng tomonidan 13 ni ayirib, hosil bo'lgan taqqoslamani ikkala tomonini 4 ga bo'lsak $x \equiv 3(mod13)$ taqqoslama hosil bo'ladi. Demak, berilgan taqqoslamaning yechimi $x = 3 + 13t, t \in Z$.

e). $11x \equiv 3(mod12)$ taqqoslamada $(11,12) = 1$ bo'lgani uchun taqqoslama yagona yechimga ega. Berilgan taqqoslamaning chap tomonidan uning modulini ayirsak, $-x \equiv 3(mod\ 12)$ yoki $x \equiv -3(mod\ 12)$ taqqoslama hosil bo'ladi. Bundan $x = -3 + 12t,\ t \in Z$ berilgan taqqoslamaning yechimi ekanligi kelib chiqadi.

f).$7x \equiv 5(mod\ 9)$ taqqoslamada $(9,7) = 1$. Demak, berilgan taqqoslama yagona yechimga ega. Bu taqqoslamaning ikkala tomonidan modul 9 ni ayiramiz. U holda $-2x \equiv -4(mod\ 9)$ hosil bo'ladi. Bundan $x \equiv 2(mod\ 9)$, ya'ni $x = 2 + 9t, t \in Z$ ekanligi kelib chiqadi.

g). Bunda $(5,8) = 1$ bo'lganligi uchun taqqoslama yagona yechimga ega. Taqqoslamaning istalgan tomoniga modulga karrali sonni qo'shish yoki ayirish mumkin. Shuning uchun ham

$5x \equiv 7+8(\text{mod}8) \rightarrow 5x \equiv 15(\text{mod}8)$.

Taqqoslamaning ikkala tomonini modul bilan o'zaro tub songa qisqartirish mumkin bo'lgani uchun $(5;15) = 5$ va $(5,8) = 1$ ekanligini e'tiborga olib oxirgi taqqoslamaning ikkala tomonini 5 ga qisqartiramiz. U holda $x \equiv 3(mod8)$ yechimni hosil qilamiz.

h). $(7,\ 15)=1$ bo'lganligi uchun bu taqqoslama yagona yechimga ega.

$7x \equiv 6+15(\text{mod}\ 15),\quad 7x \equiv 21(\text{mod}\ 15),\quad (7,\ 21)=7 \quad \text{va} \quad (7,15)=1$

ekanligini e'tiborga olib oxirgi taqqoslamaning ikkala tomonini 7ga qisqartiramiz. U holda $x \equiv 3(mod15)$ yechimni hosil qilamiz.

259. a).$13x \equiv 3(mod19)$ taqqoslamada $(13,19) = 1$ bo'lgani uchun yagona yechimga ega. Ma'lumki, agar $(a,m) = 1$bo'lsa, $ax \equiv b(mod\ m)$ taqqoslamaning yechimini $x \equiv a^{\varphi(m)-1} \cdot b(modm)$ taqqoslamadan foydalanib topish mumkin. Bizning misolimizda $a = 13, b = 3, m = 19$ bo'lgani uchun

$x \equiv 13^{\varphi(19)-1} \cdot 3(mod\ 19)$ bo'ladi. Bunda $\varphi(19)=18$ va $13^{17}=(13^2)^8 \cdot 13== 169^8 \cdot 13 =(19 \cdot 9-2)^8 \cdot 13$ bo'lgani uchun $x \equiv (-2)^8 \cdot 3 \cdot 13(\text{mod } 19) \equiv$

$$256 \cdot 133(mod\ 19) \equiv (19 \cdot 13 + 9) \cdot 3 \cdot 13(mod19) \equiv$$
$$\equiv 3 \cdot 9 \cdot 13(mod\ 19)$$
$$\equiv 3 \cdot 117(mod\ 19) \equiv (19 \cdot 6 + 3) \cdot 3(mod\ 19) \equiv 9(mod\ 19).$$

Demak, berilgan taqqoslamaning yechimi $x = 9 + 19t, t \in Z$.

b).$27x \equiv 7(mod\ 58)$ taqqoslamada $(27,58) = 1$ va $\varphi(58) = \varphi(2 \cdot 29) = \varphi(2) \cdot \varphi(29) = 28$ bo'lgani uchun $x \equiv 27^{27} \cdot 7(mod\ 58) \equiv 3^{81} \cdot 7(mod\ 58) \equiv (3^4)^{20} \cdot 21(mod\ 58) \equiv 23^{20} \cdot 21(mod\ 58) \equiv (58 \cdot 7 + 7)^{20} \cdot 21(mod\ 58) \equiv 7^{10} \cdot 21(mod\ 58) \equiv (7^3)^3 \cdot 147(mod\ 58 \equiv 343^3 \cdot (2 \cdot 58 + 31)(mod\ 58) \equiv (5 \cdot 58 + 53)^3 \cdot 31(mod\ 58) \equiv 53^3 \cdot 31(mod\ 58) \equiv 53^2 \cdot 53 \cdot 31(mod\ 58) \equiv (-5)^3 \cdot 31(mod\ 58) \equiv -9 \cdot 31(mod\ 58) \equiv -279(mod\ 58) \equiv 11(mod\ 58)$.

Demak, berilgan taqqoslamaning yechimi $x \equiv 11(mod\ 58)$, ya'ni $x = 11 + 58t, t \in Z$.

c). $5x \equiv 7(mod\ 10)$ taqqoslamada $(5,10) = 5$, lekin 7 soni 5 ga bo'linmaydi. Shuning uchun ham taqqoslama yechimga ega emas.

d). $3x \equiv 8(mod\ 13)$, bunda$(3,13) = 1$ bo'lgani uchun taqqoslama yagona yechimga ega. Bizda $a = 3, b = 8, m = 13$ va $\varphi(13)=12$ bo'lgani uchun $x \equiv 3^{11} \cdot 8(mod\ 13) \equiv (3^3)^3 \cdot 3^2 \cdot 8(mod\ 13) \equiv 72(mod\ 13) \equiv 7(mod\ 13)$. Demak berilgan toqqoslamaning yechimi $x = 7 + 13t, t \in Z$.

e).$25x \equiv 15(mod\ 17)$ da $(25,17) = 1$ bo'lgani uchun u yagona yechimga ega va $(5,17) = 1$ bo'lganidan taqqoslamaning ikkala tomonini 5 ga qisqartirish mumkin.Uholda$5x \equiv 3(mod\ 17)$ taqqoslama hosil bo'ladi. Shuning uchun ham $x \equiv 5^{\varphi(17)-1} \cdot 3(mod\ 17) \equiv 5^{15} \cdot 3(mod\ 17) \equiv (5^3)^5 \cdot 3(mod\ 17) \equiv (17 \cdot 7 + 6)^5 \cdot 3(mod\ 17) \equiv 6^5 \cdot 3(mod\ 17) \equiv (6^2)^2 \cdot 18(mod\ 17) \equiv 2^2 mod(17) \equiv 4(mod\ 17)$.

Demak, izlanayotgan yechim $x = 4 + 17t, t \in Z$.

f). $29x \equiv 35(mod\ 12)$, bunda $(29,12) = 1$ bo'lgani uchun berilgan taqqoslama yagona yechimga ega. Bu taqqoslamaning koeffitsiyentlari moduldan katta bo'lgani uchun ularni 12 moduli bo'yicha eng kichik manfiy bo'lmagan chegirmalar bilan almashtiramiz. Bunda $29 \equiv 12 \cdot 2 + 5, 35 \equiv 12 \cdot 2 + 11$ bo'lgani uchun, berilgan taqqoslamani $5x \equiv 11(mod\ 12)$ ko'rinishida yozib olish mumkin. $\varphi(12)=\varphi(2^2 \cdot 3)= =(2^2-2) \cdot 2=4$ va $x \equiv 5^{\varphi(12)-1} \cdot 11\ (mod\ 12) \equiv 5^3 \cdot 11(mod\ 12) \equiv 125 \cdot (-1)(mod 12) \equiv -5(mod 12) \equiv 7(mod\ 12)$. Demak, izlanayotgan yechim $x = 7 + 12t, t \in Z$.

g). Bu yerda $x \equiv a^{\varphi(m)-1} \cdot b(\mathrm{mod}\ m)$ formuladan foydalanamiz. Bizda $a=3$, $b=7$, $m=11$ bo'lgani uchun $x \equiv 3^{\varphi(11)-1} \cdot 7(mod 11)$ ni hosil qilamiz. Bunda $\varphi(11) = 11 - 1 = 10$ bo'lganidan $x \equiv 3^9 \cdot 7(mod 11)$. Endi oxirgi taqqoslamaning o'ng tomonidagi ifodani eng kichik musbat chegirma ko'rinishiga keltiramiz. $3^9 \cdot 7(mod 11) = (3^3)^3 \cdot 7(mod 11) \equiv 27^3 \cdot 7(mod 11) \equiv 5^3 \cdot 7(mod 11) \equiv 4 \cdot 7 \equiv 6(mod 11)$. Shunday qilib berilgan taqqoslamaning yechimi: $x \equiv 6(\mathrm{mod}\ 11)$.

260. a). Berilgan $13x \equiv 1(mod\ 27)$ taqqoslamada $(13; 27) = 1$ bo'lgani uchun u yagona yechimga ega.Bu yechimni munosib kasrlardan foydalanib, ya'ni $x \equiv (-1)^{n-1} \cdot bP_{n-1}(modm)\ (*)$ formuladan foydalanib topish uchun, avvalo P_{n-1} ni (oxirgidan oldingi munosib kasrning suratini) aniqlashimiz kerak. Buning uchun esa $\frac{m}{a} = \frac{27}{13}$ kasrni uzluksiz kasrga yoyamiz. Bunda $27 = 13 \cdot 2 + 1$, $13 = 1 \cdot 13 + 0$ lardan $\frac{27}{13} = 2 + \frac{1}{13} = (2; 13)$.

Endi P_{n-1} ni aniqlaymiz:

q_i		2	13
P_i	$P_0 = 1$	$P_1 = 2$	27

Jadvaldan $P_{n-1} = 2$ va $n = 2$. Bularni (*) ga olib borib qo'ysak $x \equiv (-1)^{2-1} \cdot 2 \cdot 1(mod\ 27) \equiv -2(mod\ 27)$, yani $x = -2 + 27t, t \in Z$.

Tekshirish: $13 \cdot (-2) \equiv 1(mod\ 27) \rightarrow 1 \equiv 1(mod\ 27)$ doimo bajariladi.

b). Berilgan $37x \equiv 25(mod\ 117)$ taqqoslamada $(37; 117) = 1$ bo'lgani uchun u yagona yechimga ega. P_{n-1} ni aniqlaymiz. Buning uchun $\frac{117}{37}$ kasrni uzluksiz kasrlarga yoyamiz. $117 = 37 \cdot 3 + 6, 37 = 6 \cdot 6 + 1$, $6 = 1 \cdot 6 + 0$ lardan $q_1=3$, $q_2 = 6$, $q_3 = 6$ ekanligini topamiz. U holda $\frac{117}{37} = (3,6,3)$ va

q_i		3	6	6
P_i	$P_0 = 1$	3	19	117

bo'lganidan $n = 3, P_{n-1} = P_2 = 19$ hamda $x \equiv (-1)^2 \cdot 19 \cdot 25(mod\ 117) \equiv 475(mod\ 117) \equiv 7(mod\ 117)$. Demak, izlanayotgan yechim $x = 7 + 117t, t \in Z$.

Tekshirish: $37 \cdot 7 \equiv 259 \equiv (117 \cdot 2 + 25)\ (mod\ 117) \equiv 25(mod\ 117)$, ya'ni yechim to'g'ri topilgan.

c) $113x \equiv 89(mod\ 311)$ taqqoslamada $(113, 311) = 1$ bo'lgani uchun u yagona yechimga ega. Endi $\frac{311}{113}$ kasrni uzluksiz kasrlarga yoyamiz: $311 = 113 \cdot 2 + 85, 113 = 85 \cdot 1 + 28,\ 85 = 28 \cdot 3 + 1,$ $28 = 1 \cdot 28 + 0$. Demak,

$q_1 = 2,\ q_2 = 1,\ q_3 = 3,\ q_4 = 28$ va $\frac{311}{113} = (2; 1; 3; 28)$ hamda

q_i		2	1	3	28
P_i	$P_0 = 1$	2	3	11	311

bo'lganligi uchun P_{n-1}=11, $n = 4$ va $x \equiv (-1)^3 \cdot 11 \cdot 89(mod\ 311) \equiv -979(mod\ 311) \equiv -46(mod\ 311)$, ya'ni $x = -46 + 311t, t \in Z$.

Tekshirish: $113 \cdot (-46)\ (mod\ 311) \equiv -5198\ (mod\ 311) \equiv (311 \cdot 16 + 222)\ (mod\ 311) \equiv -222(mod\ 311) \equiv 89\ (mod\ 311)$. Demak, taqqoslamaning yechimi to'g'ri topilgan.

d) $221x \equiv 111(mod\ 360)$. Bunda $221 = 13 \cdot 17$ va $360 = 36 \cdot 10 = 2^2 \cdot 3^2 \cdot 2 \cdot 5 = 2^3 \cdot 3^2 \cdot 5$, ya'ni $(221; 360) = 1$.Shuning uchun ham berilgan taqqoslama yagona yechimga ega. Shu yechimni topish uchun $\frac{360}{221}$ kasrni uzluksiz kasrga yoyamiz. $360 = 221 \cdot 1 + 139, 221 = 139 \cdot 1 + 82, 139 = 82 \cdot 1 + 57, 82 = 57 \cdot 1 + 25, 57 = 25 \cdot 2 + 7, 25 = 7 \cdot 3 + 4, 7 = 4 \cdot 1 + 3, 4 = 1 \cdot 3 + 1, 3 = 1 \cdot 3 + 0$.
Bulardan $q_1 = 1, q_2 = 1, q_3 = 1, q_4 = 1,\ q_5 = 2,\ q_6 = 3,\ q_7 = 1,$ $q_8 = 1,\ q_9 = 3$ va $\frac{360}{221} = (1, 1, 1, 1, 2, 3, 1, 1, 3)$ larni topamiz. Endi P_{n-1} ni aniqlaymiz.

q_i		1	1	1	1	2	3	1	1	3
P_i	P_0	1	2	3	5	13	44	57	101	360

Bundan P_{n-1}=101, $n = 9$, va $x \equiv (-1)^8 \cdot 101 \cdot 111(mod\ 360) \equiv 11100 + +111(mod\ 360) \equiv (30 \cdot 360 + 300 + 111)(mod\ 360) \equiv$

$411(mod\ 360) \equiv 51(mod\ 360)$. Demak, berilgan taqqoslamaning yechimi $x = 51 + 360t, t \in Z$.

Tekshirish: $221 \cdot 51 \equiv 11271 \equiv (360 \cdot 31 + 111) \equiv 111(mod\ 360)$.

Bu yerdan ko'rinadiki berilgan misolning yechimi to'g'ri topilgan.

e) $39x \equiv 84(mod\ 93)$ da $39 = 3 \cdot 13,\ 93 = 3 \cdot 31$, ya'ni $(39; 93) = 3$ bo'lgani va 84 soni 3 ga bo'lingani uchun berilgan taqqoslama 3 ta yechimga ega bo'ladi. Berilgan taqqoslamani 3 ga qisqartirib yagona yechimga ega bo'lgan $13x \equiv 28(mod\ 31)$ taqqoslamani hosil qilamiz. Endi $\frac{31}{13}$ kasrni uzluksiz kasrlarga yoyamiz. Bunda $31 = 13 \cdot 2 + 5,\ 13 = 5 \cdot 2 + 3, 5 = 1 \cdot 3 + 2, 3 = 1 \cdot 2 + 1,\ 2 = 1 \cdot 2$ bo'lgani uchun q_1=2, q_2=2, q_3=1, q_4=1, q_5=2 va $\frac{31}{13} = (2, 2, 1, 1, 2)$. Endi P_{n-1}ni aniqlaymiz.

q_i		2	2	1	1	2
P_i	1	2	5	7	2	31

Bundan $n = 5$, P_{n-1}=12 va $x \equiv (-1)^4 \cdot 12 \cdot 28(\text{mod } 31) \equiv 12 \cdot (-3)$ $(\text{mod } 31) \equiv -5(mod\ 31)$. Demak, berilgan taqqoslamaning yechimlari $x \equiv -5,\ 26, 57(mod\ 93)$ ya'ni $x = -5 + 93t, x = 26 + 93t,\ x = 57 + 93t,\ t \in Z$.

Tekshirish: $x_1 = -5$bo'lsa,$39 \cdot (-5) = -195\ (mod\ 93) \equiv 84(mod 93)$; $x_2 = 26$ bo'lsa, $39 \cdot 26 = 1014 \equiv 93 \cdot 23 + 84 \equiv 84(mod 93)$;

$x_3 = 57$ bo'lsa, $39 \cdot 57 = 2223 = 93 \cdot 23 + 84 \equiv 84(mod 93)$. Bulardan ko'rinadiki, uchala yechim ham to'g'ri topilgan.

f). $143x \equiv 41(mod 221)$ taqqoslamada $143 = 11 \cdot 13$, $221 = 13 \cdot 17$ bo'lgani uchun $(143, 221) = 13$, lekin 41 soni 13ga bo'linmaydi. Shuning uchun ham berilgan taqqoslama yechimga ega emas.

g). Bu yerda $x \equiv (-1)^{n-1} b P_{n-1} (\text{mod } m)$ (1)formuladan foydalanamiz. Avvalo P_{n-1} ni aniqlab olamiz. Buning uchun $\frac{m}{a} = \frac{43}{20}$kasrni uzluksiz kasrga yoyamiz va munosib kasrlarini topamiz, u holda $43 = 20 \cdot 2 + 3,\quad 20 = 3 \cdot 6 + 2,\quad 3 = 2 \cdot 1 + 1,\quad 2 = 1 \cdot 2$. Demak, $q_1 = 2$,

$q_2 = 6,\ q_3 = 1,\ q_4 = 2.\ \frac{m}{a} = \frac{43}{20} = (2, 6, 1, 2).$ Endi munosib kasrlarning suratlarini hisoblab P_{n-1}ni topamiz.

q_i		$q_1 = 2$	$q_2 = 6$	$q_3 = 1$	$q_4 = 2$
P_i	$P_0 = 1$	$P_1 = 2$	$P_2 = 13$	$P_3 = 15$	$P_4 = 43$

Demak, $n = 4, P_3 = 15$. Topilgan qiymatlarni (1) ga olib borib qo'ysak $x \equiv (-1)^3 \cdot 15 \cdot 13(mod 43) \equiv -195(mod 43) \equiv -195 + 43 \cdot 5(mod 43) \equiv 20(mod 43)$ hosil bo'ladi. **Javob:** $x = 20 + 43t, t \in Z$.

261. a). $12x \equiv 9(mod\ 15)$ taqqoslamada $(12,15) = 3$ va 9 soni 3 ga karrali bo'lgani uchun u 3 ta yechimga ega. Berilgan taqqoslamaning ikkala tomoni va modulini 3ga qisqartirsak $4x \equiv 3(mod\ 5)$ taqqoslama hosil bo'ladi. Bunda $(4,5) = 1$ bo'lgan uchun u yagona yechimga ega. Uning yechimini aniqlaymiz. $4x \equiv (3 + 5)(mod\ 5)$yoki $4x \equiv 8(mod\ 5)$. Keyingi taqqoslamaning ikkala tomonini 4 ga bo'lsak $x \equiv 2(mod\ 5)$ hosil bo'ladi. Demak, berilgan taqqoslamaning yechimlari $x \equiv 2, 7, 12(mod\ 15)$, ya'ni

$$x = 2 + 15t\ , x = 7 + 15t\ , x = 12 + 15t, t \in \mathbb{Z}.$$

b).$12x \equiv 9(mod\ 18)$taqqoslamada $(12,18) = 6$, lekin 9 soni 6 ga bo'linmaydi. Shuning uchun berilgan taqqoslama yechimga ega emas.

c). $20x \equiv 10(mod\ 25)$ taqqoslamada $(10,35) = 5$ va 25 soni 5 ga bo'linadi. Demak, taqqoslama 5 ta yechimga ega berilgan taqqoslamaning ikkala tomonini va modulini 5ga bo'lib $4x \equiv 2(mod\ 5)$ taqqoslamani hosil qilamiz. Bundan $4x \equiv (2 + 5 \cdot 2)(mod\ 5) \to 12(mod\ 5) \to x \equiv 3(mod\ 5)$. Demak, taqqoslamaning yechimlari $x \equiv 3, 3, 13, 18, 23(mod\ 15)$, ya'ni$x = 3 + 25t\ , x = 8 + 25t$,

$x = 13 + 25t, x = 18 + 25t, x = 23 + 25tt \in \mathbb{Z}$ lardan iborat bo'ladi.

d). $10x \equiv 25(mod\ 35)$ taqqoslamada $(10\ , 35) = 5$ va 25 soni 5 ga bo'linadi. Shuning uchun ham berilgan taqqoslama 5 ta yechimga ega. Berilgan taqqoslamaning ikkala tomoni va modulini 5 ga qisqartib yagona yechimga ega bo'lgan $2x \equiv 5(mod\ 7)$ taqqoslamaga ega bo'lamiz. Bundan $2x \equiv (5 - 7)(mod\ 7) \to 2x = -2(mod\ 7) \to x = -1(mod\ 7)$. Demak, berilgan taqqos-lamaning yechimlari $x \equiv$

$-1,6,13,20,27(mod\ 7)$, ya'ni $x=-1+7t$, $x=6+7t$, $x=13+7t$, $x=20+7t$, $x=27+7t, t\in\mathbb{Z}$dan iborat.

e). $39x\equiv 84(mod\ 93)$ va $(39,93)=3$ hamda 84 soni 3ga bo'linadi. Shuning uchun ham berilgan taqqoslama 3 ta yechimga ega. Berilgan taqqoslamani 3 ga bo'lib yagona yechimga ega bo'lgan $13x\equiv 28(mod\ 31)$ taqqoslamaga ega bo'lamiz. Bundan $13x\equiv(28-31)(mod\ 31)\to 13x\equiv -3(mod\ 31), 13x\equiv(-3-2\cdot 31(mod\ 31)$, $13x=-65(mod\ 31)\to x\equiv -5(mod\ 31)$. Demak, topilgan yechimlar $x\equiv -5,26,57(mod\ 93)$, ya'ni

$x=-5+93t$, $x=26+93t$, $x=57+93t, t\in\mathbb{Z}$.

f). $90x+18\equiv 0(mod\ 138)$ dan $90x\equiv -18(mod\ 138)$ bo'lgani uchun (90,138)=6 va -18 soni 6 ga bo'linadi. Demak, berilgan taqqoslama 6 ta yechimga ega. Berilgan taqqoslamani 6ga bo'lib yagona yechimga ega bo'lgan $15x\equiv -3(mod\ 23)$ taqqoslamaga kelammiz. Bundan $15x\equiv(-3++23)(mod\ 23)\to 15x\equiv 20(mod\ 23)$. Bu yerda $(15,20)=5$va $(23,5)=1$ bo'lgani uchun $3x\equiv 4(mod23)\to 3x\equiv(4+23)(mod\ 23)\to x\equiv 9(mod\ 23)$ni hosil qilamiz. Demak, berilgan taqqoslamaning yechimlari

$x\equiv 9,\ 32,\ 55,\ 78,101,124(mod\ 138)$,ya'ni$x=9+138t$, $x=32++138t$, $x=55+138t$, $x=78+138t$, $x=101+138t$, $x=124+138t, t\in\mathbb{Z}$ dan iborat.

g). Bu yerda $(15,35)=5$ va 55 soni 5 ga bo'linadi. Demak, berilgan taqqoslama 5ta yechimga ega. Berilgan taqqoslamaning ikkala tomoni va modulini 5 ga qisqartirib $3x\equiv 7(mod11)$ ni hosil qilamiz. Buni taqqoslamalarning xossalaridan foydalanib koeffitsiyentlarini almashtirish usulini yechamiz. U holda $3x\equiv 7+11(mod\ 11)$, $x\equiv 6(mod\ 11)$. Bundan berilgan taqqoslamaning yechimlari $x\equiv 6,17,28,39,50$ $(mod55)$ ekanligi kelib chiqadi.

262. a). Ma'lumki, $ax\equiv b(modm)$ taqqoslama $ax=b+my,\ y\in Z$, ga teng kuchli. Bundan $ax-my=b$, $x,y\in Z$ tenglamani hosil qilamiz. Shunday qilib $ax+m(-y)=b$, $y\in Z$, tenglama $ax\equiv b(modm)$ taqqoslamaga teng kuchli ekan. Shunga asosan $5x+4y=3\leftrightarrow 5x\equiv 3(mod\ 4)\leftrightarrow(5-4)x\equiv 3(mod\ 4)\to x\equiv 3(mod\ 4)$, ya'ni $x=3+4t$, bu holda $4y=3-5x$ bo'lgani uchun $4y=3-5(3+4t)=3-15-20t=-12-20t$, bundan $y=-3-5t, t\in\mathbb{Z}$ berilgan tenglama yechimi.

Tekshirish: $5(3+4t)+4(-3-5t)=15+20t-12-20t=3$, ya'ni topilgan yechimlar tenglamani qanoatlantiradi.

b). $17x+13y=1$ dan $17x\equiv 1(mod\ 13)\rightarrow 4x\equiv 1(mod\ 13)\rightarrow 4x\equiv -12(mod\ 13)\rightarrow x\equiv -3(mod\ 13)\rightarrow x\equiv -3(mod\ 13)\rightarrow x=$

$$=-3+13t, t\in\mathbb{Z}.$$

Endi y ni aniqlaymiz. Berilgan tenglamadan $13y=1-17x=1-17(-3+13t)=52-221t$. Bundan $y=4-17t$, $t\in\mathbb{Z}$. Shunday qilib berilgan tenglamaning yechimi $x=-3+13t, y=4-17t$, $t\in\mathbb{Z}$.

Tekshirish: $17(-3+13t)+13(4-17t)=-51+52=1$. Demak, topilgan yechimlar berilgan tenglamani qanoatlantiradi.

c). $91x-28y=35$ tenglamada $91=7\cdot 13;\ 28=2^2\cdot 7$, ya'ni $(91{,}28)=7$ va 35 soni 7ga bo'linadi. Demak, taqqoslamaning ikkala tomonini 7ga bo'lsak $13x-4y=5$ tenglama hosil bo'ladi. Bundan $13x\equiv 5(mod 4)$. Shunday qilib yechimlar $x=1+4t, y=2+13t$, $t\in\mathbb{Z}$.

Tekshirish: $13(1+4t)-4(2+13t)=13-8=5$. Demak, topilgan yechim berilgan tenglamani qanoatlantiradi.

d). $2x+3y=4\rightarrow 2x\equiv 4(mod\ 3)\rightarrow(2{,}3)=1$ va $x\equiv 2(mod\ 3)$, ya'ni $x=2+3t, t\in\mathbb{Z}$. Endi y ni aniqlaymiz. $2(2+3t)+3y=4$, $3y=-6t$, bundan $y=-2t, t\in\mathbb{Z}$. Shunday qilib berilgan tenglamaning yechimi $x=2+3t, y=-2t, t\in\mathbb{Z}$.

Tekshirish: $2(2+3t)+3(-2t)=4$. Demak, topilgan yechim berilgan tenglamani qanoatlantiradi.

e). $4x-3y=2\rightarrow 4x\equiv 2(mod\ 3)\rightarrow(4-3)x\equiv 2(mod\ 3)\rightarrow x\equiv 2(mod\ 3)$, ya'ni $x=2+3t, t\in\mathbb{Z}$. Endi x ning qiymatini tenglamaga qo'yib y ni aniqlaymiz. $y=2+4t, t\in\mathbb{Z}$. Shunday qilib berilgan tenglamaning yechimi $x=2+3t,\ y=2+4t, t\in\mathbb{Z}$.

Tekshirish: $4(2+3t)-3(2+4t)=2$, ya'ni topilgan yechim berilgan tenglamani qanoatlantiradi.

f). $3x-7y=1\rightarrow 3x\equiv 1(mod\ 7)\rightarrow(3-7)x\equiv(1+7)(mod\ 3)\rightarrow -4x\equiv 8(mod\ 3)\rightarrow x=-2(mod 7)$, ya'ni $x=-2+7t, t\in\mathbb{Z}$. U holda

$7y=3x-1=3(-2+7t)-1=-7+21t$, yoki bundan $y=-1+3t, t\in\mathbb{Z}$. Shunday qilib berilgan tenglamaning yechimi $x=-2+7t,\ y=-1+3t,\ t\in\mathbb{Z}$.

Tekshirish: $3(-2+7t)-7(-1+3t), t \in \mathbb{Z}$, ya'ni topilgan yechim berilgan tenglamani qanoatlantiradi.

g).$7x \equiv 11(mod6) \to (7-6)x \equiv (11-6)(mod6) \to x \equiv 5(mod6)$. Bundan $x = 5+6t,\ t \in Z$. Buni berilgan tenglamaga qo'ysak $7(5+6t)+6y = 11 \ \to 6y = 11-35-42t \ \to y = -4-7t,\ t \in Z$. Demak, berilgan tenglamaning yechimi $x = 5+6t,\ y = -4-7t,\ t \in Z$.

263. a) Avvalo 6-misoldagi singari berilgan $8x-13y = -6$ aniqmas tenglamaning butun sonlardagi umumiy yechimini aniqlaymiz. $8x \equiv -6(mod\ 3) \to 4x \equiv\equiv -3(mod\ 13) \to 4x \equiv 10(mod\ 13) \to 2x \equiv 5(mod\ 13) \to 2x \equiv 18(mod\ 13) \to x \equiv 9(mod\ 13)$ ya'ni $x = 9+13t, t \in \mathbb{Z}$. x ning topilgan qiymatini berilgan tenglamaga qo'yib y ni topamiz. $13y = 8(9+13t)+6,\ y = 6+8t, t \in \mathbb{Z}$. Shunday qilib berilgan to'g'ri chiziqda yotuvchi butun koordinatali nuqtalar $x = 9+13t,\ y = 6+8t, t \in \mathbb{Z}$ ekan. Endi bular orasidan $-100 \le x \le 150$ shartni qanoatlantiruvchilarni ajratib olamiz. $-100 \le 9+13t \le 150 \to -109 \le 13t \le 141 \to -\frac{109}{13} \le t \le \frac{141}{13} \to -8{,}38 \le t \le 10{,}85, t \in \mathbb{Z}$. Bu oraliqdagi butun sonlar soni $10+8+1 = 19$ ta.

b). $5x-7y = 8$ tenglamaning umumiy yechimini topamiz $5x \equiv 8(mod\ 7) \to (5-7)x \equiv 8(mod\ 7) \to -2x \equiv 8(mod\ 13) \to x \equiv -4(mod\ 7)$, ya'ni $x = -4++7t, t \in \mathbb{Z}$. Endi y ni aniqlaymiz. $7y = 5(-4+7t)-8$ bundan $y = -4+5t, t \in \mathbb{Z}$. Demak, berilgan to'g'ri chiziqda yotuvchi butun koordinatali nuqtalar $x = -4++7t, y = -4+5t, t \in \mathbb{Z}$. Endi bular orasidan $1 \le x \le 200$ shartni qanoatlantiruvchilarini ajratib olamiz. $1 \le -4+7t \le 200 \to 5 \le 7t \le 204 \to \frac{5}{7} \le t \le \frac{204}{7} \to 0{,}7 \le t \le 29{,}1\ , t \in \mathbb{Z}$. Bu oraliqdagi t ning butun qiymatlari 29ta. Shunday qilib, $x = 1$ va $x = 100$ to'g'ri chiziqlar orasida joylashgan $5x-7y = 8$ to'g'ri chiziqdagi butun koordinatali nuqtalar soni 29 ta ekan.

264. a) $f(x) = \frac{9x-1}{7}$ funksiyaning butun bo'lishi uchun $9x-1$ ifoda 7 ga bo'linishi kerak, ya'ni $9x-1 \equiv 0(mod\ 7)$ bajarilishi kerak. Bundan $9x \equiv 1(mod\ 7)$. Demak, $x = 4+7$t qiymatlarida $f(x)$ funksiya butun qiymat qabul qiladi. Haqiqatan ham $f(4+7t)=\frac{9(4+7t)-1}{7}=\frac{35+63t}{7}=5+9t\,,t\in\mathbb{Z}$.

b). $f(x)=\frac{9x-1}{15}$dan$7x\equiv 1(mod\ 15)\to 7x\equiv$(1-15)$(mod\ 15)\to 7x\equiv -14(mod\ 15)\to x\equiv -2(mod\ 15)$, ya'ni $x=-2+15t\,,t\in\mathbb{Z}$. Haqiqatan ham$f(-2+15t)=\frac{7(-2+15t)-1}{15}=\frac{-15+105t}{15}=-1+7t\,,t\in\mathbb{Z}$.

c). $2x\equiv 1(mod\ 11)\to 2x\equiv$12$(mod\ 11)\to x\equiv 6(mod\ 11)$, ya'ni

$x=6+11t\,,t\in\mathbb{Z}$. Bu qiymatda $f(6+11t)=\frac{2(6+11t)-1}{11}=\frac{-11+22t}{11}=-1+2t\,,t\in\mathbb{Z}$.

265. a). 60 kg lik qoplar sonini x, 80 kg lik qoplar sonini esa y bilan belgilab masalani $60x+80y=440$ tenglamaning natural sonlardagi yechimlarini topishga keltiramiz. Hosil bo'lgan tenglamani 20 ga qisqartib $3x+4y=22$ tenglamaga keltiramiz va bu tenglamani 6-misoldagi usul bilan yechamiz. $3x\equiv 22(mod\ 4)\to 3x\equiv 2(mod\ 4)\to 3x\equiv 6(mod\ 4)\to x\equiv 2(mod\ 4)$,ya'ni $x=2+4t\,,t\in\mathbb{Z}$ ni hosil qilamiz. Endi y ni aniqlaymiz. $4y=22-3x=22-3(2+4t)=16-12t$ yoki bundan $y=4-3t\,,t\in\mathbb{Z}$. Endi $x=2+4t\,,y=4-3t\,,t\in\mathbb{Z}$ umumiy yechimdan masalaning shartini qanoatlantiruvchi natural yechimlarni ajratib olamiz.$t=0$ da $x=2,y=4$, t=1 da $x=6,y=1$ lardan boshqa x va y larning natural qiymatlari yo'q. Demak, 2 ta 60 kg lik va 4 ta 80 kg lik qop yoki 6 ta 60 kg lik va 1 ta 80 kg lik qop kerak bo'lar ekan.

b) agar 30 so'mlik markalar sonini x bilan, 50 so'mlik markalar sonini y bilan belgilasak. Bu masalani yechishni $30x+50y=1490$ tenglamani natural sonlarda yechishga keltiriladi. Bundan $3x+5y=149\to 3x\equiv 149(mod\ 5)\to 3x\equiv 4(mod\ 5)\to 3x\equiv 9(mod\ 4)\to x\equiv 3(mod\ 5)$,ya'ni$x=3+5t\,,t\in\mathbb{Z}$ni hosil qilamiz. Topilgan qiymatni tenglamaga qo'yib y ni topamiz. $5y=149-3x=149-3(3+5t)=140+15tdany=28-3t\,,t\in\mathbb{Z}$.

Endi topilgan $x=3+5t\ y=28-3t\,,t\in\mathbb{Z}$ umumiy yechimlardan masalaning shartini qanoatlantiruvchi natural yechimlarini ajratib olamiz.

$t = 0$ da $x = 3, y = 28$, $t = 1$ da $x = 8, y = 25$, $t = 2$ da $x = 13, y = 22, t = 3$ da $x = 18$, $y = 19$, $t = 4$ da $x = 23, y = 16$, $t = 5$ da $x = 28, y = 13$,

$t = 6$ da $x = 33, y = 10$, $t = 7$ da $x = 38, y = 7, t = 8$ da $x = 43, y = 4$, $t = 9$ da $x = 48, y = 1$.

Demak, markalarni 9 xilda turlicha qilib xarid qilish mumkin ekan.

c). 200 so'mlik daftarlar sonini x bilan, 250 so'mlik daftarlar sonini y bilan belgilasak $200x + 250y = 6000$ aniqmas tenglama hosil bo'ladi. Bundan $20x + 25y = 600 \rightarrow 4x + 5y = 120 \rightarrow 4x \equiv 120(mod5) \rightarrow 4x \equiv 0(mod5) \rightarrow x \equiv 0(mod5) \rightarrow x = 5t;$ $4 \cdot 5t + 5y = 120 \rightarrow y = 24 - 4t$, $t \in Z$. Masalaning javobini jadval ko'rinishda yozamiz.

t	0	1	2	3	4	5	6
x	0	5	10	15	20	25	30
y	24	20	16	12	8	4	0

266. a) 523 sonining o'ng tomoniga yozilgan 3 ta raqamdan hosil bo'lgan sonni x bilan belgilasak, u holda $523 \cdot 10^3 + x \equiv 0(mod\ 7 \cdot 8 \cdot 9)$ bajarilishi kerak. Bundan $x \equiv -523000(mod\ 504) \equiv -(1038 \cdot 504 - 152)(mod\ 504) \equiv$

$\equiv 152(mod\ 504)$, yoki $x = 152 + 504t, t \in \mathbb{Z}$. x uchxonali son bo'lgani uchun $t = 0$ da $x = 152$, t=1 da $x = 656$ bo'lishi mumkin.

Tekshirish: 523152 soni 7, 8, 9, larga bo'linadi, shuningdek 523656 soni ham 7, 8, 9 larga bo'linadi.

b). 32 sonining o'ng tomoniga yozilgan 2 ta raqamli sonni x bilan belgilasak, u holda $32 \cdot 10^2 + x \equiv 0(mod\ 7 \cdot 3) \rightarrow x \equiv -3200(mod\ 21) \equiv -3200 + 21 \cdot 153(\text{mod } 21) \equiv -32(\text{mod } 21) \equiv 13(mod\ 21)$ yoki $x = 13 + 21t, t \in \mathbb{Z}$.

Bu yerda x ikkita raqamdan tuzilgan son bo'lgani uchun $t = 0$ da $x = 13$, $t = 1$ da x=34, t=2 da $x = 55$, t=3 da $x = 76$,t=4 da $x = 97$. Demak, izlanayotgan sonlar 3213, 3234, 3255, 3276, 3297 lardan iborat. Bularning hammasi 3 va7 ga bo'linadi.

IV.3-§.

267. 1). Birinchi taqqoslamani $x = 6 + 15t_1,\ t_1 \epsilon Z$ tenglik ko'rinishida yozib olib 2-taqqoslamadagi x ning joyiga olib borib qo'yamiz va t_1 ga nisbatan yechamiz:$6 + 15t_1 \equiv 18(mod\ 21) \to 15t_1 \equiv 12(mod\ 21)$. Bunda $(15, 21) = 3$ va $12 \vdots 3$ bo'lgani uchun taqqoslamani 3 ga qisqartirib $5t_1 \equiv 4(mod\ 7)$ ni hosil qilamiz. Bundan

$$5t_1 \equiv 4 + 3 \cdot 7)(mod\ 7) \to 5t_1 \equiv 25(mod\ 7) \to t_1 \equiv 5(mod\ 7),$$

ya'ni $t_1 = 5 + 7t_2,\ t_2 \epsilon Z$. t_1 ning bu ifodasini $x = 6 + 15t_1$ ga qo'ysak $x = 6 + 15(5 + 7t_2) = 81 + 105t_2, t_2 \epsilon Z$ hosil bo'ladi. Endi x ning ifodasini 3-taqqoslamaga qo'yib t_2 ni aniqlaymiz:$81 + 105t_2 \equiv 3(mod\ 12) \to 105t_2 \equiv -78(mod\ 12) \to (105, 12) = 3$ va $78 \vdots 3$ bo'lgani uchun taqqoslamani 3 ga qisqartirib $35t_2 \equiv -26(mod\ 4)$ ni, bundan esa $3t_2 \equiv 2(mod\ 4) \to 3t_2 \equiv 6(mod\ 4) \to t_2 \equiv 2(mod\ 4)$, ya'ni $t_2 = 2 + 4t_3, t_3 \epsilon Z$ ni hosil qilamiz. Shunday qilib $x = 81 + 105(2 + 4t_3) = 291 + 420t_3$, ya'ni $x = 291 + 420t_3,\ t_3 \epsilon Z$ berilgan sistemaning yechimiga ega bo'lamiz.

2).$x \equiv 13(mod\ 14) \to x \equiv -1(mod\ 14) \to x = -1 + 4t_1, t_1 \epsilon Z$. Buni ikkinchi taqqoslamaga qo'yib $t_1$ni aniqlaymiz:$-1 + 4t_1 \equiv 6(mod\ 35) \to 4t_1 \equiv 7(mod\ 35) \to 4t_1 \equiv 7 - 35(mod\ 35) \to 4t_1 \equiv -28(mod\ 35) \to t_1 \equiv -7(mod\ 35)$, ya'ni $t_1 = -7 + 35t_2, t_2 \epsilon Z$.

$t_1$ning bu qiymatini $x = -1 + 4t_1$ ga qo'yamiz u holda $x = -1 + 4(-7 + 35t_2) = -29 + 140t_2$. Endi x ning bu qiymatini 3-taqqoslamaga qo'yib t_2 ni topamiz: $-29 + 140t_2 \equiv 26(mod\ 45) \to 140t_2 \equiv 55(mod\ 45)$. Bunda $(140, 45) = 5$ va $55 \vdots 5$. Shuning uchun ham bu taqqoslamani 5 ga qisqartirib $28t_2 \equiv 11(mod\ 9)$ni yoki bundan $t_2 \equiv 2(mod\ 9)$ ni hosil qilamiz. Demak, $t_2 = 2 + 9t_3,\ t_3 \epsilon Z$. Shunday qilib $x = -29 + 140(2 + 9t_3) = 251 + 1260t_3\ ,\ t_3 \epsilon Z$.

3).$\begin{cases} x \equiv 19(mod\ 56) \\ x \equiv 3(mod\ 24) \\ x \equiv 7(mod\ 20) \end{cases} \to \quad x = 19 + 56t_1\ , t_1 \epsilon Z,\ 19 + 56t_1 \equiv$
$3(mod\ 24) \to 56t_1 \equiv -16(mod\ 24)$.Bunda $(56, 24) = 8$ va $16 \vdots 8$, shuning uchun ham oxirgi taqqoslamaning ikkita tomoni va modulini 8 ga qisqartirilib $7t_1 \equiv -2(mod\ 3) \to t_1 \equiv 1(mod\ 3)$, ya'ni $t_1 = 1 + 3t_2,\ t_2\ \epsilon Z$ ni hosil qilamiz . Buni $x = 19 + 56t_1$, ga qo'ysak $x = 19 + 56(1 + 3t_2) = 75 + 168t_2$ kelib chiqadi. x ning bu qiymatini 3-taqqoslamaga qo'yib t_2 ni aniqlaymiz. $75 + 168t_2 \equiv 7(mod\ 20) \to$

$168t_2 \equiv -68(mod\ 20) \rightarrow 8t_2 \equiv -8(mod\ 20) \rightarrow (8,20) = 4$ va $8 \vdots 4$ bo'lgani uchun $2t_2 \equiv -2(mod\ 5) \rightarrow t_2 \equiv -1(mod\ 5)$, ya'ni $t_2 = -1 + 5t_3$, $t_3 \epsilon Z$. Shunday qilib $x = -93 + 840t_3$, $t_3 \epsilon Z$ ni hosil qilamiz .

4). $\begin{cases} x \equiv 4(mod\ 5) \\ x \equiv 1(mod\ 12) \\ x \equiv 7(mod\ 14) \end{cases} \rightarrow \quad x = 4 + 5t_1, t_1 \epsilon Z,\ 4 + 5t_1 \equiv$ $1(mod\ 12) \rightarrow 5t_1 \equiv -3(mod\ 12) \rightarrow 5t_1 \equiv (-3 + 12 \cdot 4)(mod\ 12) \rightarrow$ $5t_1 \equiv 45(mod\ 12) \rightarrow t_1 \equiv 9(mod\ 12)$ ya'ni $t_1 \equiv 9 + 12t_2, t_2 \epsilon Z$. Bundan $x = 4 + 5(9 + 12t_2) = 49 + 60t_2$. x ning bu qiymatini 3-taqqoslamaga qo'yib t_2 ni aniqlaymiz: $49 + 60t_2 \equiv 7(mod\ 14) \rightarrow$ $60t_2 \equiv 7 - 49(mod\ 14) \rightarrow 60t_2 \equiv -42(mod\ 14)$. Bunda $(60; 14) = 2$ va $42 \vdots 2$ bo'lgani uchun $30t_2 \equiv -21(mod\ 7)$, yoki bundan $2t_2 \equiv 0(mod\ 7) \rightarrow t_2 \equiv 0(mod\ 7)$, ya'ni $t_2 = 7t_3, t_3 \epsilon Z$. Buni $x = 49 + 60t_2$ ga qo'yib $x = 49 + 60 \cdot 7t_3 = 49 + 420t_3$ni, ya'ni $x = 49 + 420t_3$, $t_3 \epsilon Z$ ni hosil qilamiz .

5). $x \equiv 13(mod\ 16) \rightarrow x = 3 + 16t_1, t_1 \epsilon Z$.

x ning bu qiymatini 2 – taqqoslamaga qo'yib $t_1$ni aniqlaymiz.

$-3 + 16t_1 \equiv 3(mod\ 10) \rightarrow 16t_1 \equiv$ $6(mod\ 10)$. Bunda $(16, 10) = 2$ va $6 \vdots 2$ bo'lgani uchun $8t_1 \equiv$ $3(mod\ 5) \rightarrow 8t_1 \equiv 8(mod\ 5) \rightarrow t_1 \equiv 1(mod\ 5)$, ya'ni $t_1 = 1 + 5t_2$, $t_2 \epsilon Z$. x ning bu qiymatini 3-taqqoslamaga qo'yib t_2 ni aniqlaymiz: $x = -3 + 16(1 + 5t_2) = 13 + 80t_2$. Buni 3-taqqoslamaga olib borib qo'yib t_2 ni topamiz. $13 + 80t_2 \equiv 9(mod\ 14) \rightarrow 80t_2 \equiv -4(mod\ 14)$. Bunda $(80,\ 14) = 24 \vdots 2$ bo'lgani uchun bo'lgani uchun taqqoslamaning ikkala tomoni va modulini 2 ga qisqartirib $40t_2 \equiv -2(mod\ 7)$ ni yoki bundan $-2t_2 \equiv -2(mod\ 7) \rightarrow t_2 \equiv 1(mod\ 7)$, ya'ni $t_2 = 1 + 7t_3, t_3 \in Z$ ga ega bo'lamiz. Demak, $x = 13 + 80t_2 = 13 + 80(1 + 7t_3) = 93 + 560t_3$, ya'ni $x = 93 + 560t_3$, $t_3 \in Z$ ni hosil qilamiz.

6). $x = 9 + 10t_1, 9 + 10t_1 \equiv 10(mod\ 15) \rightarrow 10t_1 \equiv 1(mod\ 15)$. Bundan $(10,15) = 5$, lekin 1 soni 5 ga bo'linmaydi. Demak taqqoslamalar sistemasi yechimga ega emas .

7). $x = 7 + 9t_1, t_1 \in Z$. x ning bu qiymatini 2 – taqqoslamaga qo'yib $t_1$ni aniqlaymiz. $7 + 9t_1 \equiv 2(mod\ 7) \rightarrow 2t_1 \equiv$ $2(mod\ 2) \rightarrow t_1 \equiv 1(mod\ 7) \rightarrow t_1 = 1 + 7t_2, t_2 \in Z$. Demak, $x = 7 + 9(1 + 7t_2) = 16 + 63t_2, t_2 \in Z$. Buni 3 – taqqoslamaga olib borib qo'yib t_2 ni topamiz. $16 + 63t_2 \equiv$

$3(mod\ 12) \to 63t_2 \equiv -13(mod\ 12) \to 3t_2 \equiv -1(mod\ 12)$. Bunda $(3,12) = 3$ va lekin 1 soni 3 ga bo'linmaydi. taqqoslamalar sistemasi yechimga ega emas.

8).$x = 5 + 12t_1$, $5 + 12t_1 \equiv 2(mod\ 8) \to 12t_1 \equiv -3(mod\ 8) \to 4t_1 \equiv 5(mod\ 8)$ bunda $(4,\ 8) = 4$, lekin 5 soni 8 ga bo'linmaydi shuning uchun ham taqqoslamalar sistemasi yechimga ega emas.

9).$x = 7 + 10t_1, t_1 \in Z$. x ning bu qiymatini 2 – taqqoslamaga

qo'yib $t_1$ni aniqlaymiz. $7 + 10t_1 \equiv 2(mod\ 5), 10t_1 \equiv -5(mod\ 5)$. Bunda

$(10,5) = 5$ va $5 \vdots 5$ bo'lgani uchun oxirgi taqqoslamani 5 ga qisqartirib $2t_1 \equiv -1(mod\ 1)$doimo bajariladigan taqqoslamaga ega bo'lamiz. Demak, $t_1 = t_2$ deb olish mumkin, u holda $x = 7 + 10t_2$ ni hosil qilamiz, buni 3-taqqoslamaga qo'ysak $7 + 10t_2 \equiv 8(mod\ 9) \to 10t_2 \equiv 1(mod\ 9) \to t_2 \equiv 1(mod\ 9)$, ya'ni $t_2 = 1 + 9t_3, t_3 \in Z$. Bundan $x = 7 + 10t_2 = 7 + 10(1 + 9t_3) = 17 + 90t_3, t_3 \in Z$ ni hosil qilamiz.

10). $x \equiv 8(mod\ 7) \to x \equiv 1(mod\ 7) \to x = 1 + 7t_1, t_1 \in Z$.

x ning bu qiymatini 2 – taqqoslamaga qo'yib $t_1$ni aniqlaymiz.

$1 + 7t_1 \equiv 3(mod\ 11) \to 7t_1 \equiv 2(\text{mod } 11) \to 7t_1 \equiv (2 + 11 \cdot 3)(mod\ 11) \to t_1 \equiv 5(mod\ 11)$, ya`ni $t_1 = 5 + 11t_2, t_2 \epsilon Z$. Buni $x = 1 + 7t_1$ ga olib borib qo'ysak $x = 1 + 7(5 + 11t_2) = 36 + 77t_2$. x ning bu qiymatini 3-taqqoslamaga qo'ysak $36 + 77t_2 \equiv 9(mod\ 13) \to 77t_2 \equiv -27(mod\ 13) \to (77 - 6 \cdot 13)t_2 \equiv (-27 + 2 \cdot 13)(mod\ 13) \to t_2 \equiv -1(mod\ 13) \to t_2 \equiv 1(mod\ 13) \to t_2 = 1 + 13t_3, t_3 \epsilon Z$. Shunday qilib $x = 36 + 77(1 + 13t_3) = 113 + 1001t_3, t_3 \epsilon Z$. Ya`ni $x = 113 + 1001t_3, t_3 \epsilon Z$ berilgan sistemaning yechimi .

11). Bu sistemadagi har bir taqqoslama alohida-alohida x ga nisbatan yechilgan holda berilgan. Shuning uchun ham 1-taqqoslamaning yechimlar $x = 2 + 5t_1$, $t_1 \in Z$ larning orasidan 2- taqqoslamani qanoatlantiruvchilarini ajratib olamiz. Buning uchun $x = 2 + 5t_1$ ni 2-taqqoslamaga qo'yib, t_1 ni aniqlaymiz:

$2 + 5t_1 \equiv 8(\text{mod} 11) \Rightarrow 5t_1 \equiv 6(\text{mod} 11) \Rightarrow 5t_1 \equiv -5(\text{mod} 11)$,

$t_1 \equiv -1(\text{mod}11), t_1 = -1+11t_2, t_2 \in Z$. t_1 ning topilgan ifodasini x ga olib borib qo'yamiz. U holda x=2+5(-1+11 t_2)=-3+55t_2, ya'ni $x = -3+55t_2$, $t_2 \in Z$ ga ega bo'lamiz. x ning bu ifodasini 3-taqqoslamaga olib borib qo'yib, t_2 ni aniqlaymiz.

$-3+55t_2 \equiv 12(\text{mod}15) \Rightarrow 55t_2 \equiv 15(\text{mod}15) \Rightarrow 10t_2 \equiv 0(\text{mod}15)$

bunda $(10,15)=5$ bo'lganidan $2t_2 \equiv 0(\text{mod}3)$ *yoki* $t_2 \equiv 0(\text{mod}3)$, bundan $t_2 = 15t_3, t_2 = 3+15t_3, t_2 = 6+15t_3, t_2 = 9+15t_3, t_2 = 12+15t_3, t_3 \in Z$ larni hosil qilamiz . U holda berilgan sistemaning yechimlari: $x_1 = -3+825t_3$,

$x_2 = 162+825t_3, x_3 = 327+825t_3, x_4 = 492+825t_3, x_5 = 657+825t_3$, $t_3 \in Z$ ga ega bo'lamiz.

268.1). Bizning misolimizda $m_1 = 6, m_2 = 7, m_3 = 11$, $M_1 = 77$, $M_2 = 66$, $M_3 = 42$ $\left(M_i = \frac{M}{m_i}\right)$, $b_1 = 1, b_2 = 2, b_3 = 3$, M_i' larni $M_iM_i' \equiv 1(mod\ m_i)$ $(i = 1,2,3,...)$ taqqoslamadan aniqlaymiz. $77M_1' \equiv 1(mod6) \to 5M_1' \equiv 1(mod\ 6) \to 5M_1' \equiv (1+4\cdot 6)(mod\ 6) \to M_1' \equiv 5(mod\ 6)$. Demak $M_1' = 5$; $66M_2' \equiv 1(mod\ 7) \to 3M_2' \equiv 1(mod\ 7) \to 3M_2' \equiv -6(mod\ 7) \to M_2' \equiv -2(mod\ 7); M_2' = -2$;

$42M_3' \equiv 1(mod\ 11) \to -2M_3' \equiv 12(mod\ 11) \to M_3' \equiv 6(mod\ 11), M_3' = -6$ deb olishimiz mumkin. Endi $x_0 = M_1M_1'b_1 + M_2M_2'b_2 + M_3M_3'b_3$(1) formuladan x_0 ni aniqlaymiz. $x_0 = 77\cdot 5\cdot 1 + 66\cdot(-2)\cdot 2 + 42\cdot(-6)\cdot 3 = 385 - 264 + 756 == -635$. Demak, sistemaning yechimi $x \equiv -635(mod\ 462) \equiv 289(mod\ 462)$.

2). Avvalo berilgan $2x \equiv 1(mod\ 5), x \equiv 2(mod\ 7), 3x \equiv 4(mod\ 11)$ taqqoslamalarni x ga nisbatan yechib olamiz. U holda $\begin{cases} x \equiv 3(mod\ 5) \\ x \equiv 2(mod\ 7) \\ x \equiv 5(mod\ 11) \end{cases}$ sistemaga ega bo'lamiz va bu sistemani 1-misoldagi singari mulohaza yurutib yechamiz. Bunda $M = 385$, $M_1 = 77$, $M_2 = 55$, $M_3 = 35, b_1 = 3, b_2 = 2, b_3 = 5$. M_i' larni aniqlaymiz.$77M_1' \equiv 1(mod\ 5) \to 2M_1' \equiv 1(mod\ 5) \to M_1' = 3$; $55M_1' \equiv 1(mod\ 7) \to -M_2' \equiv 1(mod\ 7) \to M_2' = -1$; $35M_3' \equiv 1(mod\ 11) \to 2M_3' \equiv 1(mod\ 11) \to M_3' = 6$; Endi x_0 ni aniqlaymiz. $x_0 = 77\cdot 3\cdot 3 + 55\cdot(-1)\cdot 2 + 35\cdot 6\cdot 5 = 693 - 110 + +1050 = 1633$ va $x \equiv$

$1633(mod\ 385) \equiv 93(mod\ 385)$. Demak, berilgan sistemaning yechimi $x \equiv 93(mod\ 385)$.

3). 2-misoldagi singari mulohaza yurutib berilgan sistemaning $x \equiv 6(mod\ 17), x \equiv 2(mod\ 5), x \equiv -2(mod\ 9)$ ko'rinishga keltirib olamiz. Bunda $m_1 = 17, m_2 = 5, m_3 = 9$ va $M = 765, M_1 = 45, M_2 = 153, M_3 = 85$, $b_1 = 6$, $b_2 = 12$, $b_3 = -2$. M_1', M_2', M_3' larni aniqlaymiz. $45M_1' \equiv 1(mod\ 17)$

$\to 11M_1' \equiv 1(mod\ 17) \to 11M_1' = (1 - 17 \cdot 2)(mod\ 17) \to M_1' \equiv -3(mod\ 17) \to M_1' = -3$; $153M_2' \equiv 1(mod\ 5) \to 3M_2' \equiv 6(mod\ 5) \to M_2' = 2$.

$85M_3' \equiv 1(mod\ 9) \to 4M_3' \equiv 10(mod\ 9) \to 2M_3' = -2$. Bularga asosan

$x_0 = 45 \cdot (-3) \cdot 6 + 153 \cdot 2 \cdot 2 + 85 \cdot (-2) \cdot (-2) = -810 + 612 + 340 = 142$. Demak, berilgan sistemaning yechimi $x \equiv 142(mod\ 765)$.

4).Yuqoridagi misollar singari mulohaza yuritib berilgan sistemani $x \equiv 4(mod\ 9)$, $x \equiv 4(mod\ 13), x \equiv 6(mod\ 11)$ ko'rinishiga keltirib olamiz.Bunda$m_1 = 9, m_2 = 13, m_3 = 11$ va $M = 1287$, $M_1 = 143$, $M_2 = 99$, $M_3 = 117$, $b_1 = 4$, $b_2 = 4$, $b_3 = 6$. M_1', M_2', M_3' larni aniqlaymiz.$143M_1' \equiv 1(mod\ 9) \to -M_1' \equiv 1(mod\ 9) \to M_1' = -1(mod\ 9) \to M_1' = -3;99M_2' \equiv 1(mod\ 13) \to 8M_2' \equiv 14(mod\ 13) \to 4M_2' \equiv 7(mod\ 13) \to M_2' \equiv 5(mod\ 13) \to M_2' = 5.117M_3' \equiv 1(mod\ 11) \to -5M_3' \equiv 1(mod\ 11) \to 6M_3' \equiv 12(mod\ 11) \to M_3' = 2$. Bularga asosan

$x_0 = 143 \cdot (-3) \cdot 4 + 99 \cdot 4 \cdot 5 + 117 \cdot 2 \cdot 6 = -1716 + 1980 + 1404 = 1668$. Demak, berilgan sistemaning yechimi $x \equiv 1668(mod\ 1287) \equiv 381(mod 1287)$.

5). $\begin{cases} 6x \equiv 1(mod\ 35) \\ 3x \equiv 4(mod\ 17) \\ 10x \equiv 7(mod\ 13) \end{cases} \leftrightarrow \begin{cases} x \equiv 6(mod\ 35) \\ x \equiv 7(mod\ 17). \\ x \equiv 2(mod\ 13) \end{cases}$

Bundan $m_1 = 35, m_2 = 17, m_3 = 13$ va $M = 7735, M_1 = 221, M_2 = 455, M_3 = 595$, $b_1 = 6$, $b_2 = 7$, $b_3 = 2$. M_1', M_2', M_3' larni aniqlaymiz. $221M_1' \equiv 1(mod\ 35) \to (221 - 6 \cdot 35)M_1' \equiv 1(mod\ 35) \to 11M_1' \equiv 1(mod\ 35) \to 24M_1' \equiv 3(mod\ 35) \to -2M_1' \equiv 3(mod\ 35) \to M_1' = 16;455M_2' \equiv 1(mod\ 17)$

$\rightarrow -4M_2' \equiv -16(mod\ 17) \rightarrow M_2' \equiv 4; 595M_3' \equiv 1(mod\ 13) \rightarrow$ $(595 - 13 \cdot 46)M_3' \equiv 1(mod\ 13) \rightarrow -3M_3' \equiv -12(mod\ 13) \rightarrow M_3' \equiv$ 4. Endi x_0 ni aiqlaymiz.

$x_0 = 221 \cdot 16 \cdot 6 + 455 \cdot 4 \cdot 7 + 595 \cdot (-1) \cdot 2 = 21216 +$ $12740 + 4760 = 38716$ va demak $x \equiv 38716(mod\ 7735) \equiv$ $(38716 - 5 \cdot 7735)(mod\ 7735) \equiv (mod\ 7735)$. Shunday qilib, $x \equiv$ $41(mod\ 7735)$ berilgan sistemaning yechimi.

$$6).\begin{cases} 8x \equiv 7(mod\ 17) \\ 5x \equiv 11(mod\ 6) \\ x \equiv -1(mod\ 19) \end{cases} \leftrightarrow \begin{cases} x \equiv 3(mod\ 17) \\ x \equiv 1(mod\ 6) \\ x \equiv -1(mod\ 19) \end{cases}.$$

Bundan $m_1 = 17, m_2 = 6, m_3 = 19$ va $M = 1938, M_1 = 114, M_2 =$ $323, M_3 = 102, b_1 = 3, b_2 = 1, b_3 = -1$. M_1', M_2', M_3' larni aniqlaymiz. $114M_1' \equiv 1(mod\ 17) \rightarrow (114 - 7 \cdot 17)M_1' \equiv 1(mod\ ) \rightarrow$ $-5M_1' \equiv 35(mod\ 17) \rightarrow M_1' = -7; 323M_2' \equiv 1(mod\ 6) \rightarrow (323 - 54 \cdot$ $6)M_2' \equiv 1(mod\ 17) \rightarrow M_2' \equiv -1; 102M_3' \equiv 1(mod\ 19) \rightarrow (102 - 19 \cdot$ $5)M_3' \equiv 1(mod\ 19) \rightarrow 7M_3' \equiv 1(mod\ 19) \rightarrow 7M_3' \equiv (1 - 3 \cdot$ $19)(mod\ 19) \rightarrow M_3' \equiv -8(mod\ 19) \rightarrow M_3' \equiv -8$. Endi x_0 ni aniqlaymiz.

$x_0 = 114 \cdot (-7) \cdot 3 + 323 \cdot (-1) \cdot 11 + 102 \cdot (-8) \cdot (-1) =$ $-2394 - 323 + 816 = -1901$. Demak, $x \equiv -1901(mod\ 1938) \equiv$ $37(mod\ 1938)$, ya`ni $x \equiv 37(mod\ 1938)$ berilgan taqqoslamalar sistemasi yechimi.

$$7).\begin{cases} 11x \equiv -4(mod\ 18) \\ 7x \equiv 1(mod\ 11) \\ 3x \equiv 5(mod\ 7) \end{cases} => \begin{cases} -7x \equiv 14(mod\ 18) \\ -4x \equiv 12(mod\ 11) \\ 3x \equiv 12(mod\ 7) \end{cases} =>$$

$\begin{cases} x \equiv -2(mod\ 18) \\ x \equiv -3(mod\ 11). \\ x \equiv 4(mod\ 7) \end{cases}$ Bundan $m_1 = 18, m_2 = 11, m_3 = 7$ va $M =$ $1386, M_1 = 77, M_2 = 126, M_3 = 198, b_1 = -2, b_2 = -3, b_3 =$ 4 M_1', M_2', M_3' larni aniqlaymiz. $77M_1' \equiv 1(mod\ 18) \rightarrow 5M_1' \equiv$ $1(mod\ 18) \rightarrow 5M_1' \equiv (1 - 2 \cdot 18)(mod\ 18) \rightarrow M_1' = -7; 126M_2' \equiv$ $1(mod\ 11) \rightarrow (126 - 11 \cdot 11)M_2' \equiv 1(mod\ 11) \rightarrow 5M_2' \equiv$ $1(mod\ 11) \rightarrow -6M_2' \equiv 12(mod\ 11) \rightarrow M_2' \equiv -2(mod\ 11) \rightarrow M_2' =$ $-2; 198M_3' \equiv 1(mod\ 7) \rightarrow (198 - 28 \cdot 7)M_3' \equiv 1(mod\ 7) \rightarrow 2M_3' \equiv$ $1(mod\ 7) \rightarrow M_3' = 4$. Endi x_0 ni aniqlaymiz. $x_0 = 77 \cdot (-7) \cdot (-2) +$ $126 \cdot (-2) \cdot (-3) + 198 \cdot 4 \cdot 4 = 1078 + 756 + +3168 = 5002$.

Demak, $x \equiv 5002(mod\ 1386) \equiv 844(mod\ 1386)$ berilgan taqqoslamalar sistemasining yechimi.

8). $\begin{cases} 21x \equiv -2(mod\ 23) \\ 12x \equiv 3(mod\ 9) \\ x \equiv 6(mod\ 11) \end{cases} \leftrightarrow \begin{cases} x \equiv 1(mod\ 23) \\ x \equiv 1(mod\ 9) \\ x \equiv 6(mod\ 11) \end{cases}$. Bundan $m_1 = 23, m_2 = 9, m_3 = 11$ va $M = 2277, M_1 = 99, M_2 = 253, M_3 = 207, b_1 = 1, b_2 = 1, b_3 = 6$. M_1', M_2', M_3' larni aniqlaymiz. $99M_1' \equiv 1(mod\ 23) \rightarrow (99 - 4 \cdot 23)M_1' \equiv 1(mod\ 23) \rightarrow 7M_1' \equiv 1(mod\ 23) \rightarrow 7M_1' \equiv (1 + 3 \cdot 23)(mod\ 23) \rightarrow 7M_1' \equiv 70(mod\ 23) \rightarrow M_1' = 10;$ $253M_2' \equiv 1(mod\ 9) \rightarrow (-28 \cdot 9 + 253)M_2' \equiv 1(mod\ 9) \rightarrow M_2' \equiv 1;$ $207M_3' \equiv 1(mod\ 11) \rightarrow (207 - 19 \cdot 11)M_3' \equiv 1(mod\ 11) \rightarrow M_3' \equiv -6$. Bulardan foydalanib x_0 ni topamiz.

$x_0 = 99 \cdot 10 \cdot 1 + 253 \cdot 1 \cdot 1 + 207 \cdot (-6) \cdot 6 = 990 + 255 - 7452 = -6209$. Demak $x \equiv -6209(mod\ 2277)$ berilgan sistemaning yechimi.

9). $\begin{cases} x \equiv 3(mod\ 29) \\ x \equiv -5(mod\ 12) \\ 2x \equiv 7(mod\ 11) \end{cases} \leftrightarrow \begin{cases} x \equiv 3(mod\ 29) \\ x \equiv -5(mod\ 12) \\ x \equiv 9(mod\ 11) \end{cases}$. Bundan $m_1 = 29, m_2 = 12, m_3 = 11$ va $M = 3828, M_1 = 132, M_2 = 319, M_3 = 318, b_1 = 3, b_2 = -5,\ b_3 = 9$. M_1', M_2', M_3' larni aniqlaymiz.

$132M_1' \equiv 1(mod\ 29) \rightarrow -13M_1' \equiv 30(mod\ 29) \rightarrow 16M_1' \equiv 30(mod\ 29) \rightarrow 8M_1' \equiv 15(mod\ 29) \rightarrow 8M_1' \equiv 44(mod\ 29) \rightarrow 2M_1' \equiv 11(mod\ 29) \rightarrow M_1' \equiv 2(mod\ 29) \rightarrow M_1' \equiv -9(mod\ 29) \rightarrow M_1' = 9;$ $319M_2' \equiv 1(mod\ 12) \rightarrow (319 - 27 \cdot 12)M_2' \equiv 1(mod\ 12) \rightarrow -5M_2' \equiv (1 + 2 \cdot 12)(mod\ 12) \rightarrow M_2' = -5;$ $348M_3' \equiv 1(mod\ 11) \rightarrow (348 - 11 \cdot 32)M_3' \equiv 1(mod\ 11) \rightarrow -4M_3' \equiv 12(mod\ 11) \rightarrow M_3' \equiv -3$. Endi x_0 ni aiqlaymiz. $x_0 = 132 \cdot (-9) \cdot 3 + 319 \cdot (-5) \cdot (-5) + 348 \cdot (-3) \cdot 9 = -3564 + 7975 - 9396 = -4985$.

Demak, $x \equiv -4985(mod\ 3828) \equiv 2671(mod\ 3828)$ berilgan taqqoslamalar sistemasining yechimi bo‘ladi.

10). $\begin{cases} 6x \equiv 5(mod\ 35) \\ x \equiv -2(mod\ 17) \\ 5x \equiv 3(mod\ 13) \end{cases} \leftrightarrow \begin{cases} x \equiv 6(mod\ 35) \\ x \equiv -2(mod\ 17) \\ x \equiv 6(mod\ 13) \end{cases}$. Bundan $m_1 = 31, m_2 = 29, m_3 = 27$ va $M = 24273, M_1 = 783, M_2 = 837, M_3 = 899, b_1 = 6, b_2 = -2, b_3 = 6$. M_1', M_2', M_3' larni aniqlaymiz. $783M_1' \equiv$

$1(mod\ 31) \to (783 - 31 \cdot 25)M_1' \equiv 1(mod\ 31) \to 8M_1' \equiv 32(mod\ 31) \to M_1' \equiv 4(mod\ 31) \to M_1' = 4;$

$837M_2' \equiv 1(mod\ 29) \to (837 - 29 \cdot 29)M_2' \equiv 1(mod\ 29) \to -4M_2' \equiv 1(mod\ 29) \to -2M_2' \equiv 15(mod\ 29) \to M_2' \equiv 22(mod\ 29) \to M_2 = 7;$

$899M_3' \equiv 1(mod\ 27) \to (899 - 27 \cdot 33)M_3' \equiv 1(mod\ 27) \to 8M_3' \equiv 1(mod\ 27) \to 8M_3' \equiv 28(mod\ 27) \to 2M_3' \equiv 7(mod\ 27) \to M_3' \equiv 17(mod\ 27) \to M_3' \equiv 10.$ Endi x_0 ni aniqlaymiz.$x_0 = 783 \cdot 46 + 837 \cdot 7 \cdot (-2) + 899 \cdot (-10) \cdot 6 = 18792 - 11718 - 53940 = 46866.$ Bundan $x \equiv -46866(mod\ 24273) \equiv 1680(mod\ 24273)$ berilgan sistemaning yechimi ekanligi kelib chiqadi.

11). Bu yerda $m_1 = 7, m_2 = 9, m_3 = 11$ va $M = 693,\ M_1 = 99,\ M_2 = 77,\ M_3 = 63,\ b_1 = 1, b_2 = 3, b_3 = 5.$ Endi M_1', M_2', M_3' larni aniqlaymiz.

$99M_1' \equiv 1(mod 7) \to (99 - 7 \cdot 14)M_1' \equiv 1(mod\ 7) \to M_1' \equiv 1(mod\ 7) \to M_1' = 1; 77M_2' \equiv 1(mod\ 9) \to (77 - 9 \cdot 8)M_2' \equiv 1(mod\ 9) \to 5M_2' \equiv 1(mod\ 9) \to 5M_2' \equiv 10(mod\ 9) \to M_2' \equiv 2(mod\ 9) \to M_2' = 2; 63M_3' \equiv 1(mod\ 11) \to (63 - 11 \cdot 6)M_3' \equiv 1(mod\ 11) \to -3M_3' \equiv 1(mod\ 11) \to -3M_3' \equiv 12(mod\ 11) \to M_3' \equiv -4(mod\ 11) \to M_3' \equiv 7(mod\ 11) \to M_3' \equiv 7.$ Endi x_0 ni aniqlaymiz.$x_0 = 99 \cdot 1 \cdot 1 + 77 \cdot 2 \cdot 3 + 63 \cdot 7 \cdot 5 = 99 + 462 + 2205 = 2766.$ Bundan $x \equiv 2766(mod\ 693) \equiv -6(mod\ 693)$ berilgan sistemaning yechimi ekanligi kelib chiqadi.

269. 1). Bu masala taqqoslamaning ta'rifiga ko'ra shunday x ni topishimiz kerakki, u $\begin{cases} x \equiv 1(mod\ 7) \\ x \equiv 2(mod\ 8) \\ x \equiv 3(mod\ 9) \end{cases}$ taqqoslamalar sistemasini qanoatlantiruvchi eng kichik natural son bo'lishi kerak. Berilgan sistemani yechamiz. Buning uchun bizga berilgan sistemada modullar o'zaro tub bo'lganligi sababli 2–misoldagi (1) formuladan foydalansak bo'ladi. Bizda $m_1 = 7, m_2 = 8, m_3 = 9$ va $M = 7 \cdot 8 \cdot 9 = 504,\ M_1 = 72, M_2 = 63,\ M_3 = 56, b_1 = 1, b_2 = 2, b_3 = 3.$ M_1', M_2', M_3' larni aniqlaymiz.$72M_1' \equiv 1(mod\ 7) \to 2M_1' \equiv 8(mod\ 7) \to M_1' \equiv 4; 63M_2' \equiv 1(mod\ 8) \to -M_2' \equiv 1(mod\ 8) \to M_2' \equiv 1(mod\ 8) \to M_2' = -1; 56M_3' \equiv 1(mod\ 8) \to 2M_3' \equiv 10(mod\ 9) \to M_3' \equiv 5(mod\ 9) \to M_3' \equiv 5.$

Bulardan foydalanib x_0 ning qiymatini aniqlaymiz:

$x_0 = 72 \cdot 41 + 63 \cdot (-1) \cdot 2 + 56 \cdot 5 \cdot 3 = 288 - 126 + 840 = 1002.$

Demak, $x \equiv 1002(mod\ 504) \equiv -6(mod\ 504)$, ya`ni $x = -6 + 504t$, $t \epsilon Z$ berilgan sistemaning umumiy yechimi. Endi shular orasidan x ning eng kichik natural son bo'ladigan qiymatini aniqlab olamiz. Agar $t \leq 0$ bo'lsa, $x < 0$ bo'ladi; $t = 1$ da $x = 498$ izlanyotgan qiymatga ega bo'lamiz.

2). $\begin{cases} x \equiv 1(mod\ 3) \\ x \equiv 2(mod\ 4) \\ x \equiv 3(mod\ 5) \end{cases} \rightarrow$ bundan $b_1 = 1,\ b_2 = 2,\ b_3 = 3,\ m_1 = 3,\ m_2 = 4,\ m_3 = 5,\ M = 60,\ M_1 = 20,\ M_2 = 15,\ M_3 = 12$. M_1', M_2', M_3'larni aniqlaymiz.

$20M_1' \equiv 1(mod\ 3) \rightarrow 2M_1' \equiv 1(mod\ 3) \rightarrow M_1' \equiv 2(mod\ 3) \rightarrow M_1' = 2;$

$15M_2' \equiv 1(mod\ 4) \rightarrow -M_2' \equiv 1(mod\ 4) \rightarrow M_2' \equiv -1(mod\ 4) \rightarrow M_2' = -1;$

$12M_3' \equiv 1(mod\ 5) \rightarrow 2M_3' \equiv 6(mod\ 5) \rightarrow M_3' \equiv 3(mod\ 5) \rightarrow M_3' \equiv 5$. Endi x_0 ni topamiz. $x_0 = 20 \cdot 2 \cdot 1 + 15 \cdot (-1) \cdot 2 + 12 \cdot 3 \cdot 3 = 40 - 30 + 108 = 118$. Demak, sistemaning umumiy yechimi $x \equiv 118(mod\ 60)$, yoki buni $x \equiv -2(mod\ 60)$, ya`ni $x = -2 + 60t$, $t \epsilon Z$ ko'rinishida yozish mumkin. Bundan izlanayotgan eng kichik natural qiymat 58 ga teng ekanligi kelib chiqadi.

3). $\begin{cases} x \equiv 3(mod\ 9) \\ x \equiv 5(mod\ 10) \\ x \equiv 6(mod\ 13) \end{cases} \rightarrow$ dan $b_1 = 3,\ b_2 = 5,\ b_3 = 6,\ m_1 = 9,\ m_2 = 10, m_3 = 13,\ M = 1170,\ M_1 = 130,\ M_2 = 117,\ M_3 = 90$. M_1', M_2', M_3' larni aniqlaymiz.$130M_1' \equiv 1(mod\ 9) \rightarrow (130 - 14 \cdot 9)M_1' \equiv 1(mod\ 9) \rightarrow 4M_1' \equiv 10(mod\ 9) \rightarrow 2M_1' \equiv 5(mod\ 9) \rightarrow M_1' \equiv 7(mod\ 9) \rightarrow M_1' = 7; 117M_2' \equiv 1(mod\ 10) \rightarrow (117 - 10 \cdot 12)M_2' \equiv -9(mod\ 10) \rightarrow M_2' \equiv 3(mod\ 10) \rightarrow M_2' = 3; 90M_3' \equiv 1(mod\ 13) \rightarrow (90 - 7 \cdot 13)M_3' \equiv 1(mod\ 13) \rightarrow M_3' \equiv -1(mod\ 5)$

$\rightarrow M_3' = -1$. Endi x_0 ni topamiz. $x_0 = 130 \cdot 7 \cdot 3 + 117 \cdot 3 \cdot 5 + 90 \cdot (-1) \cdot 6 = 2730 + 1755 - 540 = 3945$. Bu holda umumiy yechim $x \equiv 3945(mod\ 1170)$

$\equiv 435(mod\ 1170)$, ya'ni $x = 435 + 1170t$, $t \epsilon Z$. Bundan eng kichik natural yechim $x = 435$.

4). $\begin{cases} x \equiv 2(mod\ 9) \\ x \equiv 3(mod\ 10) \\ x \equiv 4(mod\ 13) \end{cases}$ $\rightarrow$ bu sistema 3-misoldagi sistemadan b_1, b_2, b_3 ning qiymatlari bilan farq qiladi. Shuning uchun ham $x_0 = 910 \cdot b_1 + 351 \cdot b_2 - 90b_3$, ya`ni $x_0 = 910 \cdot 2 + 351 \cdot 3 + 90 \cdot 4 = 1820 + 1053 - 360 = 2513$ va $x \equiv 2513(mod\ 1170) \equiv 173(mod\ 1170)$, ya`ni sistemaning umumiy yechimi $x = 173 + 1170t$, $t \epsilon Z$. Eng kichik natural yechim $x = 173$.

5). $\begin{cases} x \equiv 2(mod\ 3) \\ x \equiv 4(mod\ 7) \\ x \equiv 5(mod\ 8) \end{cases}$ $\rightarrow$ dan $b_1 = 4$, $b_2 = 4$, $b_3 = 5$, $m_1 = 3$, $m_2 = 7$, $m_3 = 8$, $M = 168$, $M_1 = 56$, $M_2 = 24$, $M_3 = 21$. M_1', M_2', M_3'larni aniqlaymiz.$56M_1' \equiv 1(mod\ 3) \rightarrow 2M_1' \equiv 1(mod\ 3) \rightarrow M_1' \equiv 2(mod\ 3) \rightarrow M_1' = 2$;$24M_2' \equiv 1(mod\ 7) \rightarrow 3M_2' \equiv 1(mod\ 7) \rightarrow 3M_2' \equiv 15(mod\ 7) \rightarrow M_2' \equiv 5(mod\ 7) \rightarrow M_2' = 5$;$21M_3' \equiv 1(mod\ 8) \rightarrow -3M_3' \equiv 9(mod\ 8) \rightarrow M_3' \equiv -3(mod\ 8) \rightarrow M_3' \equiv -3$. Shuning uchun ham $x_0 = 56 \cdot 2 \cdot 2 + 24 \cdot 5 \cdot 4 + 21 \cdot (-3) \cdot 5 = 224 + 480 - 315 = 389$ va $x \equiv 389(mod\ 168) \equiv 53(mod\ 168)$ sistemaning umumiy yechimi. Endi $x = 53 + 166t$, $t \epsilon Z$ dan eng kichik natural sonni aniqlaymiz. $t = 0$ da $x = 53$ izlanayotgan son.

6). $\begin{cases} x \equiv 4(mod\ 7) \\ x \equiv 9(mod\ 13) \\ x \equiv 1(mod\ 17) \end{cases}$ $\rightarrow$ dan$b_1 = 4, b_2 = 9, b_3 = 1, m_1 = 7, m_2 = 13$, $m_3 = 17$, $M = 7 \cdot 13 \cdot 17 = 1547$, $M_1 = 221$, $M_2 = 119$, $M_3 = 91$. M_1', M_2', M_3' larni aniqlaymiz. $221M_1' \equiv 1(mod\ 7) \rightarrow (221 - 31 \cdot 7)M_1' \equiv 1(mod\ 7) \rightarrow 4M_1' \equiv 8(mod\ 7) \rightarrow M_1' \equiv 2(mod\ 7) \rightarrow M_1' = 2$;$119M_2' \equiv 1(mod\ 13) \rightarrow -(119 - 9 \cdot 13)M_2' \equiv 1(mod\ 13) \rightarrow 2M_2' \equiv 14(mod\ 13) \rightarrow M_2 \equiv 7(mod\ 13) \rightarrow M_2' = 7$;$91M_3' \equiv 1(mod\ 17) \rightarrow (91 - 5 \cdot 17)M_3' \equiv 1(mod\ 17) \rightarrow 6M_3' \equiv -18(mod\ 17) \rightarrow M_3' \equiv 3(mod\ 17) \rightarrow M_3' \equiv 3$. Bulardan foydalanib x_0 ni topamiz. $x_0 = 221 \cdot 2 \cdot 4 + 119 \cdot 7 \cdot 9 + 91 \cdot 3 \cdot 1 = 1786 + 7497 + 273 = 9538$. Demak, $x \equiv 9538(mod\ 1547) \equiv (9538 - 6 \cdot 1547)(mod\ 1547) \equiv 256(mod\ 1547)$, ya`ni $x = 256 + 1547t$, $t \epsilon Z$

taqqoslamalar sistemasining umumiy yechimi. Bu holda eng kichik natural yechim 256 dan iborat.

7). $\begin{cases} x \equiv 9 \ (mod\ 13) \\ x \equiv 1 \ (mod\ 21) \\ x \equiv 13 \ (mod 23) \end{cases}$ bo'lgani uchun $b_1 = 9, b_2 = 1, b_3 = -10,\ m_1 = 13,\ m_2 = 21,\ m_3 = 23$ u holda $M = 6279,\ M_1 = 483,\ M_2 = 299,\ M_3 = 273$. Endi bulardan foydalanib M_1', M_2', M_3'larni aniqlaymiz.

$483M_1' \equiv 1(mod\ 13) \to (483 - 37 \cdot 13) \quad M_1' \equiv 1(mod\ 13) \to 2M_1' \equiv 14(mod\ 13) \to M_1' \equiv 7(mod\ 13) \to M_1' = 7;$ $299M_2' \equiv 1(mod\ 21) \equiv (299 - 14 \cdot 21)M_2' \equiv 1(mod\ 21) \to 5M_2' \equiv -20(mod\ 21) \to M_2' \equiv -4(mod\ 21) \to M_2' = -4;\ 273M_3' \equiv 1(mod\ 23) \to (273 - 23 \cdot 12)M_3' \equiv 1(mod\ 23) \to -3M_3' \equiv 1(mod\ 23) \to M_3' \equiv -8(mod\ 23) \to M_3' = -8.$

Bulardan foydalanib x_0 ni topamiz. $x_0 = 483 \cdot 7 \cdot 9 + 299 \cdot (-4) \cdot 1 + 273 \cdot (-8) \cdot (-10) = 30429 - 1196 + 21840 = 51073 = 8 \cdot 6279 + 841.$

Demak, $x_0 \equiv 841(mod 6279)$, ya'ni $x = 841 + 6279t, t \epsilon Z$ taqqoslamalar sistemasining umumiy yechimi. Eng kichik natural yechim 841 ga teng.

8). $\begin{cases} x \equiv 2(mod 3) \\ x \equiv 4(mod 5) \\ x \equiv 1(mod 8) \end{cases}$ dan $b_1 = -1,\ b_2 = 1,\ b_3 = 1,\ m_1 = 3, m_2 = 5,\ m_3 = 8$ deb olishimiz mumkin. Bu holda $M = 120, M_1 = 40,\ M_2 = 24,\ M_3 = 15$. Endi M_1', M_2', M_3' larni aniqlaymiz: $40M_1' \equiv 1(mod\ 3) \to M_1' \equiv 1(mod\ 3) \to M_1' = 1; 24M_2' \equiv 1(mod\ 5) \to -M_2' \equiv 1(mod\ 5) \to M_2' \equiv -1(mod\ 5) \to M_2' = -1; 15M_3' \equiv 1(mod 8) \to -M_3' \equiv 1(mod 8) \to M_3' \equiv -1(mod 8) \to M_3' = -1.$

topilganlardan foydalanib x_0 ni hisoblaymiz.

$$x_0 = 40 \cdot 1 \cdot (-1) + 24 \cdot (-1) \cdot (-1) + 15 \cdot (-1) \cdot 1 = -40 + 24 - 15 = -31.$$

Demak, $x \equiv -31(mod 120) \equiv 89(mod 120)$, ya'ni $x = 89 + 120t, t \in \mathrm{Z}$ taqqoslamalar sistemasining umumiy yechimi. Bundan masala shartini qanoatlantiradigan eng kichik natural son 89 ekanligi kelib chiqadi.

9). $\begin{cases} x \equiv 1(mod3) \\ x \equiv 4(mod5), \\ x \equiv 7(mod8) \end{cases}$ bundan ko'rinadiki, bu sistema 8-misoldagi sistemadan faqat b_1, b_2, b_3 larning qiymatlari bilan farq qiladi. Shuning uchun ham 8)-misolda qarab chiqilganiga asosan $x_0 = 40b_1 - 24b_2 - 15b_3 = 40 \cdot 1 - 24 \cdot 4 - 15 \cdot 7 = 40 - 96 - 105 = -167$ va $x \equiv -161(mod120) \equiv -41(mod120) \equiv 79(mod120)$ qaralayotgan taqqoslamalar sistemasining yechimi $x = 79 + 120t, t \in Z$ bo'lganligi uchun masala shartini qanoatlantiruvchi eng kichik natural son 79 bo'ladi.

10). $\begin{cases} x \equiv 4(mod\ 5) \\ x \equiv 6(mod\ 7), \\ x \equiv 1(mod\ 9) \end{cases}$ bo'lgani uchun $b_1 = -1, b_2 = -1, b_3 = 1$ deb olishimiz mumkin. Bizda $m_1 = 5, m_2 = 7, m_3 = 9, M = 315$, $M_1 = 63, M_2 = 45$, M_3=35. Endi M_1', M_2', M_3' larni aniqlaymiz.

$63M_1' \equiv 1(mod5) \rightarrow 3M_1' \equiv 6(mod5) \rightarrow M_1' \equiv 2(mod5) \rightarrow M_1' = 2$;

$45M_2' \equiv 1(mod7) \rightarrow 3M_2' \equiv 15(mod7) \rightarrow M_2' \equiv 5(mod7) \rightarrow M_2' = -2$;

$35M_3' \equiv 1(mod9) \rightarrow -M_3' \equiv 1(mod9) \rightarrow M_3' \equiv 1(mod9) \rightarrow M_3' = -1$.

Bularga asosan $x_0 = 63 \cdot 2 \cdot (-1) + 45 \cdot (-2) \cdot (-1) + 35 \cdot (-1) \cdot 1 = -126 + 90 - 35 = -71$ va $x \equiv -71(mod15) \equiv 244(mod315)$.

Shunday qilib, izlanayotgan natural son 244 dan iborat.

11). Bu masala taqqoslamaning ta'rifiga ko'ra shunday x ni topishimiz kerakki u, $\begin{cases} x \equiv 6(mod\ 7) \\ x \equiv 12(mod\ 13) \\ x \equiv 16(mod17) \end{cases}$ taqqoslamalar sistemasini qanoatlantiruvchi eng kichik natural son bo'lishi kerak. Berilgan sistemani yechamiz. Buning uchun bizga berilgan sistemada modullar o'zaro tub bo'lganligi sababli 2 – misolda (1) formuladan foydalansak bo'ladi. Bizda $m_1 = 7, m_2 = 13, m_3 = 17, M = 1517$, $M_1 = 63, M_2 = 45$, M_3=35. 6-misolga asosan

$x_0 = 442 \cdot b_1 + 833 \cdot b_2 + 273 \cdot b_3 = 442 \cdot (-1) + 833 \cdot (-1) + 273 \cdot (-1) = -1548$. Demak, $x \equiv -1548(mod\ 1547) \equiv$

$-1(mod\ 1547)$, ya`ni $x = 1546 + 1547t$, $t\epsilon Z$ taqqoslamalar sistemasining umumiy yechimi. Bu holda eng kichik natural yechim 1546 dan iborat.

270.1). a ning izlanayotgan qiymatini aniqlash uchun sistemani yechishga harakat qilamiz. Bunda 1 – misolda tanlangan usuldan foydalanishimiz mumkin.

$$\begin{cases} x \equiv 5(\mathrm{mod}18) \\ x \equiv 8(\mathrm{mod}21) \\ x \equiv a(\mathrm{mod}35), \end{cases} \rightarrow x = 5 + 18t_1, t_1 \in Z, 5 + 18t_1 \equiv 8(mod21) \rightarrow$$

$18t_1 \equiv 3(mod21) \rightarrow 6t_1 \equiv 1(mod7) \rightarrow -t_1 \equiv 1(mod7) \rightarrow t_1 \equiv 1,8,15\ (mod21)$.Bundan$x = 5 + 18(-1 + 21t_2) = -13 + 378t_2, t \in Z$ bo'ladi. Buni 3-tenglamaga qo'ysak $-13 + 378t_2 \equiv a(mod35) \rightarrow 378t_2 \equiv a + 13(mod35) \rightarrow (378 - 10 \cdot 35)t_2 \equiv a + 13(mod35) \rightarrow 28t_2 \equiv a + 13(mod35)$

Bunda $(28,35) = 7$ va demak, taqqoslama yechimga ega bo'lishi uchun

$a + 13 \equiv 0(mod7)$ bajarilishi kerak. Bundan $a \equiv -13(mod7) \rightarrow a \equiv 1(mod\ 7)$, ya`ni $a = 7k + 1, k \in Z$ ko'rinishda bo'lishi kerak ekanligi kelib chiqadi.

Izoh. Masalaning shartida a ning qanday qiymatida berilgan taqqoslamalar sistemasi yechimga ega, deb so'ralgan,(ya'ni sistemaning barcha yechimlarini topish so'ralmagan) shuning uchun ham a ning so'ralgan qiymatini topdik.

2)a ning izlanayotgan qiymatini aniqlash uchun sistemani yechishga harakat qilamiz. Bunda 1 – misolda tanlangan usuldan foydalanishimiz mumkin.

$$\begin{cases} x \equiv a(mod\ 7) \\ x \equiv 2(mod\ 9) \\ x \equiv 7(mod\ 11) \end{cases} \rightarrow x = 7 + 11t_1,\ t_1 \in Z.$$ Bundan $7 + 11t_1 \equiv 2(mod\ 9) \rightarrow 2t_1 \equiv -5(mod\ 9) \rightarrow 2t_1 \equiv 4(\mathrm{mod}9) \rightarrow t_1 \equiv 2(mod9)$yoki $t_1 = 2 + 9t_2,\ t_2 \in Z$. U holda $x = 7 + 11(2 + 9t_2) = 29 + 99t_2, t_2\epsilon Z$. $29 + 99t_2 \equiv a(mod7) \rightarrow 99t_2 \equiv a - 29(mod\ 7) \rightarrow t_2 \equiv a - 1(mod\ 7)$, bundan$t_2 = (a-1) + 7t_3$. Buni x ning ifodasiga olib borib qo'ysak $x = 29 + 99\big((a-1) + 7t_3\big) = 29 + +99(a-1) + 693t_3,\ t_3\epsilon Z$. Demak, berilgan sistema a – ning ixtiyoriy $a \epsilon Z$ qiymatlarida yechimga ega.

3).$x \equiv 5(mod\ 12) \to x = 5 + 12t_1 \to 5 + 12t_1 \equiv 3(mod\ 15) \to 12t_1 \equiv -2(mod\ 15) \to 12t_1 \equiv 13(mod\ 15)$. Bunda$(12,15) = 3$, lekin 13 soni 3ga bo'linmaydi. Shuning uchun ham berilgan sistema a ning birorta ham qiymatida yechimga ega emas.

4).$x = 11 + 20t_1, t \in Z$ dan $11 + 20t_1 \equiv 1(mod\ 15) \to 20t_1 \equiv -10(mod\ 15) \to 20t_1 \equiv 5(mod\ 15) \to 4t_1 \equiv 1(mod 3)$, ya'ni$t_1 = 1 + 3t_2, t_2 \in Z$ va $x = 11 + 20(1 + 3t_2) = 31 + 60t_2, t_2 \in Z$. 3-taqqoslamadan$31 + 60t_2 \equiv a(mod 18) \to 60t_2 \equiv a - 31(mod 18) \to 6t_2 \equiv a - 31(mod 18)$, bunda$(6, 18) = 6$ bo'lgani uchun berilgan taqqoslamalar sistemasi yechimga ega bo'lishi uchun $(a - 31) \vdots 6$ bo'lishi, ya'ni $a - 31 \equiv 0\ (mod\ 6)$, yoki bundan $a \equiv 1\ (mod\ 6)$ ning bajarilishi kerak ekanligi kelib chiqadi. Shunday qilib $a \equiv 6k + 1, k \in Z$ ko'rinishda bo'lsa, berilgan sistema yechimga ega bo'lar ekan.

5).$x \equiv 19(mod\ 24) \to x = 19 + 24t_1, t_1 \in Z$. x ning bu qiymatini ikkinchi taqqoslamaga qo'yib t_1 ni aniqlaymiz: $19 + 24t_1 \equiv 10(mod\ 21)$. $3t_1 \equiv -9(mod\ 21) \to t_1 \equiv -3(mod\ 7)$, ya'ni $t_1 = -3 + 7t_2, t_2 \in Z$. Bu holda $x = 19 + 24(-3 + 7t_2 = -53 + 168t_2, t_2 \in Z$. Buni 3-taqqoslamaga qo'yib t_2 ni aniqlashga harakat qilamiz. $-53 + 168t_2 \equiv a(mod\ 9) \to 168t_2 \equiv a + 53(mod\ 9) \to (168 - 18 \cdot 9)t_2 \equiv a + 53\ (mod\ 9) \to 6t_2 \equiv a - 1(mod\ 9)$. Bunda $(6.9) = 3$, demak, u yechimga ega bo'lishi uchun$(a - 1) \vdots 3$, ya'ni $a \equiv 1(mod\ 3)$ bajarilishi kerak ekan. Demak, agar $a = 3k + 1, k \in Z$ ko'rinishda bo'lsa, berilgan taqqoslamalar sistemasi yechimga ega bo'ladi.

6). $x = 6 + 15t_1, t_1 \in Z$. Buni ikkinchi taqqoslamaga qo'yib $t_1$ni aniqlaymiz: $6 + 15t_1 \equiv 18(mod\ 21) \to 15t_1 \equiv 12(mod\ 21) \to 5t_1 \equiv 4(mod\ 7) \to 5t_1 \equiv 25(mod\ 7) \to t_1 \equiv 5(mod\ 7)$, ya'ni$t_1 = 5 + 7t_2, t_2 \in Z$. Bu holda $x = 6 + 15(5 + 7t_2) = 81 + 105t_2, t_2 \in Z$. x ning bu qiymatini 3-taqqoslamaga qo'yib t_2 ni aniqlaymiz. $81 + 105t_2 \equiv a(mod 11) \to 6t_2 \equiv a - 4(mod\ 11)$. Bunda $(6,11) = 1$ bo'lgani uchun a ning ixtiyoriy butun qiymatida berilgan taqqoslama yagona yechimga ega va demak, berilgan taqqoslamalar sistemasi ham a ixtiyoriy butun qiymatida yechimga ega.

7). $x = 19 + 56t_1, 19 + 56t_1 \equiv 3(mod 24) \to 8t_1 \equiv -16(mod\ 24) t_1 \equiv -2(mod\ 3) \to t_1 \equiv 1(mod\ 3), t_1 \in Z$.Ya`ni$t_1 = 1 + 3t_2,\ t_2 \in Z$. Buni inobatga olsak $x = 19 + 56(1 + 3t_2) = 75 + 168t_2$. x ning bu qiymatini 3- taqqoslamaga qo'yib $t_2$ni aniqlaymiz:

$75 + 168t_2 \equiv a\ (mod\ 20) \to 8t_2 \equiv a + 5(mod\ 20)$. Bunda $(8; 20) = 4$ va taqqoslama yechimga ega bo'lishi uchun $(a + 5) \vdots 4$, $k \in Z$, ya'ni $a \equiv -5\ (mod\ 4)$. Yoki bundan $a \equiv 3(mod\ 4)$ shartni qanoatlantirishi kerak. Demak, agar $a = 4k + 3, k \in Z$ ko'rinishidagi butun son bo'lsa, berilgan taqqoslama yechimga ega bo'ladi.

8). $x = 3 + 5t_1, t_1 \in Z$. Buni ikkinchi taqqoslamaga qo'yib t_1 ni aniqlaymiz: $3 + 5t_1 \equiv 2(mod\ 7) \to 5t_1 \equiv -1(mod\ 7) \to 5t_1 \equiv -15(mod\ 7) \to t_1 \equiv -3(mod\ 7)$, ya`ni $t_1 = -3 + 7t_2, t_2 \in Z$. Buni x ning ifodasiga olib borib qo'ysak $x = 3 + 5(-3 + 7t_2) = -12 + 35t_2, t_2 \in Z$. $12 + 35t_2 \equiv a(mod\ 9) - t_2 \equiv a + 3(mod\ 9) \to t_2 \equiv -(a + 3)(mod\ 9)$. x ning bu qiymatini 3-taqqoslamaga qo'yib t_2 ni aniqlaymiz: $-12 + 35t_2 \equiv a(mod\ 9) \to$

$-t_2 \equiv a + 3(mod\ 9) \to t_2 \equiv -(a + 3)(mod\ 9)$. Demak, $t_2 = -(a + 3) + 9t_3,\ t_3 \in$ Z. Bundan $x = -12 + 35[-(a + 3) + 9t_3] = -12 - 35(a + 3) + 315t_3, t_3 \in$ Z. Shunday qilib berilgan taqqoslamalar sistemasi a ning ixtiyoriy butun qiymatida yechimga ega.

9). a ning izlanayotgan qiymatini aniqlash uchun sistemani yechishga harakat qilamiz. Bunda 1 – misolda tanlangan usuldan foydalanishimiz mumkin. 1-taqqoslamadan $x = 1 + 3t_1, t_1 \in$ Z. x ning bu qiymatini 2-taqqoslamaga olib borib qo'yib t_1 ni aniqlaymiz: $1 + 3t_1 \equiv 5(mod\ 7) \to 3t_1 \equiv 4(mod\ 7) \to t_1 \equiv -1(mod\ 7)$, ya'ni$t_1 = -1 + 7t_2, t_2 \in$ Z. Buni x ning ifodasiga olib borib qoyib $x = 1 + 3(-1 + 7t_2) = -2 + 21t_2, t_2 \in Z$ ga ega bo'lamiz. x ning bu qiymatini 3-taqqoslamaga qo'yib $t_2$ni aniqlaymiz: $-2 + 21t_2 \equiv a(mod\ 11) \to 10t_2 \equiv (a + 2)(mod\ 11) \to -t_2 \equiv (a + 2)(mod\ 11) \to t_2 \equiv -(a + 2)(mod\ 11)$. Demak, a ning ixtiyoriy butun qiymatida berilgan taqqoslamalar sistemasi yechimga ega.

10). 1-taqqoslamadan $x = 14 + 19t_1, t_1 \in Z$. x ning bu qiymatini 2-taqqoslamaga olib borib qo'yib t_1 ni aniqlaymiz: $14 + 19t_1 \equiv 5(mod\ 25) \to 19t_1 \equiv -9(mod\ 25) \to -6t_1 \equiv 16(mod 25) \to -3t_1 \equiv 8(mod\ 25) \to -3t_1 \equiv 33(mod\ 25) \to t_1 \equiv -11(mod\ 25) \to t_1 =$

$-11+25t_2, t_2 \in Z$. Buni x ning ifodasiga olib borib qo'yib $x = 14 + 19(-11+25t_2) = -195+475t_2, t_2 \in Z$ ga ega bo'lamiz. x ning bu qiymatini 3-taqqoslamaga qo'yib $t_2$ni aniqlaymiz:

$$-195+475t_2 \equiv a(mod 10) \to 5t_2 \equiv a+5(mod 10).$$

Bunda $(5;10)=5$. Demak, taqqoslama yechimga ega bo'lishi uchun $(a+5) \vdots 5$, ya'ni $a \equiv -5(mod 5)$ bo'lishi kerak. Bundan $a \equiv 0(mod\ 5)$, ya'ni $a = 5k, k \in Z$. Demak, agar $a = 5k, k \in Z$ ko'rinishda bo'lsa, berilgan sistema yechimga ega bo'ladi.

11). a ning izlanayotgan qiymatini aniqlash uchun sistemani yechishga harakat qilamiz. Bunda 1-misolda tanlangan usuldan foydalanishimiz mumkin. 1-taqqoslamadan $x = 5+11t_1, t_1 \in Z$. x ning bu qiymatini 2-taqqoslamaga olib borib qo'yib t_1 ni aniqlaymiz: $5+11t_1 \equiv 4(mod\ 7) \to 4t_1 \equiv -1(mod\ 7) \to -3t_1 \equiv 6(mod 7) \to t_1 \equiv -2(mod 7) \to t_1 = -2+7t_2, t_2 \in Z$. Buni x ning ifodasiga olib borib qo'yib $x = 5+11(-2+7t_2) = -17+77t_2, t_2 \in Z$ ga ega bo'lamiz. x ning bu qiymatini 3-taqqoslamaga qo'yib $t_2$ni aniqlaymiz: $-17+77t_2 \equiv a(mod 9) \to 5t_2 \equiv a+17(mod 9) \to 5t_2 \equiv a-1(mod 9)$.

Bunda $(5;9)=1$. Demak, a ning ixtiyoriy butun qiymatida berilgan taqqoslamalar sistemasi yechimga ega. $2\cdot 5t_2 \equiv 2\cdot(a-1)(mod 9) \to t_2 \equiv 2\cdot(a-1)(mod 9)$.

271.1). Buning uchun berilgan to'g'ri chiziqning kesisish nuqtasini topish kerak. Bu tenglamalarni taqqoslama ko'rinishda yozib olib uning yechimini topamiz.

$\begin{cases} x \equiv 2(mod\ 5) \\ x \equiv 1(mod\ 8) \\ x \equiv 3(mod\ 11) \end{cases} \to x = 2+5t_1, t_1 \in Z$. Buni 2-taqqoslamaga olib borib qo'yib t_1 ni aniqlaymiz: $2+5t_1 \equiv 1(mod\ 8) \to 5t_1 \equiv -1(mod\ 8) \to 5t_1 \equiv (-1-3\cdot 8)(mod\ 8) \to 5t_1 \equiv -25\ (mod\ 8) \to t_1 \equiv -5(mod\ 8)$, ya'ni $t_1 = 3+8t_2,\ t_2 \in Z$. $t_1$ning bu ifodasini x ning ifodasiga olib borib qo'ysak $x = 2+5(3+8t_2) = 17+40t_2,\ t_2 \in Z$ hosil bo'ladi. $x$ning bu qiymatini 3-taqqoslamaga qo'yib t_2 ni aniqlaymiz: $17+40t_2 \equiv 3(mod\ 11) \to -4t_2 \equiv 8(mod\ 11) \to t_2 \equiv -2(mod\ 11) \to t_2 = -2+11t_3, t_3 \in Z$. Buni x ning ifodasidagi t_2 ning o'rniga olib borib qo'ysak $x = 17+40(-2+11t_3) = -63+440t_3$, $t_3 \in Z$ hosil bo'ladi. Demak, abssissasi $x = -63+440t_3, t_3 \in Z$ nuqtadan OX o'qiga chiqarilgan perpendikular berilgan chiziqlarni butun

koordinatali nuqtalarda kesadi. Bu nuqtalarni ordinatalarini to‘g‘ri chiziq tenglamasidan topamiz.Birinchi tenglamadan$-63+440t_3=2+5y\to 5y=-65+440t_2\to y=-13+88t_3$. Ikkinchi tenglamadan $-63+440t_3=1+8y\to -64+440t_3=8y\to y=-8+55t_3$.
Uchinchi tenglamadan$-63+440t_3=3+11y\to -66+440t_3=11y\to y=-6+40t_3$. Shunday qilib, bu nuqtalarning koordinatalari $(-63+440t_3;-13+88t_3),(-63+440t_3;-8+55t_3)$,
$(-63+440t_3-6+40t_3),\ \ t_3\in Z$.

2).Buning uchun berilgan to‘g‘ri chiziqning kesisish nuqtasini topish kerak. Bu tenglamalarni taqqoslama ko‘rinishda yozib olib uning yechimini topamiz.

$\begin{cases}4x\equiv 9(mod\ 7)\\2x\equiv 15(mod\ 9)\\5x\equiv 12(mod\ 13)\end{cases}\to\begin{cases}x\equiv 4(mod\ 7)\\x\equiv 3(mod\ 9)\\x\equiv 5(mod\ 13)\end{cases}$. Endi bu sistemani yechamiz. Sistemaning 1-taqqoslamasidan $x=4+7t_1,t_1\in Z$. Buni 2-taqqoslamaga olib borib qo‘yib t_1 ni aniqlaymiz: $4+7t_1\equiv 3(mod\ 9)\to 7t_1\equiv -1(mod\ 9)\to 16t_1\equiv 8(mod\ 9)\to 2t_1\equiv 1(mod\ 9)\to t_1\equiv 5(mod\ 9)\to t_1=5+9t_2,t_2\in Z.t_1$ ning topilgan qiymatini x ning ifodasiga olib borib qoysak $x=4+7(5+9t_2)$=$39+63t_2$,$t_2\in Z$ hosil bo‘ladi. x ning bu qiymatini 3-taqqoslamaga qo‘yib $t_2$ni aniqlaymiz: $39+63t_2\equiv 5(mod13)\to -2t_2\equiv 5(mod\ 13)\to t_2\equiv -9(mod\ 13)\to\ t_2\equiv 4(mod\ 13)$, ya`ni $t_2=4+13t_3,t_3\in Z$. Buni x ning ifodasidagi t_2 ning o‘rniga olib borib qo‘ysak $x=39+63(4+13t_3)=39+252+819t_3=291+819t_3,\ t_3\in Z$ ni hosil qilamiz. Bundan $x=291+819t_3$. Demak, abssissasi $x=291+819t_3,t_3\in Z$ nuqtadan OX o‘qiga chiqarilgan perpendikular berilgan chiziqlarni butun koordinatali nuqtalarda kesadi.

3).Buning uchun berilgan to‘g‘ri chiziqning kesisish nuqtasini topish kerak. Bu tenglamalarni taqqoslama ko‘rinishda yozib olib uning yechimini topamiz.

$\begin{cases}3x\equiv 1(mod\ 5)\\2x\equiv 3(mod\ 3)\\5x\equiv 7(mod\ 7)\end{cases}\to\begin{cases}x\equiv 2(mod\ 5)\\x\equiv 0(mod\ 3)\\x\equiv 0(mod\ 7)\end{cases}$. Endi bu sistemani yechamiz.

Sistemaning 1-taqqoslamasidan $x=2+5t_1,t_1\in Z$.
Buni 2-taqqoslamaga olib borib qo‘yib t_1 ni aniqlaymiz:$2+5t_1\equiv 0(mod\ 3)\to 5t_1\equiv -2(mod\ 3)\to 2t_1\equiv 1(mod\ 3)\to 2t_1\equiv$

$4(mod\ 3) \to t_1 \equiv 2(mod\ 3) \to t_1 = 2 + 3t_2, t_2 \in Z.$ t_1 ning topilgan qiymatini$x$ ning ifodasiga olib borib qo'ysak $x = 2 + 5(2 + 3t_2) = 12 + 15t_2, t_2 \in Z$ hosil bo'ladi. x ning bu qiymatini 3-taqqoslamaga qo'yib $t_2$ni aniqlaymiz: $12 + 15t_2 \equiv 0(mod 7) \to 15t_2 \to -12(mod\ 7) \to t_2 \equiv 2(mod\ 7) \to t_2 = 2 + 7t_3, t_3 \in Z.$ Buni x ning ifodasidagi t_2 ning o'rniga olib borib qo'ysak $x = 12 + 15(2 + 7t_3) = 42 + 105t_3$ ni hosil qilamiz. Bundan $x = 42 + 105t_3, t_3 \in Z$. Demak, abtssisalari o'qining $x = 42 + 105t_3, t_3 \in Z$ nuqtadan OX o'qiga chiqarilgan perpendikular berilgan chiziqlarni butun koordinatali nuqtalarda kesadi.

4). $\begin{cases} x \equiv 2(mod\ 7) \\ x \equiv 3(mod\ 5) \\ 2x \equiv 6(mod\ 7) \end{cases} \to \begin{cases} x \equiv 2(mod\ 7) \\ x \equiv 3(mod\ 5) \\ x \equiv 3(mod\ 7) \end{cases}$. Endi bu sistemani yechamiz. Sistemaning 1-taqqoslamasidan $x = 2 + 7t_1, t_1 \in Z.$ Buni 2-taqqoslamaga olib borib qo'yib t_1 ni aniqlaymiz: $2 + 7t_1 \equiv 3(mod\ 5) \to 2t_1 \equiv 1(mod\ 5) \to t_1 \equiv 3(mod\ 5)$, ya'ni $t_1 = 3 + 5t_2, t_2 \in Z.$ t_1 ning topilgan qiymatini x ning ifodasiga olib borib qoysak $x = 2 + 7(3 + 5t_2) = 23 + 35t_2, t_2 \in Z$ hosil bo'ladi. x ning bu qiymatini 3-taqqoslamaga qo'yib $t_2$ni aniqlaymiz: $23 + 35t_2 \equiv 3(mod\ 7) \to 0 \cdot t_2 \equiv 1(mod\ 7)$. Bu taqqoslamani qanoatlantiruvchi t_2 qiymatlari mavjud emas va demak, masalaning shartini qanoatlantiruvchi nuqtalar ham mavjud emas.

Izoh: Bunday nuqtalarning mavjud emasligini $x \equiv 2(mod\ 7)$va $x \equiv 3(mod\ 7)$ taqqoslamaning bir vaqtda bajarilmasligi bilan ham asoslash mumkin.

5). $\begin{cases} 2x \equiv 1(mod\ 3) \\ x \equiv 3(mod\ 5) \\ x \equiv 2(mod\ 11) \end{cases} \to \begin{cases} x \equiv 2(\ mod\ 3) \\ x \equiv 3(mod\ 5) \\ x \equiv 2(mod\ 11) \end{cases}$. Endi bu sistemani yechamiz. Sistemaning 1-taqqoslamasidan $x = 2 + 3t_1, t_1 \in Z.$ Buni 2-taqqoslamaga olib borib qo'yib t_1 ni aniqlaymiz: $2 + 3t_1 \equiv 3(mod\ 5) \to 3t_1 \equiv 1(mod\ 5) \to t_1 \equiv 2(mod\ 5) \to t_1 = 2 + 5t_2, t_2 \in Z.$ t_1 ning topilgan qiymatini$x$ ning ifodasiga olib borib qo'ysak $x = 2 + 3(2 + 5t_2) = 8 + 15t_2, t_2 \in Z$ hosil bo'ladi. x ning bu qiymatini 3-taqqoslamaga qo'yib $t_2$ni aniqlaymiz: $8 + 15t_2 \equiv 2(mod\ 11) \to 4t_2 \equiv -6(mod\ 11) \to 2t_2 \equiv -3(mod\ 11) \to t_2 \equiv 4(mod\ 11) \to t_2 = 4 + 11t_3, t_3 \in Z$ ni hosil qilamiz. Bundan $x = 8 + 15(4 + 11t_3) = 68 +$

$165t_3, t_3 \in Z$. Demak, abtssisalari o'qining $x = 68 + 165t_3, t_3 \in Z$ nuqtasidan OX o'qiga chiqarilgan perpendikular berilgan chiziqlarni butun koordinatali nuqtalarda kesadi.

6). $\begin{cases} 11x \equiv 6(mod\ 5) \\ 10x \equiv 9(mod\ 11) \\ 12x \equiv -1(mod\ 13) \end{cases} \rightarrow \begin{cases} x \equiv 1(mod\ 5) \\ x \equiv 2(mod\ 11) \\ x \equiv 1(mod\ 13) \end{cases}$. Endi bu sistemani yechamiz. Sistemaning 1-taqqoslamasidan $x = 1 + 5t_1, t_1 \in Z$. Buni 2-taqqoslamaga olib borib qo'yib t_1 ni aniqlaymiz: $1 + 5t_1 \equiv 2(mod\ 11) \rightarrow 5t_1 \equiv$

$\equiv 1(mod\ 11) \rightarrow -6t_1 \equiv 12(mod\ 11) \rightarrow t_1 \equiv -2(mod\ 11) \rightarrow$ $t_1 = -2 + 11t_2, t_2 \in Z$. t_1 ning topilgan qiymatini x ning ifodasiga olib borib qo'ysak $x = 1 + 5(-2 + 11t_2) = -9 + 55t_2, t_2 \in Z$ hosil bo'ladi. $x$ning bu qiymatini 3-taqqoslamaga qo'yib $t_2$ni aniqlaymiz: $-9 + 55t_2 \equiv 1(mod\ 13) \rightarrow 3t_2 \equiv -3(mod\ 13\ ) \rightarrow t_2 \equiv -1(mod\ 13) \rightarrow t_2 = -1 + 13t_3, t_3 \in Z$. Buni x ning ifodasiga qo'yib $x = -9 + 55(-1 + 13t_3) = -64 + 715t_3, t_3 \in Z$ ga ega bo'lamiz. Demak, abssissalari o'qining $x = -64 + 715t_3, t_3 \in Z$ nuqtasidan OX o'qiga chiqarilgan perpendikular berilgan chiziqlarni butun koordinatali nuqtalarda kesadi.

7). $\begin{cases} 3x \equiv 5(mod\ 7) \\ 5x \equiv 4(mod\ 8) \\ 11x \equiv -2(mod\ 13) \end{cases} \rightarrow \begin{cases} x \equiv 4(mod\ 7) \\ x \equiv 4(mod\ 8) \\ x \equiv 1(mod\ 13) \end{cases}$. Endi bu sistemani yechamiz. Sistemaning 1-taqqoslamasidan $x = 4 + 7t_1, t_1 \in Z$. Buni 2-taqqoslamaga olib borib qo'yib t_1 ni aniqlaymiz: $4 + 7t_1 \equiv 4(mod\ 8) \rightarrow 7t_1 \equiv 0(mod\ 8) \rightarrow t_1 \equiv 0(mod\ 8) \rightarrow t_1 = 8t_2, t_2 \in Z$. t_1 ning topilgan qiymatini$x$ ning ifodasiga olib borib qoysak $x = 4 + 56t_2, t_2 \in Z$ hosil bo'ladi. x ning bu qiymatini 3-taqqoslamaga qo'yib $t_2$ni aniqlaymiz: $4 + 56t_2 \equiv 1(mod\ 13) \rightarrow 4t_2 \equiv -3(mod\ 13) \rightarrow t_2 = 9 + 13_3\ , t_3 \in Z$. Buni x ning ifodasiga qo'yib $x = 4 + 56(9 + 13t_3) = 508 + 728t_3, t_3 \in Z$ ga ega bo'lamiz. Demak, abtsitsalari o'qining$x = 508 + 728t_3, t_3 \in Z$ nuqtasidan OX o'qiga chiqarilgan perpendikulyar berilgan chiziqlarni butun koordinatali nuqtalarda kesadi.

8). $\begin{cases} 10x \equiv 1(mod\ 9) \\ x \equiv 3(mod\ 7) \\ x \equiv 2(mod\ 5) \end{cases} \rightarrow \begin{cases} x \equiv 1(mod\ 9) \\ x \equiv 3(mod\ 7) \\ x \equiv 2(mod\ 5) \end{cases}$. Endi bu sistemani yechamiz. Sistemaning 1-taqqoslamasidan $x = 1 + 9t_1, t_1 \in Z$. Buni 2-taqqoslamaga olib borib qo'yib t_1 ni aniqlaymiz: $1 + 9t_1 \equiv 3(mod\ 7) \to 9t_1 \equiv 2(mod\ 7) \to t_1 \equiv 1(mod\ 7) \to t_1 = 1 + 7t_2,\ t_2 \in Z$. t_1 ning topilgan qiymatini x ning ifodasiga olib borib qo'ysak $x = 1 + 9(1 + 7t_2) = 10 + 63t_2,\ t_2 \in Z$ hosil bo'ladi. $x$ning bu qiymatini 3-taqqoslamaga qo'yib $t_2$ni aniqlaymiz: $10 + 63t_2 \equiv 2(mod\ 5) \to 3t_2 \equiv -3(mod\ 5) \to t_2 \equiv -1(mod\ 5) \to t_2 = -1 + 5t_3, t_3 \in Z$. Buni x ning ifodasiga qo'yib $x = 10 + 63(-1 + 5t_3) = -53 + 315t_3, t_3 \in Z$ga ega bo'lamiz. Demak, abssissalari o'qining $x = -53 + 315t_3, t_3 \in Z$ nuqtasidan OX o'qiga chiqarilgan perpendikular berilgan chiziqlarni butun koordinatali nuqtalarda kesadi.

9). $\begin{cases} 11x \equiv 5(mod\ 17) \\ 19 \equiv 1(mod\ 37) \\ 11x \equiv 4(mod\ 7) \end{cases} \to \begin{cases} x \equiv 2(mod\ 17) \\ x \equiv 2(mod\ 37) \\ x \equiv 1(mod\ 7) \end{cases}$. Endi bu sistemani

yechamiz. Sistemaning 1-taqqoslamasidan $x = 2 + 17t_1, t_1 \in Z$. Buni 2-taqqoslamaga olib borib qo'yib t_1 ni aniqlaymiz: $2 + 17t_1 \equiv 2(mod 37) \to 17t_1 \equiv 0(mod\ 37) \to t_1 \equiv 0(mod\ 37) \to t_1 = 37t_2,\ t_2 \in Z$. t_1 *ning topilgan qiymatini* x *ning ifodasiga* olib borib qo'ysak $x = 2 + 629t_2,\ t_2 \in Z$ hosil bo'ladi. x ning bu qiymatini 3-taqqoslamaga qo'yib $t_2$ni aniqlaymiz: $2 + 629t_2 \equiv 1(mod\ 7) \to (629 - 7 \cdot 90)t_2 \equiv -1(mod\ 7) \to -t_2 \equiv -1(mod\ 7) \to t_2 \equiv 1(mod\ 7) \to t_2 = 1 + 7t_3, t_3 \in Z$. Buni x ning ifodasiga qo'yib $x = 2 + 629 \cdot (1 + 7t_3) = 631 + 4403t_3, t_3 \in Z$ ga ega bo'lamiz. Demak, abssissalari o'qining $x = 631 + 4403t_3, t_3 \in Z$ nuqtasidan OX o'qiga chiqarilgan perpendikular berilgan chiziqlarni butun koordinatali nuqtalarda kesadi.

10). $\begin{cases} x \equiv 2(mod\ 10) \\ 5x \equiv 2(mod\ 13) \\ 10x \equiv -3(mod\ 13) \end{cases} \to \begin{cases} x \equiv 2(mod\ 19) \\ x \equiv 3(mod\ 13) \\ x \equiv 1(mod\ 13) \end{cases}$. Bu sistema

ziddiyatli sistema. Shuning uchun ham bu holda masala shartini qanoatlantiruvchi nuqtalar yo'q (4-masaladan keying izohni qarang).

11). Buning uchun $\begin{cases} x - 7y = 5 \\ 3x + 8y = 7 \\ x = 11 + 3y \end{cases}$ sistemaning yechimini topish kifoya.

Bu sistema ushbu $\begin{cases} x \equiv 5(\mathrm{mod}\,7) \\ 3x \equiv 7(\mathrm{mod}\,8) \\ x \equiv 11(\mathrm{mod}\,3) \end{cases} \rightarrow \begin{cases} x \equiv 5(\mathrm{mod}\,7) \\ x \equiv 5(\mathrm{mod}\,8) \\ x \equiv 11(\mathrm{mod}\,3) \end{cases}$ taqqoslamalar sistemasiga teng kuchli. Endi shu taqqoslamalar sistemasini yechamiz. Sistemaning 1-taqqoslamasidan $x = 5 + 7t_1, t_1 \in Z$. Buni 2 taqqoslamaga olib borib qo'yib t_1 ni aniqlaymiz: $5 + 7t_1 \equiv 5(mod 8) \rightarrow 7t_1 \equiv 0(mod\ 8) \rightarrow t_1 \equiv 0(mod\ 8) \rightarrow t_1 = 8t_2,\ t_2 \in Z.$ t_1 ning topilgan qiymatini x ning ifodasiga olib borib qo'ysak $x = 5 + 56t_2,\ t_2 \in Z$ hosil bo'ladi. x ning bu qiymatini 3-taqqoslamaga qo'yib $t_2$ni aniqlaymiz: $5 + 56t_2 \equiv 11(mod 3) \rightarrow 2t_2 \equiv 0(mod\ 3) \rightarrow t_2 = 3t_3, t_3 \in Z.$ Buni x ning ifodasiga qo'yib $x = 5 + 168t_3, t_3 \in Z$ ga ega bo'lamiz. Demak, abtsitsalari o'qining $x = 5 + 168t_3, t_3 \in Z$ nuqtasidan OX o'qiga chiqarilgan perpendikular berilgan chiziqlarni butun koordinatali nuqtalarda kesadi.

272. a).$56 = 8 \cdot 7$ va $(8,7) = 1$ bo'lgani uchun masala shartiga ko'ra $4x87y6 \equiv 0(mod\ 8), 4x87y6 \equiv 0(mod\ 7)$ taqqoslamalar o'rinli bo'lishi kerak. 8 ga bo'linish belgisiga asosan birinchi taqqoslamadan $7y6 \equiv (mod\ 8) \rightarrow 7 \cdot 10^2 + 10y + 6 \equiv 0(mod\ 8) \rightarrow (-1) \cdot 2^2 + 2y - 2 \equiv 0(mod\ 8) \rightarrow 2y \equiv 6(mod\ 8) \rightarrow\ y \equiv\ 3(mod\ 4) \rightarrow y = 3 + 4k, k \in Z$.Bu yerda y raqam bo'lganligi uchun $y = 3$ va $y = 7$. y ning bu topilgan qiymatlarni yuqoridagi 2-taqqoslamaga qo'yib $4x8736 \equiv 0(mod\ 7)$ va $4x8776 \equiv 0(mod\ 7)$ larni hosil qilamiz. Bularning birinchisidan:$4 \cdot 10^5 + x \cdot 10^4 + 8 \cdot 10^3 + 7 \cdot 10^2 + 3 \cdot 10 + 6 \equiv 0(mod\ 7) \rightarrow 4 \cdot 3^5 + x \cdot 3^4 + 8 \cdot 3^3 + 7 \cdot 3^2 + 3 \cdot 3 - 1 \equiv 0(mod\ 7) \rightarrow 4 \cdot (-1) \cdot 3^2 + x \cdot 3(-1) + 8(-1) + 1 \equiv 0(mod\ ) \rightarrow -1 - 3x - 1 + 1 \equiv 0(mod\ 7) \rightarrow 3x \equiv -1\ (mod\ 7) \rightarrow x \equiv 2(mod\ 7).$ Bundan $x_1 = 2, x_2 = 9$. Endi ikkinchi taqqoslamani yechamiz:

$4 \cdot 10^5 + x \cdot 10^4 + 8 \cdot 10^3 + 7 \cdot 10^2 + 7 \cdot 10 + 6 \equiv 0(mod\ 7) \rightarrow 4 \cdot 3^5 + x \cdot 3^4 + 1 \cdot 3^3 - 1 \equiv 0(mod\ 7) \rightarrow 4 \cdot (-1) \cdot 3^2 - 3x - 1 - 1 \equiv 0(mod\ 7) \rightarrow 3x \equiv -3(mod\ 7\ ) \rightarrow x \equiv 6(mod\ 7).$ Bundan $x_3 = 6$. Endi x ning topilgan qiymatlarini olib borib o'rqiga qo'ysak $428736, 498776, 468776$ sonlarini hosil bo'ladi.

c) Shartga ko'ra

$$\begin{cases} xyz138 \equiv 0(mod\ 7) \\ 138xyz \equiv 6(mod\ 13) \\ x1y3z8 \equiv (mod 11) \end{cases}$$ bajariladi. 1-taqqoslamadan $xyz \cdot 10^3 + 138 \equiv 0(mod\ 7) \to xyz \cdot 3^3 + 5 \equiv 0(mod\ 7) \to xyz \equiv 5(mod\ 7)$. (1)

2-taqqoslamadan $138 \cdot 10^3 + xyz \equiv 6(mod\ 13) \to 8 \cdot (-3)^3 + xyz \equiv 6(mod\ 13) \to xyz \equiv 1(mod\ 13)$. (2)

(1) va (2) taqqoslamalarni birgalikda yechib xyz ni aniqlaymiz. (1) dan

$xyz = 5 + 7t_1, \quad t_1 \in Z$. Buni (2)ga olib borib qo'yamiz. U holda $5 + 7t_1 \equiv 1(mod\ 13) \to 7t_1 \equiv -4(mod\ 13) \to -6t_1 \equiv -4(mod\ 13) \to 3t_1 \equiv 2(mod\ 13) \to t_1 \equiv 5(mod\ 13) \to t_1 = 5 + 13t_2, t_2 \in Z$. Demak, $xyz = 5 + 35 + 91t_2 = 40 + 91t_2, t_2 \in Z$. Bundan $t = 1,2,3, \ldots 10$ larda uch xonali sonlar

$$x = 131, 222, 313, \ \ldots, 950 \qquad (3)$$

sonlarini hosil qilamiz. Endi 3-taqqoslamaga qaraymiz.

$$x \cdot 10^5 + 10^4 + y \cdot 10^3 + 3 \cdot 10^2 + z \cdot 10 + 8 \equiv 5(mod\ 11) \to -x + 1 - y + 3 - z + 8 \equiv 5(mod\ 11) \to x + y + z \equiv 7(mod\ 11) \to x + y + z = 7 + 11t_1, t_1 \in Z, \qquad (4)$$

bu yerda x, y, z lar raqamlar bo'lganligi uchun. $0 < x + y + z < 27$ bo'lganligi uchun (4) dan $t = 0$ va $t = 1$ da $x + y + z = 7$ va $x + y + z = 18$ larni hosil qilamiz. Endi (3) sonlar ketma-ketligidan shu shartlarni qanoatlantiruvchilarini ajratib olamiz. Ular 313,495. Demak, izlanayotgan sonlar 313138, 495138.

c).$792 = 8 \cdot 9 \cdot 11$ va shart bo'yicha $13xy45z \equiv 0(mod\ 792)$. Bu oxirgi taqqoslama

$$\begin{cases} 13xy45z \equiv 0(mod\ 8) \\ 13xy45zz \equiv 0(mod\ 9) \\ 13xy45z \equiv 0(mod\ 11) \end{cases}$$ taqqoslamalar sistemasiga teng kuchli.

1-taqqoslamadan 8 ga bo'linish belgisiga asosan $45z \equiv 0(mod\ 8) \to 450 + z \equiv 0(mod\ 8) \to z \equiv 6(mod\ 8)$. Demak $z = 6$ va uni 2 va 3-taqqoslamalarga qo'ysak: $\begin{cases} 13xy456 \equiv 0(mod\ 9) \\ 13xy456 \equiv 0(mod\ 11) \end{cases}$ hosil bo'ladi. 9 ga bo'linish belgisiga asosan bu yerdagi 1-taqqoslamadan $19 + x + y \equiv$

$0(mod\ 9) \to x + y + 1 \equiv 0(mod\ 9)$. 2- taqqoslamadan $13 \cdot 10^5 + x \cdot 10^4 + y \cdot 10^3 + 4 \cdot 10^2 + 5 \cdot 10 + 6 \equiv 0(mod\ 11) \to 2 \cdot (-1) + x - y + 4 - 5 + 6 \equiv 0(mod\ 11) \to 0(mod\ 11) \to x - y + 3 \equiv 0(mod\ 11)$. Bulardan x va y lar raqam bo'lganligi uchun $\begin{cases} x + y = 8 \\ x - y = 8 \end{cases} \to$ $x = 8, y = 0$. Shunday qilib izlanayotgan son 1380456.

273. a).2-taqqoslamadan $x = 3 + 7t_1$, u holda buni 1-taqqoslamaga qo'ysak $3 + 3y \equiv 5(mod\ 7) \to 3y \equiv 9(mod\ 7) \to y \equiv 3(mod\ 7)$.
Javob: $x = 3 + 7t_1, y = 3 + 7t_1, t_1 \in Z$.

b). $\begin{cases} 9y \equiv 15(mod\ 12\ ) \\ 7x - 3y \equiv 1(mod\ 12) \end{cases}$.

1-taqqoslamadan $3y \equiv 5(mod\ 4) \to y \equiv 3(mod\ 4) \to y \equiv 3,7,11\ (mod\ 12)$. Bundan va berilgan sistemadan quyidagi 3 ta sistemani hosil qilamiz:

$\begin{cases} y \equiv 3(mod\ 12) \\ 7x \equiv 10(mod\ 12) \end{cases}; \begin{cases} y \equiv 7(mod\ 12) \\ 7x \equiv 10(mod\ 12) \end{cases}; \begin{cases} y \equiv 11(mod\ 12) \\ 7x \equiv -2(mod\ 12) \end{cases}$.

Bular mos ravishda quyidagi sistemalarga teng kuchli:

$\begin{cases} y \equiv 3(mod\ 12) \\ x \equiv -2(mod\ 12) \end{cases}; \begin{cases} y \equiv 7(mod\ 12) \\ x \equiv -2(mod\ 12) \end{cases}; \begin{cases} y \equiv 11(mod\ 12) \\ x \equiv -2(mod\ 12) \end{cases}$.

Shunday qilib yechimlar

: $\begin{cases} x \equiv 10 \\ y \equiv 3 \end{cases} (mod\ 12);$ $\begin{cases} x \equiv 10 \\ y \equiv 7 \end{cases} (mod\ 12);$ $\begin{cases} x \equiv 10 \\ y \equiv 11 \end{cases} (mod\ 12).$

c). $\begin{cases} x \equiv 2(mod\ 4) \\ -2y \equiv -1(mod\ 4) \end{cases} \to \begin{cases} x \equiv 2(mod\ 4) \\ 2y \equiv 3(mod\ 4) \end{cases};$

bu yerdagi ikkinchi taqqoslamada $(2:4) = 2$, lekim 3 soni 2 ga bo'linmaydi taqqoslama yechimga ega emas. Shuning uchun sistema ham yechimga ega emas.

d). $\begin{cases} 9y \equiv 15(mod\ 12) \\ 3x - 7y \equiv 1(mod\ 12) \end{cases} \to \begin{cases} 3y \equiv 5(mod\ 4) \\ 3x - 7y \equiv 1(mod\ 12) \end{cases} \to$

$\begin{cases} 3y \equiv 1(mod\ 4) \\ 3x - 7y \equiv 1(mod\ 12) \end{cases} \to \begin{cases} 3y \equiv 9(mod\ 4) \\ 3x - 7y \equiv 1(mod\ 12) \end{cases}$

$\to \begin{cases} y \equiv 3(mod\ 4) \\ 3x - 7y \equiv 1(mod\ 12) \end{cases}$

$\to \begin{cases} y \equiv 3,7,11(mod\ 12) \\ 3x - 7y \equiv 1(mod\ 12) \end{cases}$.

Bundan $\begin{cases} y \equiv 3(mod\ 12) \\ 3x - 21 \equiv 1(mod\ 12) \end{cases} \rightarrow \begin{cases} y \equiv 3(mod\ 12) \\ 3x \equiv 10(mod\ 12) \end{cases}$

Bu yerda 2-taqqoslama yechimga ega emas.

$\begin{cases} y \equiv 7 \\ 3x \equiv 50\ (mod\ 12) \end{cases}$. Bu yerda ham 2-taqqoslama yechimga ega emas.

$\begin{cases} y \equiv 11(mod\ 12) \\ 3x \equiv 78(mod\ 12) \end{cases} \rightarrow \begin{cases} y \equiv 11(mod\ 12) \\ 3x \equiv 6(mod\ 12) \end{cases} \rightarrow \begin{cases} y \equiv 11(mod\ 12) \\ x \equiv 2(mod\ 4) \end{cases} \rightarrow$

$\begin{cases} y \equiv 11(mod\ 12) \\ x \equiv 2,6,10(mod\ 12) \end{cases}$.

Demak, yechimlar $\begin{cases} x \equiv 2(mod\ 12) \\ y \equiv 11(mod\ 12) \end{cases}$; $\begin{cases} x \equiv 6(mod\ 12) \\ y \equiv 11(mod\ 12) \end{cases}$;

$\begin{cases} x \equiv 10(mod\ 12) \\ y \equiv 11(mod\ 12) \end{cases}$.

e). $\begin{cases} 3x - 5y \equiv 1(mod\ 12) \\ 9y \equiv 15(mod\ 12) \end{cases} \rightarrow \begin{cases} 3x - 5y \equiv 1(mod\ 12) \\ 3y \equiv 5(mod\ 4) \end{cases} \rightarrow$

$\begin{cases} 3x - 5y \equiv 1(mod\ 12) \\ y \equiv 3(mod\ 4) \end{cases} \rightarrow \begin{cases} 3x - 5y \equiv 1(mod\ 12) \\ y \equiv 3,7,11(mod\ 4) \end{cases}$.

Bundan

$\begin{cases} 3x \equiv 16(mod\ 12) \\ y \equiv 3(mod\ 4) \end{cases} \rightarrow$ Bu yerda $(3,12) = 3$, lekin 16 soni 3 ga bo'linmaydi, ya'ni sistema yechimga ega emas.

$\begin{cases} y \equiv 7(mod\ 12) \\ 3x \equiv 36(mod\ 12) \end{cases} \rightarrow \begin{cases} y \equiv 7(mod\ 12) \\ 3x \equiv 0(mod\ 12) \end{cases} \rightarrow \begin{cases} y \equiv 7(mod\ 12) \\ x \equiv 0\ (mod\ 4) \end{cases} \rightarrow$

$\begin{cases} y \equiv 7(mod\ 12) \\ x \equiv 0,4,8\ (mod\ 12) \end{cases}$.

Bundan yechimlar $\begin{cases} y \equiv 7(mod\ 12) \\ x \equiv 0(mod\ 12) \end{cases}$; $\begin{cases} y \equiv 7(mod\ 12) \\ x \equiv 4(mod\ 12) \end{cases}$;

$\begin{cases} y \equiv 7(mod\ 12) \\ x \equiv 8\ (mod\ 12) \end{cases}$ bo'larekan.

274. a). $\begin{cases} x + 2y \equiv 3(mod\ 5) \\ 4x + y \equiv 2(mod\ 5) \end{cases}$ dagi ikkinchi taqqoslamaning ikkala tomonini 2 ga (2,5)=1 ko'paytiramiz ularni hadlab ayiramiz. U holda $-7x \equiv -1(mod\ 5) \rightarrow 3x \equiv 4(mod\ 5) \rightarrow x \equiv 3(mod\ 5)$ni hosil qilamiz. Buni berilgan sistemaga qo'ysak $y \equiv 2 - 4x(mod\ 5) \rightarrow$ $y \equiv -10(mod\ 5) \rightarrow y \equiv 0(mod\ 5)$ kelib chiqadi. Demak, yechim

$$\begin{cases} x \equiv 3(mod\ 5) \\ y \equiv 0(mod\ 5) \end{cases}.$$

b). $\begin{cases} x + 2y \equiv 0(mod\ 5) \\ 3x + 2y \equiv 2(mod\ 5) \end{cases}$ sistemadagi taqqoslamalarni hadlab ayiramiz. U holda $-2x \equiv -2(mod\ 5) \rightarrow x \equiv 1(mod\ 5)$. Buni berilgan sistemaga qo'ysak $2y \equiv -x(mod\ 5) \rightarrow 2y \equiv -1(mod\ 5) \rightarrow 2y \equiv 4(mod\ 5) \rightarrow y \equiv 2(mod\ 5)$ kelib chiqadi. Demak, yechim

$$\begin{cases} x \equiv 1(mod\ 5) \\ y \equiv 2(mod\ 5) \end{cases}.$$

c). $\begin{cases} 3x + 4y \equiv 29(mod\ 143) \\ 2x - 9y \equiv -84(mod\ 143) \end{cases} \rightarrow \begin{cases} 6x + 8y \equiv 58(mod\ 143) \\ 6x - 27y \equiv -252(mod\ 143) \end{cases}$

$\rightarrow 35y \equiv 310(mod\ 143) \rightarrow 35y \equiv 24(mod\ 143)$. Bu yerda $(35; 143) = 1$ bo'lgani uchun taqqoslama yagona yechimga ega. Bu yechimni topish uchun $\frac{143}{5}$ ni uzluksiz kasrga qo'ysak $\frac{143}{35} = (4,11,12)$ hosil bo'ladi. Bundan munosib kasrlarning suratini aniqlasak

q_i		4	11	1	2
P_i	$P_0 = 1$	4	45	49	143

Bundan $P_{n-1} = 49,\ n = 4$ bo'ladi va $y \equiv (-1)^3 \cdot 49 \cdot 24(mod\ 143) \equiv -1176(mod\ 143) \equiv (-1176 + 1144)(mod\ 143) \equiv -32(mod\ 143) \equiv 111(mod\ 143)$. Demak, $3x + 4 \cdot 111 \equiv 29(mod\ 143) \rightarrow 3x \equiv -415(mod\ 143) \rightarrow 3x \equiv 14(mod\ 143) \rightarrow 3x \equiv (14 - 143)(mod\ 143) \rightarrow$

$3x \equiv -129(mod\ 143) \rightarrow x \equiv -43(mod\ 143) \rightarrow x \equiv 100(mod\ 143)$.

Javob: $\begin{cases} x \equiv 100 \\ y \equiv 111 \end{cases}(mod\ 143)$.

d). $\begin{cases} x + 2y = 4(mod\ 5) \\ 3x + y \equiv 2(mod\ 5) \end{cases}$. Bu yerdagi ikkinchi taqqoslamaning ikkala tomonini 2 ga $(2,5) = 1$ ko'paytiramiz ularni hadlab ayiramiz. U holda $-5x \equiv 0(mod\ 5) \rightarrow x \equiv 0(mod\ 5)$. Buni berilgan sistemaga qo'ysak $y \equiv 2(mod\ 5)$ hosil bo'ladi. Demak, sistemaning yechimi $\begin{cases} x \equiv 0(mod\ 5) \\ y \equiv 2(mod\ 5) \end{cases}$.

e). $\begin{cases} x + 5y \equiv 5(mod\ 6) \\ 5x + 3y \equiv 1(mod\ 6) \end{cases}$. Bu yerdagi birinchi taqqoslamaning ikkala tomonini 5 ga $(6,5) = 1$ ko'paytiramiz va ularni hadlab ayiramiz. U holda

$$22y \equiv 24(mod\ 6) \to 4y \equiv 0(mod\ 6) \to 2y \equiv 0(mod\ 3) \to y \equiv 0(mod\ 3) \to$$

$y \equiv 0{,}3(mod\ 6)$. y ning bu qiymatlarini berilgan sistemaga qo'yib x ni aniqlaymiz: $x \equiv 2, 5(mod\ 6)$. Demak, yechim $\begin{cases} x \equiv 5(mod\ 6) \\ y \equiv 0(mod\ 6) \end{cases}$; $\begin{cases} x \equiv 2(mod\ 6) \\ y \equiv 3(mod\ 6) \end{cases}$.

f). $\begin{cases} 5x - y \equiv 3(mod\ 6) \\ 2x + 2y \equiv -1(mod\ 6) \end{cases}$. Sistemaning birinchi taqqoslamasidan $5x - y \equiv 3(mod\ 6) \to y \equiv 5x - 3(mod\ 6)$. Buni ikkinchi taqqoslamaga qo'ysak $2x + 2y \equiv -1(mod\ 6) \to 2x + 2(5x - 3) \equiv -1(mod\ 6) \to 12x \equiv 5(mod 6)$ hosil bo'ladi. Bu yerda $(12,6) = 6$, lekin 5 soni 6ga bo'linmaydi. Shuning uchun ham bu taqqoslama va demak, berilgan sistema ham yechimga ega emas.

g). $\begin{cases} x - y \equiv 2(mod\ 6) \\ 4x + 2y \equiv 2(mod\ 6) \end{cases}$.Bu yerdagi birinchi taqqoslamadan $x - y \equiv 2(mod\ 6) \to x \equiv y + 2(mod\ 6)$. Buni ikkinchi taqqoslamaga qo'ysak

$4x + 2y \equiv 2(mod\ 6) \to 4(y + 2) + 2y \equiv 2(mod\ 6) \to 6y \equiv -6(mod 6)$ hosil bo'ladi. Bu taqqoslama ayniy taqqoslama bo'lgani uchun uni y ning ixtiyoriy qiymati qanoatlantiradi. Shuning uchun ham $x \equiv y + 2(mod\ 6)$, ya'ni sistemaning yechimlari to'plami $x - y \equiv 2(mod\ 6)$ taqqoslamaning yechimlari bilan bir xil.

h). $\begin{cases} 4x - y \equiv 2(mod\ 6) \\ 2x + 2y \equiv 0(mod\ 6) \end{cases}$. Bu yerdagi birinchi taqqoslamadan

$4x - y \equiv 2(mod\ 6) \to y \equiv 4x - 2(mod\ 6)$. Buni ikkinchi taqqoslamaga qo'ysak

$$2x + 2y \equiv 0(mod\ 6) \to x + y \equiv 0(mod\ 3) \to x + 4x - 2 \equiv 0(mod\ 3) \to$$

$5x \equiv 2(mod\ 3) \to x \equiv 1(mod\ 3) \to x \equiv 1{,}4(mod\ 6)$. Demak,

sistemaning yechimlari: $\begin{cases} x \equiv 1(mod\ 6) \\ y \equiv 2(mod\ 6) \end{cases}$' $\begin{cases} x \equiv 4(mod\ 6) \\ y \equiv 2(mod\ 6) \end{cases}$.

275. *a)*. Ma'lumki, (1) dan

$Dx \equiv D_1(mod\ m)$va $Dy \equiv D_2(mod\ m)$. (*)

Agar $(m, D) = 1$ bu ikkala taqqoslama ham yagona yechimga ega.

$$x \equiv D^{\varphi(m)-1}.D_1(modm)\text{va}x \equiv D^{\varphi(m)-1}D_2(modm).$$

b). (*) dan$(D; m) = d > 1$ bo'lib D_1 va D_2 larning ikkalasi ham $d$ga bo'linsa, ularning har biri d ta yechimga ega bo'ladi. Agar D_1va D_2larning birortasi d ga bo'linmasa sistema yechimga ega emas.Shunday qilib, berilgan sistemaning yechimga bo'lmasligi sharti (*) dagi D_1 yoki D_2larning birortasining $(D; m) = d$ ga bo'linmasligidir.

c). $D \equiv D_1 \equiv D_2 \equiv 0(mod\ m)$ bajarilsa, (1) dagi 2-taqqoslama birinchisining natijasi bo'ladi. Haqiqatan ham 1-taqqoslamaning ikkala tomonini a_2 ko'paytirsak

$$a_1a_2x + b_1a_2y \equiv c_1a_2(mod\ m) \qquad (2)$$

hosil bo'ladi. $D \equiv 0$ va$D_2 \equiv 0(mod\ m)$ lardan

$$\left.\begin{matrix} a_2b_1 \equiv a_1b_2 \\ a_2b_1 \equiv a_1c_2 \end{matrix}\right\}(mod\ m).$$

U holda (2) dan $a_1a_2x + a_1b_2y \equiv a_1c_2(mod\ m)$. Bu yerda $(a_1, m) = 1$ bo'lganligi uchun oxirgi taqqoslamaning ikkala tomonini a_1 ga qisqartirib $a_2x + by \equiv c_2(mod\ m)$ni, ya'ni (1) dagi 2-taqqoslamani hosil qilamiz.

IV. 4-§.

276. *a)*. Avvalo berilgan taqqoslamaning koeffitsiyentlaridan modulga karrali sonlarni chiqarib soddalashtiramiz. U holda quyidagiga ega bo'lamiz: $x^{10} - 2x + 1 \equiv 0(mod\ 5)$.Bu taqqoslamada Ferma teoremasiga ko'ra, $x^5 \equiv x(mod\ 5)$ ekanligidan foydalanib darajasini pasaytiramiz: $(x^5)^2 - 2x + 1 \equiv 0(mod\ 5) \to x^2 - 2x + 1 \equiv 0(mod\ 5)$. Demak, berilgan taqqoslama oxirgi taqqoslamaga teng kuchli ekanligidan oxirgi taqqoslamani yechamiz: $(x-1)^2 \equiv 0(mod\ 5) \to x - 1 \equiv 0(mod\ 5) \to x \equiv 1(mod\ 5)$. Shunday qilib berilgan taqqoslamaning yechimi $x \equiv 1(mod\ 5)$ dan iborat.

Izoh:$x^2 - 2x + 1 \equiv 0(mod\ 5)$ taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi chegirmalarni qo'yib sinab ko'rish yo'li bilan ham yechish mumkin .

Tekshirish: $6^2 - 2 \cdot 6 + 1 \equiv 25(mod\ 5) \equiv 0(mod\ 5)$. Demak, topilgan yechim berilgan taqqoslamani qanoatlantiradi .
Javob:$x \equiv 1(mod\ 5)$.

*b).*Bu taqqoslamada Ferma teoremasiga ko'ra, $x^5 \equiv x(mod\ 5)$ ekanligidan foydalanib darajasini pasaytiramiz: U holda quyidagiga ega bo'lamiz: $x^5 - 2x^3 + x^2 - 2 \equiv 0(mod\ 3) \rightarrow x^3 \cdot x^2 + x^3 + x^2 + 1 \equiv 0(mod\ 3) \rightarrow x^3 + x + x^2 + 1 \equiv x + x + x^2 + 1 \equiv x^2 + 2x + 1 \equiv 0(mod\ 3) \rightarrow (x+1)^2 \equiv 0(mod\ 3) \rightarrow x + 1 \equiv 0(mod\ 3)$. Bundan $x \equiv -1(mod\ 3)$.

Tekshirish: $(-1)^5 - 2(-1)^3 + (-1)^2 - 2 \equiv 0(mod\ 3)$. Demak, topilgan yechim berilgan taqqoslamani qanoatlantiradi.
Javob: $x \equiv -1(mod\ 3)$.

d)Avvalo berilgan taqqoslamani $x^3 \cdot x^2 - x^3 \cdot x - x + 1 \equiv x^3 - x^2 - x + 1 \equiv 0(mod\ 3)$ ko'rinishda yozib olamiz va bunda $x^2(x-1) - (x-1) = (x-1)(x^2-1) = (x-1)^2(x+1)$ bo'lgani uchun berilgan taqqoslama

$(x-1)^2(x+1) \equiv 0(mod\ 3)$ ga teng kuchli. Bundan $x_1 \equiv 1(mod\ 3)$ va $x_2 \equiv -1(mod\ 3)$ lar berilgan taqqoslamaning yechimlari ekanligi kelib chiqadi.

Tekshirish:1)$1^5 - 1^4 - 1 + 1 \equiv 0(mod\ 3)$;

2)$(-1)^5 - (-1)^4 + 1 - 1 \equiv 0(mod\ 3)$. Demak, topilgan yechim berilgan taqqoslamani qanoatlantiradi.
Javob:$x_1 \equiv 1(mod\ 3)$va$x_2 \equiv -1(mod\ 3)$.

d). Avvalo berilgan taqqoslamani $x^5 \cdot x^2 - x^5 \cdot x + 2 \equiv 0(mod\ 5) \rightarrow x^3 - x^2 + 2 \equiv 0(mod\ 5)$ ko'rinishda yozib olamiz. Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm1, \pm2$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x_1 \equiv -1(mod\ 5)$va$x_2 \equiv -2(mod\ 5)$ lar berilgan taqqoslamani qanoatlantirishini topamiz.

Tekshirish:

1)$(-1)^7 - (-1)^6 + 5 \cdot (-1)^2 - 3 = -1 - 1 + 5 - 3 = 0 \equiv 0(mod\ 5)$;

2) $(-2)^7 - (-2)^6 + 5 \cdot (-2)^2 - 3 = 64(-3) + 20 - 3 = 65(-3) + 20 \equiv 0(mod\ 5)$. Demak, topilgan yechim berilgan taqqoslamani qanoatlantiradi.

Javob: $x_1 \equiv -1(mod\ 5)$va$x_2 \equiv -2(mod\ 5)$.

e). Avvalo berilgan taqqoslamani $x^5+x^4+x^3-x^2-2\equiv 0(mod\,5)\to x+x^4+x^3-x^2-2\equiv 0(mod\,5)\to x^4+x^3-x^2+x-2\equiv 0(mod\,5)$ ko'rinishda yozib olamiz. Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm1,\pm2$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x_1\equiv 2(mod\,5)$va$x_2\equiv -2(mod\,5)$ lar berilgan taqqoslamani qanoatlantirishini topamiz.

Tekshirish:

1)$2^5+2^4+2^3-2^2-2=32+16+8-4-2=50\equiv 0(mod\,5)$;

2) $(-2)^5+(-2)^4+(-2)^3-(-2)^2-2=-32+16-8-4-2=-30\equiv 0(mod\,5)$. Demak, topilgan yechim berilgan taqqoslamani qanoatlantiradi.

Javob: $x_1\equiv 2(mod\,5)$va$x_2\equiv -2(mod\,5)$.

f).Berilgan taqqoslamani $x^7-6\equiv 0(mod\,5)\to x^5\cdot x^2-6\equiv x^3-6\equiv 0(mod\,5)$ ko'rinishda yozib olamiz.

Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm1,\pm2$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x\equiv 1(mod\,5)$ berilgan taqqoslamani qanoatlantirishini topamiz.

Tekshirish:$1-6=-5\equiv 0(mod\,5)$. Demak, topilgan yechim berilgan taqqoslamani qanoatlantiradi. **Javob**: $x\equiv 1(mod\,5)$.

g).Berilgan taqqoslamani $x^8+2x^7+x^5-x+3\equiv 0(mod\,5)\to x^5\cdot x^3+2x^5\cdot x^2+x^5-x+3\equiv 0(mod\,5)\to x^4+2x^3+x-x+3\equiv 0(mod\,5)\to\ 2x^3+4\equiv 0(mod\,5)\to x^3+2\equiv 0(mod\,5)$ ko'rinishda yozib olamiz.Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm1,\pm2$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x\equiv 2(mod\,5)$ berilgan taqqoslamani qanoatlantirishini topamiz.

Tekshirish: $2^8+2^8+2^5-2+3=512+32+1=545\equiv 0(mod\,5)$.

Demak , topilgan yechim berilgan taqqoslamani qanoatlantiradi.

Javob: $x\equiv 2(mod\,5)$.

h).Berilgan taqqoslamani $6x^4+17x^2-16\equiv 0(mod\,3)\to 2x^2-1\equiv 0(mod\,3)$ ko'rinishda yozib olamiz. Bu taqqoslamani 3 moduli bo'yicha chegirmalarning to'la sistemasidagi 0, ±1 ni qo'yib tanlash usuli bilan yechamiz. Bu sonlarning birortasi ham berilgan taqqoslamani qanoatlantirmaydi. Shuning uchun ham berilgan taqqoslama yechimga ega emas. **Javob:** taqqoslama yechimga ega emas.

i). Berilgan taqqoslamani $4x^7 - 2x^3 + 8 \equiv 0(mod\ 5) \to -x^5 \cdot x^2 - 2x^3 - 2 \equiv (mod\ 5) \to -x^3 - 2x^3 - 2 \equiv 0(mod\ 5) \to 3x^3 + 2 \equiv 0(mod\ 5) \to x^3 - 1 \equiv 0(mod\ 5)$ ko'rinishda yozib olamiz. Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm 1, \pm 2$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x \equiv 1(mod\ 5)$ berilgan taqqoslamani qanoatlantirishini topamiz.

Tekshirish : $4 \cdot 1^7 - 2 \cdot 1^3 + 8 = 10 \equiv 0(mod\ 5) \equiv 0(mod\ 5)$.
Demak , topilgan yechim berilgan taqqoslamani qanoatlantiradi.
Javob: $x \equiv 1(mod\ 5)$.

j). Berilgan taqqoslamani $3x^7 - 2x^6 + 2x^2 + 13 \equiv 0(mod\ 5) \to -2x^5 \cdot x^2 - 2x^5 \cdot x + 2x^2 - 2 \equiv (mod\ 5) \to -2x^3 - 2x^2 + 2x^2 - 2 \equiv 0(mod\ 5) \to x^3 + 1 \equiv 0(mod\ 5)$ ko'rinishda yozib olamiz. Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm 1, \pm 2$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x \equiv -1(mod\ 5)$ berilgan taqqoslamani qanoatlantirishini topamiz.

Tekshirish: $3 \cdot (-1)^7 - 2 \cdot (-1)^6 + 2 \cdot (-1)^2 + 13 = -3 - 2 + 2 + 13 = 10 \equiv 0(mod\ 5) \equiv 0(mod\ 5)$. Demak , topilgan yechim berilgan taqqoslamani qanoatlantiradi. **Javob:** $x \equiv -1(mod\ 5)$.

277. *a).* Berilgan taqqoslamani $f(x) = x^3 + 4x^2 - 3 \equiv 0(mod\ 5) \to x^3 - x^2 + 2 \equiv 0(mod\ 5)$ ko'rinishda yozib olamiz. Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm 1, \pm 2$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x_1 \equiv -1(mod\ 5) va x_2 \equiv -2(mod\ 5)$ lar berilgan taqqoslamaning yechimlari bo'lgani uchun $x^3 + 4x^2 - 3 \equiv (x+2)(x+1)h(x) \equiv 0(mod\ 5)$ bo'lishi kerak. $h(x)$ ni aniqlash uchun $x^3 + 4x^2 - 3$ ni $(x+2)(x+1) = x^2 + 3x + 2$ ga bo'lamiz va biz quyidagiga ega bo'lamiz:$f(x) = (x+2)(x+1)(x+1) + (-5x+5)$, ya'ni
$f(x) \equiv (x+2)(x+1)^2(mod\ 5)$.
Javob: $f(x) \equiv (x+2)(x+1)^2(mod\ 5)$.

b). Berilgan taqqoslamani soddalashtirib $f(x) = x^4 + x^3 - x^2 + x - 2 \equiv 0(mod\ 5) \to x^3 - x^2 + x - 1 \equiv 0(mod\ 5)$ ko'rinishda yozib olamiz. $f(x) = x^3 - x^2 + x - 1 \equiv 0(mod\ 5)$ ni quyidagicha yozish mumkin: $f(x) = x^2(x-1) + (x-1) = (x-1)(x^2+1) \equiv 0(mod\ 5)$. Endi $x^2 + 1 \equiv 0(mod\ 5)$ taqqoslamaning yechimini izlaymiz. Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm 1, \pm 2$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x_1 \equiv 2(mod\ 5)$va$x_2 \equiv -2(mod\ 5)$lar berilgan taqqoslamaning yechimlari. Shuning uchun ham $f(x) \equiv (x+2)(x-1)(x-2)(mod\ 5)$. **Javob:** $f(x) \equiv (x+2)(x-1)(x-2)(mod\ 5)$.

c). Berilgan $x^4 + x + 4 \equiv (mod\ 11)$ taqqoslamani 11 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar

$$0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5 \qquad (1)$$

larni qo'yib tanlash usuli bilan yechamiz. U holda $x \equiv 2(mod\ 11)$ berilgan taqqoslamaning bitta yechimi ekanligini topamiz. U holda $x^4 + x + 4 = (x-2)(x^3 + 2x^2 + 4x - 2) + 22(mod\ 11) \equiv (x-2)(x^3 + 2x^2 + 4x - 2)(mod\ 11)$ ni hosil qilamiz. Endi $x^3 + 2x^2 + 4x - 2 \equiv 0(mod\ 11)$ taqqoslamanig yechimini izlaymiz. (1) dagi chegirmalarni tekshirib ko'ramiz. U holda $x_2 \equiv 2(mod\ 11)$, $x_3 \equiv 3(mod\ 11), x_4 \equiv 4(mod\ 11)$ lar uning yechimi ekanligini topamiz. Demak, $x^4 + x + 4 \equiv (x-2)^2(x-3)(x-4)(mod\ 11)$ bo'lar ekan.

Javob: $f(x) \equiv (x-2)^2(x-3)(x-4)(mod\ 11)$.

d). Berilgan $x^2 + 2x + 2 \equiv 0(mod 5)$ taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0 , $\pm 1, \pm 2$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x_1 \equiv 1(mod\ 5)$, $x_2 \equiv 2(mod\ 5)$ lar taqqoslamaning yechimlari bo'ladi. Shuning uchun ham $x^2 + 2x + 2 = (x-1)(x-2)(mod 5)$.

Javob: $f(x) \equiv (x-1)(x-2)(mod 5)$.

e). Berilgan $3x^3 - 1 \equiv 0(mod\ 5)$ taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0 , $\pm 1, \pm 2$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x_1 \equiv -2(mod\ 5)$ taqqoslamani qanoatlantirishini topamiz. Shuning uchun ham $3x^3 - 1 \equiv (x+2)(3x^2 - 6x + 2)(mod\ 5)$. Endi $3x^2 - x + 2 \equiv 0(mod 5)$ taqqoslamaning yechiminizlaymiz. Bu taqqoslama yechimga emas. Shuning uchun ham $3x^2 - 1 \equiv (x+2)(3x^2 - x + 2)(mod\ 5)$.

Javob:$f(x) \equiv (x+2)(3x^2 - x + 2)(mod 5)$.

f).$f(x) = 2x^4 + x^3 - 3x^2 + 2x - 2 \equiv 0(mod 11)$ ni qaraymiz. Bu taqqoslamani 11 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x_1 \equiv 1(mod\ 11)$ ning berilgan taqqoslamani qanoatlantirishini ko'ramiz. $f(x)$ ni $(x-1)$ ga bo'lamiz. U holda $f(x) = (x-1)(2x^3 +$

$3x^2+2)\equiv 0(mod11)$ ga ega bo'lamiz. Endi $2x^3+3x^2+2\equiv 0(mod11)$ taqqoslamaning yechimini izlaymiz. Bu taqqoslamani tanlash usuli bilan yechib uning yechimi yo'q ekanligiga ishonch hosil qilamiz. Shunday qilib, $f(x)=(x-1)(2x^3+3x^2+2)\equiv 0(mod11)$. **Javob:** $f(x)=(x-1)(2x^3+3x^2+2)\equiv 0(mod11)$.

g). $f(x)=x^4-7x^3+13x^2+21x+23\equiv 0(mod7)$ ni qaraymiz. Buni soddalashtirib $f(x)=x^4-x^2+2\equiv 0(mod7)$ ni hosil qilamiz. Bu taqqoslamani 7 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm1, \pm2, \pm3$ larni qo'yib tanlash usuli bilan yechamiz. U holda $x_1\equiv 2(mod\ 7)$, $x_2\equiv -2(mod\ 7)$ larning berilgan taqqoslamani qanoatlantirishini ko'ramiz. Bundan foydalanib $f(x)$ ni $(x+2)(x-2)=x^2-4$ ga bolib, $f(x)=x^4-x^2+2\equiv(x+2)(x-2)(x^2+3)+14\equiv(x+2)(x-2)(x^2+3)(mod7)$ ni hosil qilamiz. Endi $x^2+3\equiv 0(mod7)$ ning yechimini izlaymiz. Bu yerda $x^2\equiv -3(mod7)\rightarrow x^2\equiv 4(mod7)$ bo'lgani uchun $x_3\equiv 2(mod\ 7)$, $x_4\equiv -2(mod\ 7)$ lar oxirgi taqqoslamaning yechimi bo'ladi. Demak, $f(x)\equiv(x+2)^2(x-2)^2(mod7)$.

Javob: $f(x)\equiv(x+2)^2(x-2)^2(mod7)$.

h). $f(x)=2x^4+x^3-3x^2+2x-2\equiv 0(mod5)$ ni qaraymiz. Bu taqqoslamani 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm1, \pm2$ larni qo'yib, tanlash usuli bilan yechamiz. U holda $x_1\equiv 1(mod\ 5)$, $x_2\equiv 2(mod\ 5)$ larning berilgan taqqoslamani qanoatlantirishini ko'ramiz. Bundan foydalanib $f(x)$ ni $(x-1)(x-2)=x^2-3x+2$ ga bo'lib $f(x)=(x-1)(x-2)(2x^2+7x+14)+30(x-1)(mod5)\equiv(x-1)(x-2)(2x^2+2x-1)(mod5)$ ni hosil qilamiz. Endi $2x^2+2x-1\equiv 0(mod5)$ ning yechimlarini izlaymiz. Bu taqqoslama yechimga ega emas. Shuning uchun ham $f(x)\equiv(x-1)(x-2)(2x^2+2x-1)(mod5)$ deb yoza olamiz.

Javob: $f(x)\equiv(x-1)(x-2)(2x^2+2x-1)(mod5)$.

i). $f(x)=2x^3+5x^2-2x-3\equiv 0(mod7)$ ni qaraymiz. Buni soddalashtirib $f(x)=2x^3-2x^2-2x-3\equiv 0(mod7)$ ni hosil qilamiz. Bu taqqoslamani 7 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm1, \pm2, \pm3$ larni qo'yib tanlash usuli bilan yechamiz. U holda bu sonlarning birortasi ham berilgan taqqoslamani qanoatlantirmasligini ko'ramiz, ya'ni taqqoslama yechimga ega emas. Shuning uchun ham $f(x)$ ko'paytuvchilarga ajralmaydi. **Javob:** $f(x)$ ko'paytuvchilarga ajralmaydi.

j).$f(x) = x^4 - 2x^2 + x + 4 \equiv 0(mod7)$ ni qaraymiz. Bu taqqoslamani 7 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar 0, $\pm1, \pm2, \pm3$ larni qo'yib tanlash usuli bilan yechamiz. U holda $x \equiv 2(mod\ 7)$ ning berilgan taqqoslamani qanoatlantirishini ko'ramiz. Bundan foydalanib $f(x)$ ni$x - 2$ ga bo'lib $f(x) = (x-2)(x^3 + 2x^2 + 2x + 5) + 14 \equiv (x-2)(x^3 + 2x^2 + 2x - 2)(mod7)$ ni hosil qilamiz. Endi $x^3 + 2x^2 + 2x - 2 \equiv 0(mod7)$ ning yechimlarini izlaymiz. $x \equiv 3(mod\ 7)$ uning yechimi bo'lgani uchun oxirgu taqqoslama $x - 3$ ga bo'linadi, ya'ni $x^3 + 2x^2 + 2x - 2 = (x-3)(x^2 + 5x + 17) + 49\ (mod7) \equiv (x-3)(x^2 - 2x + 3)(mod7)$. $x^2 - 2x + 3 \equiv 0(mod7)$ ni qaraymiz. Bu taqqoslama yechimga ega emas, ya'ni ko'paytuvchilarga ajralmaydi. Shunday qilib, $f(x) = (x-2)(x-3)(x^2 - 2x + 3)(mod7)$.

Javob: $f(x) = (x-2)(x-3)(x^2 - 2x + 3)(mod7)$.

277. *a).* Avvalo, berilgan taqqoslamani soddalashtiramiz. U holda quyidagiga ega bo'lamiz: $f(x) \equiv 8x^{13} \cdot x^7 - (13+2)x^{13} \cdot x^6 + 7x^{13} \cdot x^5 + (13 \cdot 2 + 2)\,x^{13} \cdot x^4 - 4x^{13} \cdot x^3 + (2 \cdot 13 + 4)x^{13} \cdot x^2 + 10x^6 - 4x^3 + (13+10)x^2 - (13+8)x - 11 \equiv 8x^8 - 2x^7 + 7x^6 + 2x^5 - 4x^4 + 4x^3 - 3x^6 - 4x^3 - 3x^2 + 5x + 2(mod13) \equiv -5x^8 - 2x^7 + 4x^6 + 2x^5 - 4x^4 - 3x^2 + 5x + 2(mod13)$.

Demak, biz$-5x^8 - 2x^7 + 4x^6 + 2x^5 - 4x^4 - 3x^2 + 5x + 2 \equiv 0(mod13)$ taqqoslamani yechishimiz kerak. $m = 13$ moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm1, \pm2, \pm3, \pm4, \pm5, \pm6$ larni sinab ko'rsak ularning birortasi ham berilgan taqqoslamani qonoatlantirmaydi. Demak, taqqoslama yechimga ega emas.

Javob: taqqoslama yechimga ega emas.

b).Avvalo, berilgan taqqoslamani soddalashtiramiz. U holda quyidagiga ega bo'lamiz: $f(x) \equiv x^7 \cdot x^3 + x^7 \cdot x + x^7 - x^4 - x^2 + 4x - 3 \equiv x^4 + x^2 + x - x^4 - x^2 + 4x - 3 \equiv 5x - 3(mod7)$. Demak, biz $5x - 3 \equiv 0(mod7)$ taqqoslamaning yechimlarini izlashimiz kerak: $5x \equiv 3(mod7) \rightarrow$

$5x \equiv 10(mod7)$. Bu yerda $(5,7) = 1$ bo'lgani uchun $x \equiv 2(mod7)$.

Javob: $x \equiv 2(mod7)$.

c). Avvalo, berilgan taqqoslamani soddalashtiramiz. U holda quyidagiga ega bo'lamiz: $f(x) \equiv x^{101} + 3x^{15} + x^{11} - 3x^5 + 9x^2 + 10x - 5 \equiv (x^{11})^9 \cdot x^2 + 3x^{11} \cdot x^4 + x^{11} - 3x^5 - 2x^2 - x - 5 \equiv x + 3x^5 + x - 3x^5 - 2x^2 - x - 5 \equiv -2x^2 + x - 5 \equiv 0(mod 11)$. Demak, berilgan taqqoslama $2x^2 - 5x + 5 \equiv 0(mod 11)$ ga teng kuchli ekan. 11 modul bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm, \pm2, \pm3, \pm4, \pm5$ larni sinab ko'rish yo'li bilan yechsak, $x_1 \equiv 2(mod 11)$va$x_2 \equiv 4(mod 11)$ yechimlarga ega bo'lamiz.

Javob: $x_1 \equiv 2(mod 11)$va$x_2 \equiv 4(mod 11)$.

d).Berilgan taqqoslamani soddalashtiramiz. U holda quyidagiga ega bo'lamiz: $f(x) \equiv 2x^{35} - 17x^{15} + 13x^8 - 3x^5 + 12x + 5 \equiv 0(mod 11) \to 2(x^{10})^3 \cdot x^5 + 5x^{10} \cdot x^5 + 2x^8 - 3x^5 + x + 5 \equiv 2x^5 + 5x^5 + 2x^8 - 3x^5 + x + 5 \equiv 2x^8 + 4x^5 + x + 5 \equiv 0(mod 11)$. Demak, berilgan taqqoslama$2x^8 + 4x^5 + x + 5 \equiv 0(mod 11)$ ga teng kuchli. 11 modul bo'yicha $0, \pm1, \pm2, \pm3, \pm4, \pm5$ larni sinab ko'rish yo'li bilan yechsak $x_1 \equiv 3(mod 11)$ va $x_2 \equiv 5(mod 11)$ lar taqqoslamaning yechimlari ekanligiga ishonsch hosil qilamiz.

Javob: $x_1 \equiv 3(mod 11)$ va $x_2 \equiv 5(mod 11)$.

e). Berilgan $x^{12} - 2x^7 + x^3 + 1 \equiv 0(mod 5)$ soddalashtirib

$$(x^5)^2 \cdot x^2 - 2x^5 \cdot x^2 + x^3 + 1 \equiv 0(mod 5) \to x^4 - 2x^3 + x^3 + 1 \equiv 0(mod 5)$$

$\to x^4 - x^3 + 1 \equiv 0(mod 5)$. Demak, berilgan taqqoslama $x^4 - x^3 + 1 \equiv 0(mod 5)$ ga teng kuchli. 5 modul bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm1, \pm2$ larni sinab ko'rish yo'li bilan yechsak, $x \equiv -2(mod 5)$ yechimga ega bo'lamiz. **Javob:** $x \equiv -2(mod 5)$.

279. Bizga ma'lumki, p-tub modul bo'yicha $x^p - x$ ni $f(x)$ ga bo'lishdan chiqqan qoldiq $R(x)$ ning barcha koeffitsiyentlari p ga bo'linishi kerak. $R(x)$ ni aniqlaymiz: $x^7 - x = (x^3 + ax + b)(x^4 - ax^2 - bx + a^2) + 2abx^2 + (b^2 - a^3 - 1)x - a^2b$. Demak,

$$\begin{cases} 2ab \equiv 0(mod 7) \\ b^2 - a^3 - 1 \equiv (mod 7) \\ a^2b \equiv (mod 7) \end{cases}$$

bajaralishi kerak, shartga ko'ra $a \not\equiv 0(mod 7)$ va $b \not\equiv 0(mod 7)$ bo'lgani uchun $ab \not\equiv (mod 7)$, ya`ni birinchi shart bajarilmaydi. Shuday qilib berilgan taqqoslama 3 ta yechimga ega bo'la olmaydi.

280. $x^p - x$ ni $x^n - a$ ga bo'lib $x^p - x = (x^n - a)\cdot(x^{p-n} - ax^{p-2n}) + ax^{p-n} - x$ ni hosil qilamiz. Birinchi qoldiq $ax^{p-n} - x$ ga teng. Bo'lish jarayonidagi ikkinchi qoldiq $a^2x^{p-2n} - x$ va hokazo k-qo'ldiq $a^kx^{p-kn} - x$ larni hosil qilamiz. Faraz etaylik k- qoldiq oxirgisi bo'lsin. U holda

$R(x) = a^kx^{p-kn} - x$ bo'ladi. 279-misolga asosan $x^n \equiv a(modp)$ ning n ta yechimga ega bo'lishi uchun $R(x)$ ning barcha koeffitsiyentlari p ga bo'linishi kerak. Agar $p - nk > 1$ bo'lsa, a^k va 1 koeffitsiyentlar p ga bo'linmaydi va demak, bu holda $x^n \equiv a(modp)$ taqqoslama n ta yechimga ega bo'lmaydi.

Agarda $p - nk = 1$ bo'lsa $R(x) = (a^k - 1)x$ bo'lib $x^n \equiv a(modp)$ taqqoslamaning n ta yechimga ega bo'lishi uchun $a^k - 1 \equiv 0(modp)$ yoki

$$a^k \equiv 1(modp) \to a^{\frac{p-1}{n}} \equiv 1(modp) \qquad (1)$$

bajarilishi kerak ekan. Shunday qilib, $x^n \equiv a(modp),\ n < p$ va $(a,p) = 1$ taqqoslamaning n ta yechimga ega bo'lishi uchun $\frac{p-1}{n}$ butun son bo'lib (1) shartning bajarilishi zarur va yetarli ekan.

281. *a).* $x^3 \equiv 1(mod7)$ ni qaraymiz. 280-misoldagi $a^{\frac{p-1}{n}} \equiv 1(modp)$ shartni tekshiramiz $1^{\frac{7-1}{3}} \equiv 1(mod7)$ bajariladi. Demak, berilgan taqqoslama 3 ta yechimga ega. Endi shu yechimlarni topamiz. Buning uchun 7 modul bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm1, \pm2, \pm3$ larni sinab ko'rish yo'li bilan yechsak, u holda $x_1 \equiv 1,\ x_2 \equiv 2,\ x_3 \equiv 3(mod7)$ larning berilgan taqqoslamani qanoatlantirishini ko'ramiz.

Javob: $x_1 \equiv 1,\ x_2 \equiv 2,\ x_3 \equiv 3(mod7)$.

b). $x^2 \equiv 2(mod5)$ ni qaraymiz. (1) dan $2^{\frac{5-1}{2}} \equiv 2^2 \equiv 4(mod5)$. Demak, berilgan taqqoslama yechimga ega emas.

Javob: taqqoslama yechimga ega emas.

c).$x^5 \equiv 10(mod11)$ ni qaraymiz. (1) dan $a^{\frac{11-1}{5}} \equiv 10^2 \equiv 1(mod11)$, Demak, berilgan taqqoslama 5 ta yechimga ega. Endi shu yechimlarni topamiz. Buning uchun 11 modul bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm1, \pm2, \pm3, \pm4, \pm5$ larni sinab ko'rish yo'li bilan yechsak, u holda $x_1 \equiv -1, x_2 \equiv +2, x_3 \equiv -3, x_4 \equiv -4, x_5 \equiv -5(mod11)$ larning berilgan taqqoslamani qanoatlantirishini ko'ramiz.

Javob: $x_1 \equiv -1, x_2 \equiv 2, x_3 \equiv -3, x_4 \equiv -4, x_5 \equiv -5(mod11)$.

d).$x^4 \equiv 1(mod11)$ ni qaraymiz. (1) dan $1^{\frac{11-1}{4}} \equiv 1^{\frac{5}{2}} \equiv 1$ bo'lishi kerak. Lekin bu yerda $\frac{p-1}{n}$ butun son emas, shuning uchun ham berilgan taqqoslama 4 ta yechimga ega deya olamiz. Taqqoslamaning yechimlarini topamiz. Buning uchun

$0, \pm1, \pm2, \pm3, \pm4, \pm5$ larni qo'yib tekshiramiz. U holda berilgan taqqoslama 2 ta $x_1 \equiv -2(mod11)$, $x_2 \equiv 2(mod11)$ yechimlarga ega ekanligiga ishonch hosil qilamiz.

Javob: $x_1 \equiv -2(mod11)$, $x_2 \equiv 2(mod11)$.

e)$x^6 \equiv 3(mod7)$ ni qaraymiz. (1) dan$3^{\frac{7-1}{6}} \equiv 3(mod7)$. Demak berilgan taqqoslama yechimga ega emas.

f) **Javob:** taqqoslama yechimga ega emas.

f).$x^4 \equiv 3(mod13)$ ni qaraymiz. (1) dan $3^{\frac{13-1}{4}} \equiv 3^3 \equiv 1(mod13)$. Demak, berilgan taqqoslama 4 ta yechimga ega. Taqqoslamaning yechimlarini topamiz. Buning uchun $0, \pm1, \pm2, \pm3, \pm4, \pm5, \pm6$ larni qo'yib tekshiramiz. U holda $x_1 \equiv -2$, $x_2 \equiv 2$, $x_3 \equiv -3$, $x_4 \equiv 3(mod13)$ larning berilgan taqqoslamaning yechimi ekanligiga ishonch hosil qilamiz.

282. Vilson teoremasiga asosan p tub soni uchun

$$(p-1)! + 1 \equiv 0(modp) \rightarrow (p-2)!\,(p-1) \equiv -1(modp) \rightarrow$$
$$(p-2)! \equiv 1(modp)$$

bajariladi.

283. Faraz etaylik $p$va $p+2$ lar tub sonlar bo'lsin. Vilson teoremasiga ko'ra $(p-1)! + 1 \equiv 0(modp)$. Buning ikkala tomonini 4 ga ko'paytirib hosil bo'lgan taqqoslamani $p \equiv 0(modp)$ ayniy taqqoslama qo'shamiz. U holda

$$4 \cdot [(p-1)! + 1] + p \equiv 0(modp) \qquad (2)$$

taqqoslamaga ega bo'lamiz. Endi $p + 2 \equiv 0(modp+2)$ taqqoslamani qaraymiz. Bundan $p \equiv -2(modp+2)$.Buning ikkala tomonini $(p+1)$ ga ko'paytirsak

$$p(p+1) \equiv -2(p+1) = -2\big((p+2) - 1\big) \equiv -2(p+2) + 2$$
$$\equiv 2(modp+2)$$

hosil bô'ladi, ya'ni $p(p+1) \equiv 2(modp+2)$. Oxirgi taqqoslamaning ikkala tomonini $(p-1)!\cdot 2$ ga ko'paytirib, hosil bo'lgan taqqoslamaning ikkala tomoniga $4+p$ ni qo'shamiz. U holda

$$2\cdot(p+1)!+4+p \equiv (p-1)!\cdot 4+4+p(mod\, p+2)$$
$$\to 2[(p+1)!+1]+(p+2) \equiv 4[(p-1)!+1] + p(modp + 2). \quad (3)$$

Agar p tub son bo'lsa, Vilson teoremasiga ko'ra $(p+1)!+1 \equiv 0(modp+2)$ bo'lishi kerak. Shuning uchun ham (3) dan

$$4[(p-1)!+1]+p \equiv 0(modp+2) \quad (4)$$

kelib chiqadi. (2)va (4) dan

$$4[(p-1)!+1]+p \equiv 0(modp(p+2)) \quad (5)$$

ni hosil qilamiz.

Aksincha, agar (5) shart bajarilsa, (4) ning bajarilishi kelib chiqadi. Bundan (3) ga asosan $(p+1)!+1 \equiv 0(modp+2)$ hosil bo'ladi. Bu esa Vilson teoremasiga asosan $p+2$ soni tub p son degani.

284. $p=4n+1$ tub son bo'lsin. U holdaVilson teoremasiga asosan:

$(p-1)! \equiv -1(modp) \to (4n)! \equiv -1(modp)$. Bu yerda
$1\cdot 2\cdot 3\cdot\dots\cdot 2n\cdot(2n+1)(2n+2)\cdot\dots\cdot(4n-1)\cdot 4n =$
$(2n)!\,(p-2n)(p-2n+1)\dots(p-2)(p-1) =$
$(2n)!\,[(-1)(-2)\dots(-2n)+pt](modp) \equiv (-1)^{2n}((2n)!)^2\,(modp) \equiv$
$((2n)!)^2\,(modp)$ bo'lgani uchun $((2n)!)^2 \equiv -1(modp)$ ga bo'lamiz.

285.*a)*. Vilson teoremasiga asosan

$$(p-1)! \equiv -1(modp). \quad (6)$$

Bundan $a(p-1)! \equiv -a(modp)$. Ferma teoremasiga asosan

$$a^p \equiv a(modp). \quad (7)$$

Bu ikki taqqoslamani hadlab qo'shsak $a^p+a(p-1)! \equiv 0(modp)$hosil bo'ladi.

b). (6) va (7) ni hadlab ko'paytirsak

$$a^p(p-1)! \equiv -a(modp) \to a^p(p-1)!+a \equiv 0(modp)$$

hosil bo'ladi.

286. $p>2$tub son bo'lsin. U holda (6) dan$(p-1)!+1 =$
$(p-2)!\,(p-1)+1 = (p-2)!\,p-(p-2)!+1 \equiv$
$0(modp)$yoki$(p-2)!-1 \equiv 0(modp)$.

287. $x^p - x = f(x)Q(x) + R(x)$ (8) ayniyantni qaraymiz. Bu yerda $Q(x)$ va $R(x)$ lar butun koeffitsiyentli ko'phadlar bo'lib $\deg Q(x) = p - n$, $\deg R(x) \leq n - 1$. Agar $f(x) \equiv 0(mod p)$ taqqoslama n ta yechimga ega bo'lsa, u yechimlar $R(x) \equiv 0(mod p)$ni *ham qanoatlantirishi kerak.* deg.$R(x) \leq n - 1$bo'lgani uchun 2-teoremaning natijasiga asosan $R(x)$ ning barcha koeffitsiyentlari p ga bo'linishi kerak.

Aksincha, $R(x)$ning barcha koeffitsiyentlari p gabo'linsa, (8) dan $f(x)Q(x) \equiv 0(mod p)$ hosil bo'ladi. Demak, $f(x) \equiv 0(mod p)$ taqqoslama n ta yechimga ega.

288. *a).*x^5-x ning berilgan $x^2 + 2x + 2$ ko'phadga bo'lib qoldiqni topamiz:

$$x^5 - x = (x^2 + 2x + 2) \cdot (x^3 - 2x^2 + 2x) + (-5x).$$

Bundan $R(x) = -5x$ bo'lib -5 soni 5 ga bo'linadi. Berilgan taqqoslama 2 ta yechimga ega. Haqiqatan ham, 5 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm 1, \pm 2$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x_1 \equiv 1(mod\ 5)$ va $x_2 \equiv 2(mod\ 5)$ lar berilgan $x^2 + 2x + 2 \equiv 0(mod\ 5)$ taqqoslamani qanoatlantirishini topamiz. **Javob:**$x_1 \equiv 1(mod 5)$, $x_2 \equiv 2\ (mod 5)$.

b). Avvalo, berilgan $3x^3 - 4x^2 - 2x - 4 \equiv 0(mod 7)$ taqqoslamani bosh hadining koeffitsiyenti 1 ga teng bo'lgan taqqoslama bilan almashtiramiz.

$3a \equiv 1(mod 7) \rightarrow a \equiv 5(mod 7) \rightarrow a \equiv -2(mod 7)$bo'lgani uchun berilgan taqqoslamaning ikkala tomonini (-2) ga ko'partiamiz:

$$-6x^3 + 8x^2 + 4x + 8 \equiv x^3 + x^2 - 3x + 1(mod 7)$$

bo'lgani uchun berilgan taqqoslama $x^3 + x^2 - 3x + 1 \equiv 0(mod 7)$ga teng kuchli. Endi $R(x)$ ni aniqlaymiz: $x^7 - x = (x^3 + x^2 - 3x + 1) \cdot (x^4 - x^3 + 4x^2 - 8x + 21) + (-49x^2 + 70x - 21)$ bo'lgani uchun $R(x) = -49x^2 + 70x - 21$. $R(x)$ ning barcha koeffitsiyentlari 7 ga karrali, shuning uchun ham berilgan $x^3 + x^2 - 3x + 1 \equiv 0(mod 7)$ taqqoslama 3 ta yechimga ega. Bu yerda 7 moduli bo'yicha chegirmalarning to'la sistemasidagi sonlar $0, \pm 1, \pm 2, \pm 3$ ni qo'yib tanlash usuli bilan yechamiz. U holda $x_1 \equiv 1$, $x_2 \equiv 2$, $x_3 \equiv 3\ (mod 7)$ lar uni qanotlantiradi.

Javob:$x_1 \equiv 1$, $x_2 \equiv 2$, $x_3 \equiv 3\ (mod 7)$.

IV.5-§.

289. 1). Berilgan taqqoslama ushbu

$$\begin{cases} 3x^3+4x^2-7x-6\equiv 0(mod3) \\ 3x^3+4x^2-7x-6\equiv 0(mod5) \end{cases}$$

sistemaga teng kuchli. Bu sistemani soddalashtiramiz:

$$\begin{cases} x^2-x\equiv 0(mod3) \\ 2x^3+x^2+2x+1\equiv 0(mod5). \end{cases}$$

Bu sistemadagi 1-taqqoslamaning yechimlari $x\equiv 0,1\ (mod3)$; ikkinchisiniki esa $x\equiv -2,2(mod5)$ lardan iborat, natijada quyidagi 4ta sistemaga ega bo'lamiz:

$$a)\begin{cases} x\equiv 0(mod3) \\ x\equiv -2(mod5) \end{cases}\quad b)\begin{cases} x\equiv 0(mod3) \\ x\equiv 2(mod5) \end{cases}$$

$$c)\begin{cases} x\equiv 1(mod3) \\ x\equiv -2(mod5) \end{cases}\quad d)\begin{cases} x\equiv 1(mod3) \\ x\equiv 2(mod5) \end{cases}.$$

a)dan $\begin{cases} x=3t_1 \\ 3t_1\equiv -2(mod5) \end{cases}\rightarrow\begin{cases} t_1\equiv 1(mod\ 5) \\ t_1=1+5t_2,\ t_2\in Z. \end{cases}\rightarrow x=3(1+5t_2)=3+15t_2, t_2\in Z;$

b)dan $\begin{cases} x=3t_1 \\ 3t_1\equiv 2(mod5) \end{cases}\rightarrow\begin{cases} x\equiv 3t_1 \\ t_1\equiv -1(mod5) \end{cases}\rightarrow x=3(-1+5t_2)=-3+15t_2, t_2\in Z$

c) dan $\begin{cases} x=1+3t_1 \\ 1+3t_1\equiv -2(mod5) \end{cases}\rightarrow\begin{cases} x\equiv 1+3t_1 \\ 3t_1\equiv -3(mod5) \end{cases}\rightarrow$

$\begin{cases} x=1+3t_1 \\ t_1=-1+5t_2 \end{cases}$

$$\rightarrow x=1+3(-1+5t_2)=-2+15t_2, t_2\in Z.$$

d) dan $\begin{cases} x=1+3t_1 \\ 1+3t_1\equiv 2(mod5) \end{cases}\rightarrow\begin{cases} x=1+3t_1 \\ 3t_1\equiv 1(mod5) \end{cases}\rightarrow\begin{cases} x=1+3t_1 \\ t_1=2+5t_2 \end{cases}$

$$\rightarrow x=1+3(2+5t_2)=7+15t_2, t_2\in Z.$$

Shunday qilib berilgan taqqoslama 4 ta yechimga; $x\equiv 3,-3,-2,7\ (mod15)$ega ekan.

Javob: $x\equiv 3,-3,-2,7\ (mod15)$.

2). Berilgan taqqoslama $6x^3-3x^2-13x-10\equiv 0(mod30)$ ushbu taqqoslamalar

$$\begin{cases} 6x^3-3x^2-13x-10\equiv 0(mod2) \\ 6x^3-3x^2-13x-10\equiv 0(mod3) \\ 6x^3-3x^2-13x-10\equiv 0(mod5) \end{cases}$$

sistemasiga teng kuchli. Bu sistema

$$\begin{cases} x^2 - x \equiv 0(mod2) \\ 2x - 1 \equiv 0(mod3) \\ x^3 + 2x^2 + 2x \equiv 0(mod5) \end{cases}$$

ga teng kuchli. Bu yerdagi birinchi taqqoslamaning yechimi $x \equiv 0{,}1(mod2)$, ikkinchisiniki $x \equiv 2(mod3)$, uchunchisiniki esa $x \equiv 0, 1, 2(mod5)$ dan iborat.

Bulardan

a)$\begin{cases} x \equiv 0(mod2) \\ x \equiv 2(mod3) \\ x \equiv 0(mod5) \end{cases}$ b)$\begin{cases} x \equiv 1(mod2) \\ x \equiv 2(mod3) \\ x \equiv 0(mod5) \end{cases}$

c)$\begin{cases} x \equiv 0(mod2) \\ x \equiv 2(mod3) \\ x \equiv 1(mod5) \end{cases}$d) $\begin{cases} x \equiv 0(mod2) \\ x \equiv 2(mod3) \\ x \equiv 2(mod5) \end{cases}$

e)$\begin{cases} x \equiv 1(mod2) \\ x \equiv 2(mod3) \\ x \equiv 1(mod5) \end{cases}$ f)$\begin{cases} x \equiv 1(mod2) \\ x \equiv 2(mod3) \\ x \equiv 2(mod5) \end{cases}$

chiziqli taqqoslamalar sistemalariga ega bo'lamiz.

a) dan $\begin{cases} x = 2t_1 \\ 2t_1 \equiv 2(mod3) \\ x \equiv 0(mod5) \end{cases} \to \begin{cases} x = 2t_1 \\ t_1 = 1 + 3t_2 \\ x \equiv 0(mod5) \end{cases} \to \begin{cases} x = 2 + 6t_2 \\ x \equiv 0(mod5) \end{cases} \to$

$$2 + 6t_2 \equiv 0(mod5) \to \ 3t_2 \equiv -1(mod5) \to 3t_2 \equiv 9(mod5)t_2 = 3 + 5t_3, \ \ t_3 \in Z \to x = 2 + 6(3 + 5t_3) \ = 20 + 30t_3$$

b) dan

$$\begin{cases} x = 1 + 2t_1 \\ 1 + 2t_1 \equiv 2(mod3) \\ x \equiv 0(mod5) \end{cases} \to \begin{cases} x = 1 + 2t_1 \\ t_1 = 2 + 3t_2 \\ x \equiv 0(mod5) \end{cases} \to \begin{cases} x = 1 + 4 + 6t_2 \\ 5 + 6t_2 = 0(mod5) \\ t_2 \equiv 5t_3 \end{cases}$$

$$x = 5 + 6(5t_3) = 5 + 30t_3.$$

c) dan $\begin{cases} x = 2t_1 \\ 2t_1 \equiv 2(mod3) \\ x \equiv 1(mod5) \end{cases} \to \begin{cases} x = 2t_1 \\ t_1 = 1 + 3t_2 \\ x \equiv 1(mod5) \end{cases} \to \begin{cases} x = 2 + 6t_2 \\ 2 + 6t_2 \equiv 1(mod5) \\ t_2 \equiv -1(mod5) \end{cases}$

$$x = 2 + 6(-1 + 5t_3) = -4 + 30t_3.$$

d) dan $\begin{cases} x = 5 + 6t_2 \\ x \equiv 2(mod5) \end{cases} \to 2 + 6t_2 \equiv 2(mod5) \to 6t_2 \equiv 0(mod5)$

$$\to t_2 \equiv 0(mod5) \to t_2 = 5t_3 \to x = 2 + 30t_3.$$

e) dan $\begin{cases} x = 5 + 6t_2 \\ 5 + 6t_2 \equiv 1(mod5) \end{cases} \rightarrow \begin{cases} t_2 \equiv -4(mod5) \\ t_2 \equiv 1 + 5t_3 \end{cases} \rightarrow$

$x = 5 + 6(1 + 5t_3) \equiv 11 + 30t_3\,, t_3 \in Z.$

f) dan $\begin{cases} x = 5 + 6t_2 \\ 5 + 6t_2 \equiv 2(mod5) \end{cases} \rightarrow \begin{cases} x = 5 + 6t_2 \\ t_2 \equiv 2 + 5t_3 \end{cases} \rightarrow x = 17 + 30t_3,$
$t_3 \in Z.$

Shunday qilib, berilgan taqqoslamaning yechimlari
$x \equiv -13, -10, -4, 2, 5, 11\ (mod30)$ lardan iborat ekan.

Javob:$x \equiv -13, -10, -4, 2, 5, 11\ (mod30).$

3).Berilgan taqqoslama $x^4 - 33x^2 + 8x - 26 \equiv 0(mod35)$ ushbu taqqoslamalar

$$\begin{cases} x^4 - 33x^2 + 8x - 26 \equiv 0(mod5) \\ x^4 - 33x^2 + 8x - 26 \equiv 0(mod7) \end{cases}$$

$$\rightarrow \begin{cases} x^4 + 2x^2 + 3x - 1 \equiv 0(mod5) \\ x^4 + 2x^2 + x + 2 \equiv 0(mod7) \end{cases}$$

sistemasiga teng kuchli. Bu yerda birinchi taqqoslamaning yechimi $x \equiv 1(mod5)$; ikkinchisining yechimi esa $x \equiv 2(mod7)$ dan iborat. Bulardan $\begin{cases} x \equiv 1(mod5) \\ x \equiv 2(mod7) \end{cases}$ sistemaga kelamiz. Buni yechib

$\begin{cases} x = 1 + 5t_1 \\ 1 + 5t_1 \equiv 2(mod7) \end{cases} \rightarrow \begin{cases} x = 1 + 5t_1 \\ 5t_1 \equiv 1(mod7) \end{cases} \rightarrow \begin{cases} x = 1 + 5t_1 \\ 5t_1 \equiv 15(mod7) \end{cases}$

$\rightarrow \begin{cases} x = 1 + 5t_1 \\ t_1 \equiv 3(mod7) \end{cases} \rightarrow x = 1 + 5(3 + 7t_2) \rightarrow x = 16 + 35t_2,\ t_2 \in$
$Z.$

Demak,berilgan sistemaning yechimi $x \equiv 16(mod35)$ dan iborat.

Javob: $x \equiv 16(mod35).$

4).Berilgan taqqoslama $x^5 - 3x^4 + 5x^3 + 9x^2 + 4x - 12 \equiv 0(mod42)$ ushbu taqqoslamalar

$$\begin{cases} x^5 - 3x^4 + 5x^3 + 9x^2 + 4x - 12 \equiv 0(mod2) \\ x^5 - 3x^4 + 5x^3 + 9x^2 + 4x - 12 \equiv 0(mod3) \\ x^5 - 3x^4 + 5x^3 + 9x^2 + 4x - 12 \equiv 0(mod7) \end{cases}$$

sistemasiga teng kuchli. Bu sistemadagi taqqoslamalarni soddalashtirib quyidagi sistemaga ega bo'lamiz:

$$\begin{cases} (x^2)^2 \cdot x - (x^2)^2 + x^2 \cdot x + x^2 \equiv 0(mod2) \\ (x^3) \cdot x^2 - x^3 + x \equiv 0(mod3) \\ x^5 - 3x^4 - 2x^3 + 2x^2 - 3x + 2 \equiv 0(mod7) \end{cases} \rightarrow$$

$$\begin{cases} x^3 + x^2 \equiv 0(mod2) \\ x \equiv 0(mod3) \\ x^5 - 3x^4 - 2x^3 + 2x^2 - 3x + 2 \equiv 0(mod7) \end{cases}.$$

Birinchi taqqoslamani ixtiyoriy $x \in Z$ soni qanoatlantiradi. Ikkinchisining yechmi $x \equiv 0(mod3)$ dan iborat. Uchinchi taqqoslamani yechamiz. 0, $\pm1, \pm2, \pm3$ lardan faqat 2 uni qanoatlantiradi. Demak, bu taqqoslama yagona $x \equiv 2(mod7)$ yechimga ega. Shunday qilib, ushbu

$$\begin{cases} x \equiv 0(mod3) \\ x \equiv 2(mod7) \end{cases}$$

sistemani hosil qildik. Buni yechib

$$\begin{cases} x = 3t \\ 3t \equiv 2(mod7) \end{cases} \rightarrow \begin{cases} x = 3t \\ t = 3 + 7t \end{cases} \rightarrow x = 3 + 21t$$

$x \equiv 3, 24\ (mod42)$ ga ega bo'lamiz. **Javob:**$x \equiv 3, 24\ (mod42)$.

5). Berilgan taqqoslama $x^5 + x^4 - 3x^3 + 2x - 2 \equiv 0(mod77)$ ushbu taqqoslamalar

$$\begin{cases} x^5 + x^4 - 3x^3 + 2x - 2 \equiv 0(mod7) \\ x^5 + x^4 - 3x^3 + 2x - 2 \equiv 0(mod11) \end{cases}$$

sistemasiga teng kuchli. Bu sistemadagi taqqoslamalarni yechib quyidagilarga ega bo'lamiz: 0, ±1, ±2, ±3 lardan birortasi ham qanotlantirmaydi. Berilgan taqqoslama yechimga ega emas.

Javob: taqqoslama yechimga ega emas.

6).Berilgan taqqoslama $3x^3 + 6x^2 + x + 10 \equiv 0(mod15)$ ushbu taqqoslamalar

$$\begin{cases} 3x^3 + 6x^2 + x + 10 \equiv 0(mod3) \\ 3x^3 + 6x^2 + x + 10 \equiv 0(mod5) \end{cases}$$

sistemasiga teng kuchli. Bu sistemadagi birinchi taqqoslamani tanlash usuli bilan yechamiz. Buning uchun $0, \pm1$ larni unga qo'yib tekshirib ko'rish kifoya. holda $x \equiv -1(mod3)$ ning birinchi taqqoslamani qanoatlantirishini ko'ramiz. Sistemadagi ikkinchi taqqoslamaning yechimlari esa $x \equiv 0,1,2(mod5)$ lardan iborat. Natijada quyidagi 3 ta sistemaga ega bo'lamiz:

$$\text{a)}\begin{cases} x \equiv -1(mod3) \\ x \equiv 0(mod5) \end{cases} \text{b)}\begin{cases} x \equiv -1(mod3) \\ x \equiv 1(mod5) \end{cases} \text{c)}\begin{cases} x \equiv -1(mod3) \\ x \equiv 2(mod5) \end{cases}.$$

Shuning uchun ham berilgan taqqoslamaning yechimini $x \equiv x_0 = M_1M_1'b_1 + M_2M_2'b_2(\mathrm{mod}15)$ ko'rinishda izlash mumkin. Bizda $M_1 = 5$, $M_2 = 3$ bo'lgani uchun $5M_1' \equiv 1(mod3) \to M_1' = 2$ va $3M_2' \equiv 1(mod5) \to M_2' = 2$ ga ega bo'lamiz. Bulardan foydalanib $x \equiv x_0 = 5 \cdot 2b_1 + 3 \cdot 2b_2(mod15) \equiv -5b_1 + 6b_2(mod15)$. Endi bunda $b_1 = -1$, $b_2 = 0, 1, 2$ deb olib berilgan taqqoslamaning yechimlari $x_1 \equiv 5(mod15)$, $x_2 \equiv 11(mod15), x_3 \equiv 2(mod15)$ ni hosil qilamiz.

Javob: $x \equiv 2{,}5{,}11(mod15)$.

7). Berilgan taqqoslama $37x \equiv 17(mod180)$ da $180 = 36 \cdot 5$ bo'lgani uchun ushbu taqqoslamalar $\begin{cases} 37x \equiv 17(mod5) \\ 37x \equiv 17(mod36) \end{cases} \to$ $\begin{matrix} x \equiv 1(mod5) \\ x \equiv 17(mod36) \end{matrix}$ sistemasiga teng kuchli. Shuning uchun ham berilgan taqqoslamaning yechimini $x \equiv x_0 = M_1M_1'b_1 + M_2M_2'b_2(\mathrm{mod}15)$ ko'rinishda izlash mumkin. Bizda $M_1 = 36$, $M_2 = 5$ bo'lgani uchun $36M_1' \equiv 1(mod5) \to M_1' = 1$ va $5M_2' \equiv 1(mod36) \to M_2' = -7$ ga ega bo'lamiz. Bulardan foydalanib $x \equiv x_0 = 36 \cdot b_1 + 5 \cdot (-7)b_2(mod15) \equiv 36b_1 - 35b_2(mod180)$. Endi bunda $b_1 = 1$, $b_2 = 17$ deb olib berilgan taqqoslamaning yechimlari $x \equiv -19(mod180)$, ni hosil qilamiz. **Javob:** $x \equiv -19(mod180)$.

290.1). $4x^3 - 8x - 13 \equiv 0(mod\ 27)$ taqqoslamani yechishimiz kerak. Bu yerda$27 = 3^3$ bo'lgani uchun avvalo, $4x^3 - 8x - 13 \equiv 0(mod3)$ taqqoslamani qaraymiz. Bundan $x^3 + x - 1 \equiv 0(\mathrm{mod}\ 3)$. Bu taqqoslamaning yechimlarini topish uchun 3 moduli bo'yicha chegirmalarning to'la sistemadagi sonlar $0, \pm 1$ ni qo'yib tekshirib ko'ramiz. U holda $x \equiv -1(mod\ 3)$ uning yechimi ekanlini ko'ramiz.

Endi bu $x \equiv -1 + 3t_1$ yechimni nazariy qismdagi

$$\frac{f(x_1)}{p} + t_1 f'(x_1) \equiv 0(modp) \quad (7)$$

ga olib borib qo'yamiz: U holda, bizda $p = 3$, $x_1 = -1, f(-1) = -9, f'(x) = 12x^2 - 8$ va$f'(-1) = 4$ bo'lgani uchun $\frac{-9}{3} + 4t_1 \equiv$

$0(mod\ 3) \to t_1 \equiv 3(mod\ 3) \to t_1 = 3 + 3t_2 \to x = -1 + 9 + 9t_2 = 8 + 9t_2$. Endi yana (7) dan foydalanib (nazariy qismdagi (9) ga qarang)

$$\frac{f(x_2)}{p^2} + t_2 f'(x_2) \equiv 0(mod\ p) \quad (1)$$

ni tuzib olamiz. Bizda $p = 3, x_2 = 8. f(8) = 1971, f'(8) = 760$ bo'lgani uchun $\frac{1971}{9} + 760t_2 \equiv 0(mod\ 3) \to t_2 \equiv -219(mod\ 3) \to t_2 \equiv 0(mod\ 3)$, ya'ni $t_2 = 3t_3$. Bu qiymatni x ning ifodasiga olib borib qo'ysak $x = 8 + 9(3t_3) = 8 + 27t_3, t_3 \epsilon Z$, ya'ni $x \equiv 8(mod\ 27)$ berilgan taqqoslamaning yechimiga ega bo'lamiz. **Javob:** $x \equiv 8(mod\ 27)$.

2). $f(x) = x^4 - 3x^3 + 2x^2 - 5x - 10 \equiv 0(mod\ 343)$ taqqoslamani yechishimiz kerak. Bu yerda $343 = 7^3$ bo'lgani uchun avvalo, $f(x) \equiv 0(mod 7)$ taqqoslamani qaraymiz. Bu $x^4 - 3x^3 = 2x^2 + 2x - 3 \equiv 0(mod\ 7)$ ga teng kuchli. Bu taqqoslamaning yechimlarini topish uchun 7 moduli bo'yicha chegirmalarning to'la sistemadagi sonlar $0, \pm 1, \pm 2, \pm 3$ ni qo'yib tekshirib ko'ramiz. U holda $x \equiv 3(mod\ 7)$ uning yechimi ekanlini ko'ramiz. Demak, $x \equiv 3(mod\ 7)$ berilgan taqqoslamaning yechimi. Endi bu $x \equiv 3 + 7t_1$ yechimni nazariy qismdagi $\frac{f(x_1)}{p} + t_1 f'(x_1) \equiv 0(mod p)$ (7) ga olib borib qo'yamiz:

U holda, bizda $f(3) = 3^4 \text{-} 3 \cdot 27 + 18 - 15 - 10 = -7; f'(x) = 4x^3 - 9x^2 + 4x - 5$ va $f'(3) = 108 - 81 + 12 - 5 = 34$ bo'lgani uchun (7) dan $\frac{(-7)}{7} + 34t_1 \equiv 0(mod\ 7)$

$\to 34t_1 \equiv 1(mod\ 7) \to t_1 \equiv -1(mod\ 7)$, ya'ni $t_1 \equiv -1 + 7t_2$. Buni x ning ifodasiga qo'ysak $x = 3 + 7(-1 + 7t_2) = -4 + 49t_2$ hosil bo'ladi. Endi bundan foydalanib (9) ni tuzamiz. Bunda $f(-4) = (-4)^4 - 3 \cdot (-4)^3 + 2 \cdot .16 + 20 - 10 = 256 + 192 + 32 + 20 - 10 = 490. f'(-4) = -256 - 9 \cdot 16 - 16 - 5 = -421$ bo'lgani uchun $\frac{490}{49} + t_2(-421) \equiv 0(mod\ 7) \to -421t_2 \equiv -10(mod\ 7) \to -t_2 \equiv -3(mod\ 7) \to t_2 \equiv 3(mod\ 7) \to t_2 = 3 + 7t_3$, $t_3 \in Z$. Hosil bo'lgan qiymatni x ning ifodasiga olib borib qo'ysak $x = -4 + 49(3 + 7t_3) = -4 + 147 + 343t_3 \equiv 143 + 343t_3, t_3 \epsilon Z$ ga ega bo'lamiz, ya'ni $x \equiv 143(mod\ 343)$ berilgan taqqoslamaning yechimiga ega bo'lamiz. **Javob:** $x \equiv 143(mod\ 343)$.

3). $f(x) = x^4 - 4x^3 + 2x^2 + x + 6 \equiv 0(mod\ 25)$ taqqoslamani yechishimiz kerak. Bu yerda $25 = 5^2$ bo'lgani uchun avvalo, $f(x) \equiv 0(mod 5)$ taqqoslamani, ya'ni $f(x) = x^4 - 4x^3 + 2x^2 + x + 6 \equiv 0(mod\ 5)$ ni qaraymiz. Bu $x^4 + x^3 + 2x^2 + x + 1 \equiv 0(mod\ 5)$ ga teng kuchli. Bu taqqoslamaning yechimlarini topish uchun 5 moduli bo'yicha chegirmalarning to'la sistemadagi sonlar $0, \pm 1, \pm 2$ ni qo'yib tekshirib ko'ramiz. U holda $x \equiv 2(mod\ 5)$ va $x \equiv -2(mod\ 5)$ lar uning yechimlari ekanligini ko'ramiz.

Endi avvalo $x = 2 + 5t_1$ yechimni nazariy qismdagi (7)-formulaga olib borib qo'yamiz. U holda, bizda $p = 5, x_1 = 2, f(2) = 0, f'(x) = 4x^3 + 12x^2 + 4x + 1$ va $f'(2) = -7$ bo'lgani uchun (7) dan $\frac{0}{5} + 4t_1 \equiv 0(mod\ 3) \to t_1 \equiv 0(mod\ 5) \to t_1 = 5t_2 \to x = 2 + 5 \cdot 5t_2 = 2 + 25t_2, t_2 \in Z$ yoki $x_1 \equiv 2(mod\ 25)$. Ikkinchi yechimi $x \equiv -2(mod\ 5)$ uchun $\frac{f(-2)}{5} + t_1 f'(-2) \equiv 0(mod\ 5)$. Bunda $f(-2) = 16 + 32 + 8 - 2 + 6 = 60$, $f'(-2) = -32 - 48 - 8 + 1 = -87$ bo'lgani uchun $\frac{60}{5} + t_1(-87) \equiv 0(mod\ 5) \to 12 + 3t_1 \equiv 0(mod\ 5) \to 3t_1 \equiv 3(mod\ 5) \to t_1 \equiv 1(mod\ 5)$, ya'ni $t_1 = 1 + 5t_2 \to x = -2 + 5(1 + 5t_2) = 3 + 25t_2$. Demak, berilgan taqqoslamaning ikkinchi yechimi $x_2 \equiv 3(mod\ 25)$.

Javob: $x_1 \equiv 2(mod\ 25)$, $x_2 \equiv 3(mod\ 25)$.

4). $9x^2 + 29x + 62 \equiv 0(mod\ 64)$ taqqoslamani yechishimiz kerak. Bu yerda $64 = 2^6$ bo'lgani uchun avvalo, $f(x) \equiv 0(mod 2)$ taqqoslamani, ya'ni $f(x) = 9x^2 + 29x + 62 \equiv 0(mod\ 2)$ ni qaraymiz. Bu $x^2 + x \equiv 0(mod\ 2)$ ga teng kuchli. Bu taqqoslamaning yechimlarini topish uchun 2 moduli bo'yicha chegirmalarning to'la sistemadagi sonlar $0, 1$ ni qo'yib tekshirib ko'ramiz. U holda $x \equiv 0(mod\ 2)$ va $x \equiv 1(mod\ 2)$ lar uning yechimlari ekanligini ko'ramiz.

a). $x \equiv 0(mod\ 2) \to x \equiv 2t_1$ yechim uchun $\frac{f(0)}{2} + f'(0)t_1 \equiv 0(mod\ 2)$ dan $f(0) = 62; f'(x) = 18x + 29$, $f'(0) = 29$ bo'lgani uchun $31 + 29t_1 \equiv 0(mod\ 2) \to t_1 \equiv -1(mod\ 2) \to t_1 = -1 + 2t_2$, $t_2 \in Z. x = 2(-1 + 2t_2) = -2 + 4t_2$, $t_2 \in Z$. $x = -2 + 4t_2$ dan foydalanib (9) ga asoslanib quyidagilarga ega bo'lamiz: $\frac{f(-2)}{4} + t_2 f'(-2) \equiv 0(mod\ 2)$, bunda $f(-2) = 36 - 58 + 62 = 40$, $f'(-2) = -7$ bo'lgani uchun $10 - 7t_2 \equiv 0(mod\ 2) \to t_2 \equiv 0(mod 2) \to t_2 = 2t_3 \to x = -2 + 4(2t_3) = -2 + 8t_3$, $t_3 \in Z$.

Endi $x = -2 + 8t_3$, $t_3 \in Z$dan foydalanib navbatdagi qadamni amalga oshiramiz. U holda $\frac{f(-2)}{8} + t_3 f'(-2) \equiv 0(mod\ 2)$ni hosil qilamiz. Bunda $f(-2) = 40$, $f'(-2) = -7$ bo'lgani uchun $5 - 7t_3 \equiv 0(mod\ 2) \to 7t_3 \equiv 5(mod\ 2) \to t_3 \equiv 1(mod\ 2), t_3 = 1 + 2t_4 \to$

$x = -2 + 8(1 + 2t_4) = 6 + 16t_4, t_4 \epsilon Z$.Endi $x = 6 + 16t_4,\ t_4 \in Z$dan foydalanib navbatdagi qadamni amalga oshiramiz. U holda $\frac{f(6)}{16} + t_4 f'(6) \equiv 0(mod\ 2)$ ga ega bo'lamiz. Bu yerda $f(6) = 9 \cdot 36 + 29 \cdot 6 + 62 = 324 + 174 + 62 = 560, f'(6) = 18 \cdot 6 + 29 = 108 + 29 = 137$ bo'lgani uchun $35 + t_4 \cdot 137 \equiv 0(mod\ 2) \rightarrow\ t_4 \equiv 1(mod\ 2) \rightarrow t_4 = 1 + 2t_5 \rightarrow x = 6 + 16(1 + 2t_5) = 22 + 32t_5\ , t_5 \epsilon Z$.

Endi $x = 22 + 32t_5,\ t_5 \in Z$dan foydalanib oxirgi qadamni amalga oshiramiz.U holda $\frac{f(22)}{32} + t_5 f'(22) \equiv 0(mod\ 2)$ ga ega bo'lamiz. Bu yerda $f(22) = 9 \cdot 22^2 + 29 \cdot 22 + 62 = 4356 + 638 + 62 = 5056, f'(22) = 18 \cdot 22 + 29 = 425$ bo'lgani uchun $\frac{5056}{32} + 425t_5 \equiv 0(mod\ 2) \rightarrow 158 + t_5 \equiv 0(mod\ 2) \rightarrow t_5 \equiv 0(mod\ 2) \rightarrow t_5 = 2t_6 \rightarrow x = 22 + 32(2t_6) = 22 + 64t_6, t_6 \epsilon Z$. Demak, berilgan taqqoslamaning bitta yechimi $x \equiv 22(mod\ 64)$ ekan.

b). Endi $x \equiv 1(mod\ 2)$ ga mos yechimini izlaymiz: $x = 1 + 2t_1$yechim uchun $\frac{f(1)}{2} + f'(1)t_1 \equiv 0(mod\ 2)$ni tuzib olamiz. Bu yerda $f(1) = 9 + 29 + 62 = 100,\ f'(1) = 18 + 29 = 47$ bo'lgani uchun $50 + 47t_1 \equiv 0(mod\ 2)$

$t_1 \equiv 0(mod\ 2)$, $t_1 = 2t_2 \rightarrow x = 1 + 2t_1 = 1 + 4t_2$ .x ning bu qiymatidan foydalanib quyidagilarga ega bo'lamiz: $\frac{f(1)}{4} + f'(1)t_2 \equiv 0(mod\ 2) \rightarrow\ 25 + 47t_2 \equiv 0(mod\ 2) \rightarrow t_2 \equiv -1(mod\ 2 \rightarrow t_2 = -1 + 2t_3 \rightarrow x = 1 + 4(-1 + 2t_3) = -3 + 8t_3$. x ning bu topilgan qiymatidan foydalanib quyidagiga ega bo'lamiz: $\frac{f(-3)}{8} + f'(-3)t_3 \equiv 0(mod\ 2)$. Bunda$f(-3) = 9 \cdot 9 + 29(-3) + 62 = 81 - 87 + 62 = 56$ va$f'(-3) = 18(-3) + 29 = -54 + 29 = -25$ ekanligini e'tiborga olib $7 - 25t_3 \equiv 0(mod\ 2) \rightarrow 1 + t_3 \equiv 0(mod\ 2) \rightarrow t_3 = 1 + 2t_4 \rightarrow x = -3 + 8(1 + 2t_4) = 5 + 16t_4$. $x = 5 + 16t_4$ dan foydalanib navbatdagi qadamni amalga oshiramiz. U holda $\frac{f(5)}{16} + f'(5)t_4 \equiv 0(mod\ 2)$. Bunda $f(5) = 9 \cdot 25 + 29 \cdot 5 + 62 = 225 + 145 + 62 = 432$ va $f'(5) = 18 \cdot 5 + 29 = 119$ bo'lgani uchun $27 + 119t_4 \equiv 0(mod\ 2) \rightarrow t_4 \equiv -1(mod\ 2) \rightarrow t_4 = -1 + 2t_5 \rightarrow x = 5 + 16(-1 + 2t_5) = -11 + 32t_5$. x ning bu qiymatidan foydalanib t_5 ni topish uchun quyidagi taqqoslamani hosil qilamiz: $\frac{f(-11)}{32} + f'(-11)t_5 \equiv$

$0(mod\,2)$. Bu yerda $f(-11) = 9\cdot 121 - 29\cdot 11 + 62 = 1089 - 319 + 62 = 832$ va$f'(-11) = 18\cdot(-11) + 29 = -169$ ekanligini e'tiborga olib $26 - 169t_5 \equiv 0(mod\,2) \to t_5 \equiv 0(mod\,2) \to t_5 = 2t_6 \to x = -11 + 32(2t_6) = -11 + 64t_6,\ t_6 \epsilon Z$. Demak, berilgan taqqoslamaning ikkinchi yechimi $x \equiv -11(mod\,64)$ dan iborat ekan.

Javob: $x \equiv 22(mod\,64),\ x \equiv 53(mod\,64)$.

5). $6x^3 - 7x - 11 \equiv 0(mod\,125)$ taqqoslamani qaraymiz. Bu yerda $125 = 5^3$ bo'lgani uchun 5 moduli bo'yicha taqqoslamani qaraymiz.

$6x^3 - 7x - 11 \equiv 0(mod\,5) \to x^3 - 2x - 1 \equiv 0(mod\,5)$.

Bu taqqoslamani 5 moduli bo'ycha chegirmalarning to'la sistemasi $0, \pm 1, \pm 2$ dagi sonlarni qo'yib sinab ko'rish usuli bilan yechamiz. U holda $x_1 \equiv -1(mod\,5), x_2 \equiv -2(mod\,5)$ larning qaralayotgan taqqoslamaning yechimi ekanligini topamiz.

a). Avvalo, $x \equiv -1(mod\,5)$ ya'ni $x = -1 + 5t_1$ yechimni qaraymiz. (7) ga asosan $\frac{f(-1)}{5} + f'(-1)t_1 \equiv 0(mod\,5)$ taqqoslamani hosil qilamiz. Bu yerda

$f(-1) = -6 + 7 - 11 = -10$, $f'(-1) = 18\cdot(-1)^2 - 7 = 11$ bo'lgani uchun $-2 + 11t_1 \equiv 0(mod\,5) \to t_1 \equiv 2(mod\,5) \to t_1 = 2 + 5t_2 \to x = -1 + 5(2 + 5t_2) = 9 + 25t_2$ ni hosil qilamiz. Endi 25 moduli bo'yicha taqqoslamadan 125 moduli bo'yicha taqqoslamaga o'tish uchun $\frac{f(9)}{25} + f'(9)t_2 \equiv 0(mod\,5)$ ni tuzib olamiz. Bunda $f(9) = 6\cdot 9^3 - 7\cdot 9 - 11 = 6\cdot 729 - 63 - 11 = 4300$ va$f'(9) = 18\cdot 9^2 - 7 = 18\cdot 81 - 7 = 1451$ ekanligini e'tiborga olsak $172 + 1451t_2 \equiv 0(mod\,5)$ ga ega bo'lamiz. Bundan $t_2 \equiv -2(mod\,5) \to t_2 = -2 + 5t_3 \to x = 9 + 25(-2 + 5t_3) = -41 + 125t_3$. Demak, berilgan taqqoslamaning bitta yechimi $x \equiv -41(mod\,125)$.

b). Endi $x \equiv -2(mod\,5)$ yechimni qaraymiz. Bundan $x = -2 + 5t_1$ va (7) ga asosan $\frac{f(-2)}{5} + f'(-2)t_1 \equiv 0(mod 5)$. Bunda $f(-2) = 6\cdot(-8) - 7\cdot(-2) - 11 = -48 + 14 - 11 = -45$ va $f'(-2) = 18\cdot 4 - 7 = 65$ ekanligini inobatga olsak, $-9 + 65t_1 \equiv 0(mod\,5) \to 65t_1 \equiv 9(mod\,5)$. Bu yerda $(65, 5) = 5$, lekin 9 soni 5 ga bo'linmaydi shuning uchun bu taqqoslama yechimga ega emas.

Javob: $x \equiv -4(mod\,125)$.

6). $x^3+3x^2-5x+16\equiv 0(mod\ 125)$ taqqoslamani qaraymiz. Bu yerda $125=5^3$ bo'lgani uchun 5 moduli bo'yicha taqqoslamani qaraymiz.

$x^3+3x^2-5x+16\equiv 0(mod\ 5)\rightarrow x^3+3x^2+1\equiv 0(mod\ 5)$.

Bu taqqoslamani 5 moduli bo'ycha chegirmalarning to'la sistemasi $0,\pm1,\pm2$ dagi sonlarni qo'yib sinab ko'rish usuli bilan yechamiz. U holda $x_1\equiv 1(mod\ 5)$,$x_2\equiv -2(mod\ 5)$ larning qaralayotgan taqqoslamaning yechimi ekanligini topamiz.

$a)$. $x\equiv 1(mod\ 5)$ ni, ya'ni $x=-1+5t_1$ yechimni qaraymiz. (7) ga asosan, ya'ni $\frac{f(1)}{5}+f'(1)t_1\equiv 0(mod\ 5)$. Bu yerda $f(1)=15$, $f'(1)=3\cdot1^2+6\cdot1-5=4$ bo'lgani uchun $3+4t_1\equiv 0(mod\ 5)\rightarrow 4t_1\equiv 2(mod\ 5)\rightarrow 2t_1\equiv 1(mod\ 5)\rightarrow t_1\equiv 3(mod\ 5)\rightarrow$ $t_1=3+5t_2\rightarrow x=1+5(3+5t_2)=16+25t_2$. x ning bu qiymatidan foydalanib t_2 ni topish uchun quyidagi taqqoslamani hosil qilamiz: $\frac{f(16)}{25}+f'(16)t_2\equiv 0(mod\ 5)$. Bu yerda $f(16)=16^3+3\cdot16^2-5\cdot 16+16=16(256+48-4)=4800$ va $f'(16)=3\cdot256+96-5=859$ ekanligini e'tiborga olsak $192+859t_2\equiv 0(mod\ 5)$ hosil bo'ladi. Bundan $t_2\equiv 2(mod\ 5)\rightarrow t_2=2+5t_3\rightarrow x=16+25(2+5t_3)=66+125t_3\rightarrow x\equiv 66(mod\ 125)$. Demak, berilgan taqqoslamaning bitta yechimi $x\equiv 66(mod\ 125)$ dan iborat.

b). Endi $x\equiv -2(mod\ 5)$ yechimni qaraymiz: $x=-2+5t_1$ dan (7) ga asosan

$\frac{f(-2)}{5}+f'(-2)t_1\equiv 0(mod\ 5)$. Bunda $f(-2)=-8+12+10+16=30$ va $f'(-2)=3\cdot4+6\cdot(-2)-5=-5$ ekanligini e'tiborga olib $6-5t_1\equiv 0(mod\ 5)$ ni hosil qilamiz. Bundan $5t_1\equiv 1(mod\ 5)$, bunda $(5,5)=5$ bo'lib, 1 soni 5 ga bo'linmagani uchun taqqoslama yechimga ega emas.

Javob: $x\equiv 66(mod\ 125)$.

7). $x^4+4x^3+2x^2+x+12\equiv 0(mod\ 625)$ taqqoslamani qaraymiz. Bu yerda $625=5^4$ bo'lgani uchun 5 moduli bo'yicha taqqoslamani qaraymiz.

$x^4+4x^3+2x^2+x+12\equiv 0(mod5)\rightarrow x^4-x^3+2x^2+x+2\equiv 0(mod\ 5)$. Bu taqqoslamani 5 moduli bo'ycha chegirmalarning to'la sistemasi $0,\pm1,\pm2$ dagi sonlarni qo'yib sinab ko'rish usuli bilan yechamiz. U holda $x_1\equiv 1(mod\ 5)$, $x_2\equiv -1(mod\ 5)$ va $x_3\equiv$

$2(mod\ 5)$ larning qaralayotgan taqqoslamaning yechimi ekanligini topamiz.

a). $x \equiv 1(mod\ 5)$ yechimni qaraymiz. $x = 1 + 5t_1$ dan (7) ga asosan, ya'ni $\frac{f(1)}{5} + f'(1)t_1 \equiv 0(mod\ 5)$. Bu yerda $f(1) = 20,\ f'(1) = 21$ bo'lgani uchun $4 + 21t_1 \equiv 0(mod\ 5) \to t_1 \equiv 1(mod\ 5) \to t_1 = 1 + 5t_2 \to x = 1 + 5(1 + 5t_2) = 6 + 25t_2$. x ning bu qiymatidan foydalanib t_2 ni topish uchun quyidagi taqqoslamani hosil qilamiz: $\frac{f(6)}{25} + f'(6)t_2 \equiv 0(mod\ 5)$. Bunda $f(6) = 90$ va

$f'(6) = 4 \cdot 6^3 + 12 \cdot 6^2 + 4 \cdot 6 + 1 = 824 + 432 + 25 = 1281$ bo'lgani uchun $90 + 1281t_2 \equiv 0(mod\ 5) \to t_2 \equiv 0(mod\ 5) \to t_2 = 5t_3 \to x = 6 + 125t_3$. Endi x ning bu qiymatidan foydalanib t_3 ni topish uchun quyidagi taqqoslamani hosil qilamiz: $\frac{f(6)}{125} + f'(6)t_3 \equiv 0(mod\ 5)$. Bundan $18 + 1281t_3 \equiv 0(mod\ 5) \to t_3 \equiv 2(mod\ 5) \to t_3 = 2 + 5t_4 \to x = 6 + 125(2 + 5t_4) = 256 + 625t_4$. Demak, berilgan taqqoslamaning bitta yechimi $x = 256 + 625t_4$.

b) Endi $x \equiv -1(mod\ 5)$, ya'ni $x = -1 + 5t_1$ yechimni qaraymiz. Bu holda (7) ga asosan $\frac{f(-1)}{5} + f'(-1)t_1 \equiv 0(mod\ 5)$ni hosil qilamiz. Bu yerda $f(-1) = 1 - 4 + 2 - 1 + 12 = 10$ va $f'(-1) = -4 + 12 - 4 + 1 = 5$ bo'lgani uchun

$2 + 5t_1 \equiv 0(mod\ 5) \to 5t_1 \equiv 3(mod\ 5)$ ga ega bo'lamiz. Bu yerda $(5,5) = 5$ va 3 soni 5 ga bo'linmaydi shuning uchun taqqoslama yechimga ega emas.

c) $x \equiv 2(mod 5)$, ya'ni $x = 2 + 5t_1$ yechimni qaraymiz. Bu holda (7) ga asosan $\frac{f(2)}{5} + f'(2)t_1 \equiv 0(mod\ 5)$ni hosil qilamiz. Bu yerda $f(2) = 16 + 32 + 8 + 2 + 12 = 70$ va $f'(x) = 4x^3 + 12x^2 + 4x + 1 \to f'(2) = 32 + 48 + 9 = 89$ bo'lgani uchun $14 + 89t_1 \equiv 0(mod 5) \to 4t_1 \equiv 1(\text{mod } 5) \to t_1 \equiv -1(\text{mod } 5) \to t_1 = -1 + 5t_2 \to x = 2 + 5(1 + 5t_2) = -3 + 25t_2$ ni hosil qilamiz. x ning bu qiymatidan foydalanib t_2 ni topish uchun quyidagi taqqoslamani hosil qilamiz: $\frac{f(-3)}{25} + f'(-3)t_2 \equiv 0(mod\ 5)$. Bu yerda $f(-3) = 81 - 108 + 18 - 3 + 12 = 0$ va $f'(-3) = 4 \cdot (-27) + 12 \cdot 9 - 12 + 1 = -108 + 108 - 11 = -11$ bo'lgani uchun $-11t_2 \equiv 0(\text{mod } 5) \to t_2 \equiv 0(\text{mod } 5) \to t_2 = 5t_3 \to x = -3 + 25 \cdot 5t_3 = -3 + 125t_3$. Bundan foydalanib t_3 ni topish uchun $\frac{f(-3)}{125} + f'(-3)t_3 \equiv 0(mod\ 5)$ taqqoslamani tuzib olamiz. Bu yerdan $-11t_3 \equiv 0(\text{mod } 5) \rightarrow t_3 = 5t_4 \rightarrow$ $x = -3 + 625t_4$ kelib chiqadi.

Javob: $x = 256 + 625t_4\ x = -3 + 625t_4,\ t_4 \in Z.$

8). $f(x) = 2x^4 + 5x - 1,\ f(x) \equiv 0\ (\text{mod } 27)$ taqqoslamani yeching. $27 = 3^3$.

$f(x) \equiv 0(\text{mod} 3)$, $(0, \pm 1)$ taqqoslama bitta $x \equiv 1(\text{mod} 3)$ yechimga ega. Bu yerda $f'(x) = 8x^3 + 5$ va $f'(x_1) = 13$ va 13 soni 3 ga bo'linmaydi. Demak, A holga to'g'ri keladi.

$x = 1 + 3t_1,\ f(1) + 3t_1 \cdot f'(1) \equiv 0(\text{mod} 9),\ 6 + 3t_1 \cdot 13 \equiv 0(\text{mod} 9), 13t_1 \equiv -2(\text{mod} 3),\ t_1 \equiv -2(\text{mod} 3),$

$t_1 = -2 + 3t_2$. Demak, $x = 1 + 3(-2 + 3t_2) = -5 + 9t_2,\ f(-5) + 9t_2 f'(-5) \equiv 0(\text{mod } 27)$

$1224 + 9t_2(-995) \equiv 0(\text{mod } 27),\ -995t_2 \equiv -136(\text{mod } 3),\ t_2 \equiv -1(\text{mod } 3),\ t_2 = -1 + 3t_3,\ t_3 \in Z$

$x = -5 + 9(-1 + 3t_3) = -14 + 27t_3$. Demak, $x \equiv 13(\text{mod } 27)$

berilgan taqqoslamaning yechimi.

291. 1). $x^4 + 4x^3 + 2x^2 + x + 12 \equiv 0(mod\ 45)$ ni qaraymiz.

Bu taqqoslama

$$\begin{cases} x^4 + 4x^3 + 2x^2 + x + 12 \equiv 0(mod\ 5) \\ x^4 + 4x^3 + 2x^2 + x + 12 \equiv 0(mod\ 9) \end{cases}$$

ga teng kuchli. Buning birinchisining yechimlari: $x \equiv \pm 1(mod\ 5)$ va $x \equiv 2(mod\ 5)$. (290.1) misolning ishlanishiga qarang). Endi sistemadagi ikkinchi taqqoslamani yechamiz. Buning uchun avvalo, $x^4 + 4x^3 + 2x^2 + x + 12 \equiv 0(mod\ 3)$ taqqoslamani yechamiz. Bu taqqoslama $x^4 + x^3 - x^2 + x \equiv 0(mod\ 3)$ ga teng kuchli. Buni 3 moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm 1$ dagi chegirmalarni sinab ko'rish usuli bilan yechsak $x \equiv 0(mod\ 3)$, ya'ni $x = 3t_1$ uning yechimi ekanligini topamiz. Bu holda (7) ga asosan $\frac{f(0)}{3} + f'(0)t_1 \equiv 0(mod\ 3)$ ni hosil qilamiz. Bu yerda $f(0) = 12$ va $f'(0) = 1$ bo'lgani uchun $4 + t_1 \equiv 0(\text{mod } 3) \rightarrow t_1 \equiv -1(\text{mod } 3) \rightarrow t_1 = -1 + 3t_2 \rightarrow x \equiv -3 + 9t_2$ ni hosil qilamiz. Natijada biz berilgan taqqoslama quyidagi taqqoslamalar sistemasiga ekvivalent deya olamiz:

$$a)\begin{cases} x \equiv 1(mod\ 5) \\ x \equiv -3(mod\ 9) \end{cases};\quad b)\begin{cases} x \equiv -1(mod\ 5) \\ x \equiv -3(mod\ 9) \end{cases};$$

$$c)\begin{cases} x \equiv 2(mod\ 5) \\ x \equiv -3(mod\ 9) \end{cases}.$$

Bu sistemalarni yechamiz.U holda :

$$a)\ \begin{cases} x = 1 + 5t_1 \\ 1 + 5t_1 \equiv -3(mod\ 9) \end{cases} \to$$

$$\begin{cases} x = 1 + 5t_1 \\ 5t_1 \equiv -4(mod\ 9) \end{cases} \to \begin{cases} x = 1 + 5t_1 \\ t_1 \equiv 1(mod\ 9) \end{cases} \to$$

$x = 1 + 5(1 + 9t_2) = 6 + 45t_2$, ya'ni $x \equiv -21(\text{mod } 45)$.

$$b)\begin{cases} x = -1 + 5t_1 \\ -1 + 5t_1 \equiv -3(mod\ 9) \end{cases} \to \begin{cases} x = -1 + 5t_1 \\ 5t_1 \equiv -4(mod\ 9) \end{cases} \to$$

$$\begin{cases} x = -1 + 5t_1 \\ t_1 \equiv -4 + 9t_2 \end{cases} \to$$

$x = -1 + 5(-4 + 9t_2) = 6 + 45t_2 \to x \equiv -21(\text{mod } 45)$.

$$c)\begin{cases} x = 2 + 5t_2 \\ 2 + 5t_1 \equiv -3(mod\ 9) \end{cases} \to \begin{cases} x = 2 + 5t_1 \\ t_1 = -1 + 9t_2 \end{cases} \to x = 2 +$$

$5(-1 + 9t_2) = -3 + 43t_2 \to x \equiv -3(mod\ 45)$.

Javob: $x \equiv 6, 24, 42(mod\ 45)$.

2). $f(x) = x^4 - 3x^3 - 4x^2 - 2x - 2 \equiv 0(mod\ 50)$ taqqoslamani qaraymiz. Bu yerda $50 = 2 \cdot 5^2$ bo'lgani uchun bu taqqoslama

$$\begin{cases} f(x) \equiv 0(mod\ 2) \\ f(x) \equiv 0(mod\ 25) \end{cases}$$

ga teng kuchli. Buning birinchisining yechimlari: $x \equiv 0(mod\ 2)$ va $x \equiv 1(mod\ 5)$. Endi $f(x) = 0(mod\ 25)$ taqqoslamani yechamiz. Buning uchun esa $f(x) \equiv x^4 + 2x^3 + 2x^2 + 3x - 2 \equiv 0(mod\ 5)$ ni qaraymiz. Buni 5 moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm1, \pm2$ dagi chegirmalarni sinab ko'rish usuli bilan yechsak $x \equiv -1, 1, 2(mod\ 5)$ lar uning yechimi ekanligini topamiz.

a). Avvalo, $x \equiv 1(mod\ 5)$, ya'ni $x = 1 + 5t_1$ yechimni qaraylik. Bu holda (7) ga asosan $\frac{f(1)}{5} + f'(1)t_1 \equiv 0(mod\ 5)$ ni hosil qilamiz. Bu yerda $f(1) = 10$ va $f'(1) = 4 \cdot 1^3 - 9 \cdot 1^2 - 8 \cdot 1 - 2 = -15$ bo'lgani uchun $2 - 15t_1 \equiv 0(mod\ 5)$, $15t_1 \equiv 2(mod\ 5) \to (15,5) = 5$, lekin 2 soni 5ga bo'linmaydi shuning uchun taqqoslama yechimga ega emas.

b). $x \equiv -1(mod\ 5)$ ni, ya'ni $x = -1 + 5t_2$ ni qaraymiz. (7) ga asosan

$\frac{f(-1)}{5} + f'(-1)t_1 \equiv 0(mod\ 5)$. Bunda $f'(-1) = 1 + 3 - 4 + 2 - 2 = 0$ va

$f'(-1)=4\cdot(-1)^3-9\cdot(-1)^2-8\cdot(-1)-2=-7$ bo'lgani uchun $-7t_1\equiv 0(mod\,5)\to t_1\equiv 0(mod\,5)\to t_1=5t_2\to x=-1+5(5t_2)=-1+25t_2\ \to x\equiv -1(mod\,25)$.

c) Endi $x\equiv 2(\text{mod }5)$, ya'ni $x=2+5t$ yechimni qaraylik. Bu holda (7) ga asosan $\frac{f(2)}{5}+f'(2)t_1\equiv 0(mod\,5)$ ni hosil qilamiz. Bu yerda $f(2)=-30$ va $f'(2)=-22$ bo'lgani uchun $-6-22t_1\equiv 0(mod\,5)\ \to\ 3t_1\equiv 1(mod\,5)\to t_1\equiv 2(mod\,5)\to\ t_1=2+5t_2\to x=2+5(2+5t_2)=12+25t_2$.

Demak, berilgan taqqoslamani yechishni

$$\begin{cases} x\equiv 0(mod\,2)\\ x\equiv -1(mod\,25)\end{cases};$$

$$\begin{cases} x\equiv 0(mod\,2)\\ x\equiv 12(mod\,25)\end{cases};\ \begin{cases} x\equiv 1(mod\,2)\\ x\equiv -1(mod\,25)\end{cases};\begin{cases} x\equiv 2(mod\,2)\\ x\equiv 12(mod\,25)\end{cases}$$

taqqoslamalar sistemalarini yechishga keltirdik. Buning birinchisidan

$$\begin{cases} x=2t_1\\ 2t_1\equiv -1(mod\,25)\end{cases}\to\begin{cases} x=2t_1\\ t_1\equiv 12(mod\,25)\end{cases}\to x=24+50t_2.$$

Ikkinchisidan $\begin{cases} x=2t_1\\ 2t_1\equiv 12(mod\,25)\end{cases}\to\begin{cases} t_1\equiv 6(mod\,25)\\ x=12+50t_2\end{cases}$.

Uchinchisidan

$$\begin{cases} x=1+2t_1\\ 1+2t_1\equiv -1(mod\,25)\end{cases}\to\begin{cases} x=2t_1\\ t_1\equiv -1(mod\,25)\end{cases}\to x=-1+50t_2.$$

To'rtinchisidan $\begin{cases} x=1+2t_1\\ 1+2t_1\equiv 12(mod\,25)\end{cases}\to\begin{cases} x=1+2t_1\\ t_1=18+25t_2\end{cases}\to$

$$x=1+2(18+25t_2)=37+50t_3.$$

Javob: $x\equiv 12,24,37,49$ (mod 50).

3). $f(x)=x^5-5x^4-5x^3+25x^2+4x-20\equiv 0(mod\,147)$ taqqoslamani qaraymiz. Bu yerda $147=7^2\cdot 3$ bo'lgani uchun bu taqqoslama

$$\begin{cases} f(x)\equiv 0(mod\,3)\\ f(x)\equiv 0(mod\,7^2)\end{cases}$$

ga teng kuchli. Buning birinchisidan $f(x)=x^5-5x^4-5x^3+25x^2+4x-20\equiv 0(mod\,3)\to x^5+x^4+x^3+x^2+x+1\equiv 0(mod\,3)\to x+x^2+x+x^2+x+1=2x^2+3x+1\equiv 2x^2+1\equiv 0(mod\,3)$. Uning yechimlari: $x\equiv -1(mod\,3)$va $x\equiv 1(mod\,3)$.

Endi $f(x)=0(mod\,49)$ taqqoslamani yechamiz. Buning uchun esa $f(x)\equiv 0(mod\,7)$ ning yechimini topishimiz kerak. Bundan x^5+2x^4+

$2x^3+4x^2+4x+1\equiv 0(mod\ 7)$. Buni 7moduli bo'yicha chegirmalarning to'la sistemasi $0,\pm1,\pm2,\pm3$ dagi chegirmalarni sinab ko'rish usuli bilan yechsak $x\equiv -1,1,-2,2(mod\ 7)$ lar uning yechimi ekanligini topamiz. Bu 7 moduli bo'yicha topilgan yechimlardan 49 moduli bo'yicha yechimga o'tish uchun ularning har birini alohida-alohida qarab chiqamiz.

a). $x\equiv 1(\text{mod }7)$ yechim uchun (7) dan $\frac{f(1)}{7}+f'(1)t_1\equiv 0(mod\ 7)$ ni hosil qilamiz. Bu yerda $f(1)=0$ va $f'(1)=5\cdot1^4-20\cdot 1^3-15\cdot1^2+50\cdot1+4=24$ bo'lgani uchun $24t_1\equiv 0(mod\ 7)\to$ $t_1\equiv 0(mod\ 7)\to t_1=7t_2\to$

$x=1+49t_2\to x\equiv 1(\text{mod }49)$.

b). Endi ikkinchi yechimni $x\equiv -1(\text{mod }7)\to x=-1+7t_1$ qaraymiz.

(7) dan $\frac{f(-1)}{7}+f'(-1)t_1\equiv 0(mod\ 7)$ ni hosil qilamiz. Bu yerda $f(-1)=0$ va $f'(-1)=-36$ bo'lgani uchun $36t_1\equiv 0(\text{mod }7)\to$ $t_1=7t_2\to x=-1+49t_2\ \to\ x\equiv -1(\text{mod }49)$.

c). Uchinchi yechim $x\equiv 2(mod\ 7)\to x=2+7t_1$uchun (7) dan $\frac{f(2)}{7}+f'(2)t_1\equiv 0(mod\ 7)$. Bunda$f(2)=0$ va$f'(2)=-36$ bo'lgani uchun $-36t_1\equiv 0(mod\ 7)\to t_1=7t_2\to x=2+49t_2\ \to x\equiv 2(mod\ 49)$.

e). To'rtinchi yechim $x\equiv -2(\text{mod }7)\to x=-2+7t_1$uchun $\frac{f(-2)}{7}+f'(-2)t_1\equiv 0(mod\ 7)$, bunda$f(-2)=0$va $f'(-2)=84$ bo'lgani uchun $84t_1\equiv 0(mod\ 7)\,, t_1\epsilon Z\ \to x=-2+49t_2$ ga ega bo'lamiz.

Bulardan

$$\begin{cases}x\equiv -1(mod\ 3)\\ x\equiv 1(mod\ 49)\end{cases}, \begin{cases}x\equiv -1(mod\ 3)\\ x\equiv -1(mod\ 49)\end{cases}, \begin{cases}x\equiv -1(mod\ 3)\\ x\equiv 2(mod\ 49)\end{cases},$$

$$\begin{cases}x\equiv -1(mod\ 3)\\ x\equiv -2(mod\ 49)\end{cases},$$

$$\begin{cases}x\equiv 1(mod\ 3)\\ x\equiv 1(mod\ 49)\end{cases}, \begin{cases}x\equiv 1(mod\ 3)\\ x\equiv -1(mod\ 49)\end{cases},$$

$$\begin{cases}x\equiv 1(mod\ 3)\\ x\equiv 2(mod\ 49)\end{cases}, \begin{cases}x\equiv 1(mod\ 3)\\ x\equiv -2(mod\ 49)\end{cases}$$

chiziqli taqqoslamalar sistemalariga ega bo'lamiz.

Bularni yechib:

$$a_1)\begin{cases} x = -1+3t_1 \\ -1+3t_1 \equiv 1(mod\ 49) \end{cases} \rightarrow$$

$$\begin{cases} x = -1+3t_1 \\ 3t_1 \equiv 2(mod\ 49) \end{cases} \rightarrow \begin{cases} t_1 = 17+49t_2 \\ x = 50+147t_2 \end{cases}.$$

$$a_2)\begin{cases} x = -1+3t_1 \\ -1+3t_1 \equiv -1(mod\ 49) \end{cases} \rightarrow \begin{cases} t_1 = 49t_2 \\ x = -1+147t_2 \end{cases}.$$

$$a_3)\begin{cases} x = -1+3t_1 \\ -1+3t_1 \equiv 2(mod\ 49) \end{cases} \rightarrow \begin{cases} t_1 \equiv 1(mod\ 49) \\ x = 2+147t_2 \end{cases}.$$

$$a_4)\begin{cases} x = -1+3t_1 \\ -1+3t_1 \equiv -2(mod\ 49) \end{cases} \rightarrow \begin{cases} 3t_1 \equiv -1(mod\ 49) \\ x = -1+3t_1 \end{cases} \rightarrow$$

$$\begin{cases} t_1 \equiv 16+49t_2 \\ x = -1+48+147t_2 \end{cases} \rightarrow x = 47+147t_2.$$

$$a_5)\begin{cases} x = 1+3t_1 \\ 1+3t_1 \equiv 1(mod\ 49) \end{cases} \rightarrow \begin{cases} t_1 \equiv 0(mod\ 49) \\ x = 1+147t_2 \end{cases}.$$

$$a_6)\begin{cases} x = 1+3t_1 \\ 1+3t_1 \equiv -1(mod\ 49) \end{cases}$$

$$\rightarrow \begin{cases} 3t_1 \equiv -2(mod\ 49) \\ x = 1+3t_1 \end{cases} \rightarrow \begin{cases} t_1 \equiv -17+49t_2 \\ x = -50+147t_2 \end{cases}.$$

$$a_7)\quad \begin{cases} x = 1+3t_1 \\ 1+3t_1 \equiv 2(mod\ 49) \end{cases}$$

$$\rightarrow \begin{cases} 3t_1 \equiv 1(mod\ 49) \\ x = 1+3t_1 \end{cases}$$

$$\rightarrow \begin{cases} t_1 \equiv -16+49t_2 \\ x = -47+147t_2 \end{cases}. \quad a_8)\begin{cases} x = 1+3t_1 \\ 1+3t_1 \equiv -2(mod\ 49) \end{cases}$$

$$\rightarrow \begin{cases} 3t_1 \equiv -3(mod\ 49) \\ x = 1+3t_1 \end{cases} \rightarrow \begin{cases} t_1 = -1+49t_2 \\ x = -2+147t_2 \end{cases}.$$

Javob: $x \equiv -50, -47, -2, -1, 1, 2, 47, 50\ (mod\ 147)$.

4). $f(x) = x^5 + 3x^4 - 7x^3 + 4x^2 + 4x - 10 \equiv 0(mod\ 175)$ taqqoslamani qaraymiz. Bu yerda $175 = 7 \cdot 5^2$ bo'lgani uchun bu taqqoslama

$$\begin{cases} f(x) \equiv 0(mod\ 7) \\ f(x) \equiv 0(mod\ 25) \end{cases} \qquad (2)$$

ga teng kuchli. Buning birinchisidan $x^5 + 3x^4 - 3x^2 - 3x - 3 \equiv 0(mod\ 7)$. Buni 7 moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm 1, \pm 2, \pm 3$ dagi chegirmalarni sinab ko'rish usuli bilan yechsak $x \equiv 1, -2, -3(mod\ 7)$ laruning yechimi ekanligini topamiz.

Endi (1) dagi ikkinchi taqqoslamani yechamiz. Buning uchun avvalo, $f(x) \equiv 0(\mathrm{mod}\ 25)$ ni, ya'ni $x^5 + 3x^4 - 7x^3 + 4x^2 + 4x - 10 \equiv 0(mod\ 5)$ ni qaraymiz. Bu taqqoslama $3x^4 - 2x^3 - x^2 \equiv 0(mod\ 5) \rightarrow x^2(3x^2 - 2x - 1) \equiv 0(mod\ 5)$ ga teng kuchli. Buni 5 moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm 1, \pm 2$ dagi chegirmalarni sinab ko'rish usuli bilan yechsak $x \equiv 0,1,-2(mod\ 5)$ lar uning yechimi ekanligini topamiz.

Shuning uchun ham birinchi yechim $x \equiv 0(mod\ 5)$ uchun (7) ga asosan $\frac{f(0)}{5} + f'(0)t_1 \equiv 0(mod\ 5)$. Bu yerda $f(0) = -10$ va $f'(0) = 4$ bo'lganidan

$$-2 + 4t_1 \equiv 0(mod\ 5) \rightarrow\ 4t_1 \equiv 2(mod\ 5) \rightarrow\ 2t_1 \equiv 1(mod\ 5) \rightarrow t_1 \equiv 3(mod\ 5) \rightarrow t_1 = 3 + 5t_2\ \rightarrow\ x = 15 + 25t_2.$$

Ikkinchi yechim $x \equiv 1(mod\ 5)$ uchun $\frac{f(1)}{5} + f'(1)t_1 \equiv 0(mod\ 5)$. Bu yerda $f(1) = -5$ va $f'(1) = 8$ bo'lganidan $-1 + 8t_1 \equiv 0(mod\ 5) \rightarrow\ 8t_1 \equiv 1(mod\ 5) \rightarrow\ -2t_1 \equiv 6(mod\ 5) \rightarrow t_1 \equiv -3(mod\ 5) \rightarrow t_1 = -3 + 5t_2\ \rightarrow\ x = 1 + 5t_1 = 1 - 15 + 25t_2 = -14 + 25t_2\ \rightarrow x \equiv 11(mod 25)$.

Uchunchi yechim $x \equiv -2(mod\ 5)$ uchun $\frac{f(-2)}{5} + f'(-2)t_1 \equiv 0(mod\ 5)$. Bu yerda $f(-2) = 70$ va $f'(-2) = -112$ bo'lganidan $14 - 112t_1 \equiv 0(mod\ 5) \rightarrow\ -1 +\ 3t_1 \equiv 0(mod5) \rightarrow\ 3t_1 \equiv 1(mod5) \rightarrow 3t_1 \equiv 6(mod5) \rightarrow t_1 \equiv 2(mod5) \rightarrow x = -2 + 5t_1 = -2 + 5(2 + 5t_2) = 7 + 25t_2$.

Shunday qilib, berilgan taqqoslamani yechishni quyidagi 1-darajali bir noma'lumli taqqoslamalar sistemalarni yechishga keltirdik:

$$a_1)\begin{cases} x \equiv 1(mod\ 7) \\ x \equiv 15(mod\ 25)\end{cases}, a_2)\begin{cases} x \equiv -2(mod\ 7) \\ x \equiv 15(mod\ 25)\end{cases}, \quad a_3)\begin{cases} x \equiv -3(mod\ 7) \\ x \equiv 15(mod\ 25)\end{cases}$$

$$a_4)\begin{cases} x \equiv 1(mod\ 7) \\ x \equiv 11(mod\ 25)\end{cases}, a_5)\begin{cases} x \equiv -2(mod\ 7) \\ x \equiv 11(mod\ 25)\end{cases}, \quad a_6)\begin{cases} x \equiv -3(mod\ 7) \\ x \equiv 11(mod\ 25)\end{cases}$$

$$a_7)\begin{cases} x \equiv 1(mod\ 7) \\ x \equiv 7(mod\ 25)\end{cases}, a_8)\begin{cases} x \equiv -2(mod\ 7) \\ x \equiv 7(mod\ 25)\end{cases}, \quad a_9)\begin{cases} x \equiv -3(mod\ 7) \\ x \equiv 7(mod\ 25)\end{cases}.$$

Bularni yechib: $a_1)\begin{cases} x = 1 + 7t_1 \\ 1 + 7t_1 \equiv 15(\mathrm{mod}\ 25)\end{cases} \rightarrow$

$$\begin{cases} 7t_1 \equiv 14(\mathrm{mod}\ 25) \\ x = 1 + 7t_1\end{cases} \rightarrow \begin{cases} t_1 \equiv 2(\mathrm{mod}\ 25) \\ x = 1 + 7t_1\end{cases} \rightarrow \begin{cases} t_1 \equiv 2 + 25t_2 \\ x = 1 + 7t_1\end{cases} \rightarrow x = 1 + 7(2 + 25t_2) = 15 + 175t_2.$$

ya'ni $x \equiv 15(\text{mod } 175)$.

$$a_2)\begin{cases} x = -2 + 7t_1 \\ -2 + 7t_1 \equiv 15(mod\ 25) \end{cases} \to \begin{cases} 7t_1 \equiv 17(mod\ 25) \\ x = -2 + 7t_1 \end{cases} \to$$

$$\begin{cases} t_1 \equiv 6(\text{mod } 25) \\ x = -2 + 7(6 + 25t_2) = 40 + 175t_2 \end{cases}, \text{ ya'ni } x \equiv 40(\text{mod } 175).$$

$$a_3)\begin{cases} x = -3 + 7t_1 \\ -3 + 7t_1 \equiv 15(mod\ 25) \end{cases} \to \begin{cases} 7t_1 \equiv 18(mod\ 25) \\ x = -3 + 7t_1 \end{cases} \to$$

$$\begin{cases} t_1 \equiv -1(mod\ 25) \\ x = -3 + 7(-1 + 25t_2) = -10 + 175t_2 \end{cases} \to x \equiv -10(\text{mod } 175).$$

$$a_4)\begin{cases} x \equiv 1(mod\ 7) \\ x \equiv 11(mod\ 25) \end{cases} \to \begin{cases} x \equiv 1 + 7t_1 \\ 1 + 7t_1 \equiv 11(mod\ 25) \end{cases}$$

$$\to \begin{cases} x \equiv 1 + 7t_1 \\ 7t_1 \equiv 10(mod\ 25) \end{cases} \to$$

$$\begin{cases} x \equiv 1 + 7t_1 \\ t_1 \equiv 5(mod\ 25) \end{cases} \to x \equiv 1 + 7(5 + 25t_2) = 36 + 175t_2.$$

$$a_5)\begin{cases} x \equiv -2(mod\ 7) \\ x \equiv 11(mod\ 25) \end{cases}$$

$$\to \begin{cases} x \equiv -2 + 7t_1 \\ -2 + 7t_1 \equiv 11(mod\ 25) \end{cases} \to \begin{cases} x \equiv -2 + 7t_1 \\ 7t_1 \equiv 13(mod\ 25) \end{cases} \to$$

$$\begin{cases} x \equiv -2 + 7t_1 \\ t_1 \equiv 9(mod\ 25) \end{cases} \to x \equiv -2 + 7(9 + 25t_2) = 61 + 175t_2.$$

$$a_6)\begin{cases} x \equiv -3(mod\ 7) \\ x \equiv 11(mod\ 25) \end{cases}$$

$$\to \begin{cases} x \equiv -3 + 7t_1 \\ -3 + 7t_1 \equiv 11(mod\ 25) \end{cases} \to \begin{cases} x \equiv -3 + 7t_1 \\ 7t_1 \equiv 14(mod\ 25) \end{cases} \to$$

$$\begin{cases} x \equiv -3 + 7t_1 \\ t_1 \equiv 2(mod\ 25) \end{cases} \to x \equiv -3 + 7(2 + 25t_2) = 11 + 175t_2.$$

$$a_7)\begin{cases} x \equiv 1(mod\ 7) \\ x \equiv 7(mod\ 25) \end{cases}$$

$$\to \begin{cases} x \equiv 1 + 7t_1 \\ 1 + 7t_1 \equiv 7(mod\ 25) \end{cases} \to \begin{cases} x \equiv 1 + 7t_1 \\ 7t_1 \equiv 6(mod\ 25) \end{cases} \to$$

$$\begin{cases} x \equiv 1 + 7t_1 \\ t_1 \equiv 8(\text{mod } 25) \end{cases} \to x \equiv 1 + 7(8 + 25t_2) = 57 + 175t_2.$$

$$a_8)\begin{cases} x \equiv -2(mod\ 7) \\ x \equiv 7(mod\ 25) \end{cases}$$

$$\to \begin{cases} x \equiv -2 + 7t_1 \\ -2 + 7t_1 \equiv 7(mod\ 25) \end{cases} \to \begin{cases} x \equiv -2 + 7t_1 \\ 7t_1 \equiv 9(mod\ 25) \end{cases} \to$$

$$\begin{cases} x \equiv -2 + 7t_1 \\ t_1 \equiv 12(mod\ 25) \end{cases} \to x \equiv -2 + 7(12 + 25t_2) = 82 + 175t_2.$$

$$a_9)\begin{cases} x \equiv -3(mod\ 7) \\ x \equiv 7(mod\ 25) \end{cases}$$

$$\to \begin{cases} x \equiv -3 + 7t_1 \\ -3 + 7t_1 \equiv 7(mod\ 25) \end{cases} \to \begin{cases} x \equiv -3 + 7t_1 \\ 7t_1 \equiv 10(mod\ 25) \end{cases} \to$$

$$\begin{cases} x \equiv -3 + 7t_1 \\ t_1 \equiv 5(mod\ 25) \end{cases} \to x \equiv -3 + 7(5 + 25t_2) = 32 + 175t_2.$$

Javob: $x \equiv -10, 11, 15, 32, 36, 40, 57, 61\ 82(\text{mod } 175)$.

5). $f(x) = x^4 - 4x^3 + 2x^2 + x + 6 \equiv 0(mod135)$ taqqoslamani qaraymiz. Bu yerda $135 = 3^3 \cdot 5$ bo'lgani uchun bu taqqoslama

$$\begin{cases} f(x) \equiv 0(mod\ 5) \\ f(x) \equiv 0(mod\ 27) \end{cases} \quad (1)$$

ga teng kuchli. Buning birinchisidan $x^4 - 4x^3 + 2x^2 + x + 6 \equiv x^4 + x^3 + 2x^2 + x + 1 \equiv 0(mod\ 5)$. Buni 5 moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm1, \pm2$ dagi chegirmalarni sinab ko'rish usuli bilan yechsak $x \equiv 2, -2(mod\ 5)$ larining yechimi ekanligini topamiz.

Endi (1) dagi ikkinchi taqqoslamani yechamiz. Buning uchun avvalo, $f(x) \equiv 0(\text{mod } 3)$ ni, ya'ni $x^4 - 4x^3 + 2x^2 + x + 6 \equiv 0(mod\ 3)$ ni qaraymiz. Bu taqqoslama $x^4 - 4x^3 + 2x^2 + x + 6 \equiv 0(mod\ 3) \to x^4 - x^3 - x^2 + x \equiv 0(mod3) \to x(x^3 - x^2 - x + 1) \equiv 0(mod3) \to x(-x^2 + 1) \equiv 0(mod3)$. Bundan $x \equiv 0, \pm1(mod3)$ lar berilgan taqqoslamaning yechimi ekanligini topamiz.

$a_1).x \equiv 0(mod3)$ - ni qaraymiz. $x = 3t_1$ bo'lgani uchun 290-misoldagi (7) –formulaga asosan t_1 ga nisbatan $\frac{f(0)}{3} + f'(0)t_1 \equiv 0(mod3)$ taqqoslamaga ega bo'lamiz. Bunda$f(0) = 6$ va $f'(x) = 4x^3 - 12x^2 + 4x + 1$, $f'(0) = 1$ bo'lgani uchun$2 + t_1 \equiv 0(mod3) \to t_1 \equiv 1(mod3) \to t_1 \equiv 1 + 3t_2 \to x = 3 + 9t_2$. x ning bu qiymatidan foydalanib navbatdagi qadamni amalga oshiramiz. U holda t_2 ga nisbatan $\frac{f(3)}{9} + f'(3)t_2 \equiv 0(mod3)$ taqqoslamaga ega bo'lamiz. Bunda $f(3) = 0, f'(3) = 13$ b'olgani uchun $13t_2 \equiv 0(mod3)$ ga ega bo'lamiz. Bundan $t_2 = 3t_3 \to x = 3 + 27t_3 \in Z$.

a_2). Endi ikkinchi yechim$x \equiv 1(mod3)$ ni qaraymiz. Bundan $x = 1 + 3t_1$ bo'lgani uchun 290-misoldagi (7) –formulaga asosan t_1 ga nisbatan $\frac{f(1)}{3} + f'(1)t_1 \equiv 0(mod3)$ taqqoslamaga ega bo'lamiz.

Bunda $f(1) = 6$ va $f'(x) = f'(1) = -3$ bo'lgani uchun $2 - 3t_1 \equiv 0(mod3) \to 3t_1 \equiv 2(mod3)$ ni hosil qilamiz. Bu yerda $(3,3) = 3$, lekin 2 soni 3ga bo'linmaydi. Shuning uchun ham oxirgi taqqoslama yechimga ega emas.

a_3). Uchunchi yechim $x \equiv -1(mod3)$ ni qaraymiz. Bu holda $f(-1) = 12$, $f'(-1) = -19$ bo'lgani uchun $\frac{f(-1)}{3} + f'(-1)t_1 \equiv 0(mod3)$ dan $4 - 19t_1 \equiv 0(mod3) \to 19t_1 \equiv 4(mod3) \to t_1 \equiv 1(mod3) \to t_1 = 1 + 3t_2$ ni hosil qilamiz. Buni x ning ifodasiga olib borib qo'ysak $x = -1 + 3t_1 = 2 + 9t_2$ ifoda hosil bo'ladi. x ning bu ifodasidan foydalanib keyingi qadamni amalga oshiramiz. U holda t_2 ga nisbatan $\frac{f(2)}{9} + f'(2)t_2 \equiv 0(mod3)$ taqqoslama kelib chiqadi. Bunda $f(2) = 0$, $f'(2) = -7$ bo'lgani uchun $-7t_2 \equiv 0(mod3) \to t_2 \equiv 0(mod3) \to t_2 = 3t_3 \to x = 2 + 27t_3$ ga ega bo'lamiz.

Shunday qilib berilgan taqqoslamani yechishni quyidagi birinchi darajali taqqoslamalar sistemasini yechishga keltirdik:

$$b_1)\begin{cases} x \equiv -2(mod5) \\ x \equiv 3(mod27) \end{cases} \qquad b_2)\begin{cases} x \equiv -2(mod5) \\ x \equiv 2(mod27) \end{cases}$$

$$b_3)\begin{cases} x \equiv 2(mod5) \\ x \equiv 3(mod27) \end{cases} \qquad b_4)\begin{cases} x \equiv 2(mod5) \\ x \equiv 2(mod27) \end{cases}.$$

Endi bularning har birining yechimini izlaymiz.

b_1) dan

$$\begin{cases} x = -2 + 5t_1 \\ -2 + 5t_1 \equiv 3(mod27) \end{cases} \to \begin{cases} x = -2 + 5t_1 \\ 5t_1 \equiv 5(mod27) \end{cases} \to \begin{cases} x = -2 + 5t_1 \\ t_1 \equiv 1(mod27) \end{cases} \to$$

$$\begin{cases} x = -2 + 5t_1 \\ t_1 \equiv 1 + 27t_2 \end{cases} \to x = 3 + 135t_2,\ t_2 \in Z.$$

b_2) dan

$$\begin{cases} x = -2 + 5t_1 \\ -2 + 5t_1 \equiv 2(mod27) \end{cases} \to \begin{cases} x = -2 + 5t_1 \\ 5t_1 \equiv 4(mod27) \end{cases} \to \begin{cases} x = -2 + 5t_1 \\ 5t_1 \equiv 85(mod27) \end{cases} \to$$

$$\begin{cases} x = -2 + 5t_1 \\ t_1 \equiv 17 + 27t_2 \end{cases} \to x = 83 + 135t_2,\ t_2 \in Z.$$

b_3) dan

$$\begin{cases} x = 2 + 5t_1 \\ 2 + 5t_1 \equiv 3(mod27) \end{cases} \to \begin{cases} x = 2 + 5t_1 \\ 5t_1 \equiv 1(mod27) \end{cases} \to \begin{cases} x = 2 + 5t_1 \\ 5t_1 \equiv 55(mod27) \end{cases} \to$$

$$\begin{cases} x = 2 + 5t_1 \\ t_1 \equiv 11 + 27t_2 \end{cases} \to x = 57 + 135t_2,\ t_2 \in Z.$$

b_4) dan

$$\begin{cases} x = 2 + 5t_1 \\ 2 + 5t_1 \equiv 2(mod27) \end{cases} \rightarrow \begin{cases} x = 2 + 5t_1 \\ 5t_1 \equiv 0(mod27) \end{cases} \rightarrow \begin{cases} x = 2 + 5t_1 \\ t_1 \equiv 0(mod27) \end{cases} \rightarrow$$

$$\begin{cases} x = 2 + 5t_1 \\ t_1 \equiv 27t_2 \end{cases} \rightarrow x = 2 + 135t_2,\ t_2 \in Z.$$

Javob: $x \equiv 2,3,57,83(mod135)$.

6).$f(x) = 4x^3 + 7x^2 - 7x - 10 \equiv 0(mod225)$ taqqoslamani qaraymiz. Bu yerda $225 = 3^2 \cdot 5^2$ bo'lgani uchun bu taqqoslama

$$\begin{cases} f(x) \equiv 0(mod\ 3^2) \\ f(x) \equiv 0(mod\ 5^2) \end{cases} \qquad (4)$$

ga teng kuchli.

a). $f(x) = 4x^3 + 7x^2 - 7x - 10 \equiv 0(mod3^2)$ taqqoslamani yechamiz. Buning uchun $4x^3 + 7x^2 - 7x - 10 \equiv 0(mod3) \rightarrow x^3 + x^2 - x - 1 \equiv 0(mod3) \rightarrow x^2 - 1 \equiv 0(mod3) \rightarrow (x + 1)(x - 1) \equiv 0(mod3)$ taqqoslamani yechishimiz kerak. Buning yechimlari $x \equiv -1, 1(mod3)$ dan iborat.

a_1) $x = 1 + 3t_1$ yechimni qaraymiz. Bundan foydalanib t_1 ga nisbatan $\frac{f(1)}{3} + f'(1)t_1 \equiv 0(mod3)$ taqqoslamani tuzib olamiz. Bunda $f(1) = -6$, $f'(x) = 12x^2 + 14x - 7$, $f'(1) = 19$ bo'lgani uchun$-2 + 19t_1 \equiv 0(mod3) \rightarrow t_1 \equiv -1(mod3) \rightarrow t_1 = -1 + 3t_2 \rightarrow x = -2 + 9t_2$.

a_2) $x = -1 + 3t_1$ yechimni qaraymiz. Bundan foydalanib t_1 ga nisbatan $\frac{f(-1)}{3} + f'(-1)t_1 \equiv 0(mod3)$ taqqoslamani tuzib olamiz. Bunda $f(-1) = 0$, $f'(-1) = -9$ bo'lgani uchun$-9t_1 \equiv 0(mod3) \rightarrow 0 \cdot t_1 \equiv 0(mod3)$.Bu taqqoslama ayniy taqqoslama bo'lib t_1 ning ixtiyoriy qiymatini qanoatlantiradi.

Endi (1) dagi ikkinchi taqqoslamani $4x^3 + 7x^2 - 7x - 10 \equiv 0(mod5) \rightarrow -x^3 + 2x^2 - 2x \equiv 0(mod5)$ni qaraymiz. Bundan $x(-x^2 + 2x - 2) \equiv 0(mod5) \rightarrow x \equiv 0, -1, -2(mod5)$.

b_1) $x \equiv 0(mod5) \rightarrow x = 5t_1$ ni qaraymiz. Bundan foydalanib t_1 ga nisbatan $\frac{f(0)}{5} + f'(0)t_1 \equiv 0(mod5)$ taqqoslamani tuzib olamiz. Bunda $f(0) = -10$, $f'(0) = -7$ bo'lgani uchun$-2 - 7t_1 \equiv 0(mod5) \rightarrow 3t_1 \equiv -3(mod5) \rightarrow t_1 = -1 + 5t_2 \rightarrow x = -5 + 25t_2$.

b_2) $x \equiv -1(mod5) \rightarrow x = -1 + 5t_1$ yechimni qaraymiz. Bundan foydalanib t_1 ga nisbatan $\frac{f(-1)}{5} + f'(-1)t_1 \equiv 0(mod5)$ taqqoslamani tuzib olamiz. Bunda $f(-1) = 0$, $f'(-1) = -9$ bo'lgani uchun $-9t_1 \equiv 0(mod5) \rightarrow t_1 \equiv 0(mod5) \rightarrow t_1 = 5t_2 \rightarrow x = -1 + 25t_2$.

$b_3)$ $x \equiv -2(mod5) \rightarrow x = -2 + 5t_1$ yechimni qaraymiz. Bundan foydalanib t_1 ga nisbatan $\frac{f(-2)}{5} + f'(-2)t_1 \equiv 0(mod5)$ taqqoslamani tuzib olamiz. Bunda $f(-2) = 0$, $f'(-2) = 13$ bo'lgani uchun $13t_1 \equiv 0(mod5) \rightarrow t_1 \equiv 0(mod5) \rightarrow t_1 = 5t_2 \rightarrow x = -2 + 25t_2$.

Bulardan quyidagi chiziqli tenglamalar sistemasiga kelamiz.

$$c_1)\begin{cases} x \equiv 7(mod9) \\ x \equiv -5(mod25) \end{cases}; c_2)\begin{cases} x \equiv 7(mod9) \\ x \equiv -1(mod25) \end{cases};$$

$$c_3)\begin{cases} x \equiv 7(mod9) \\ x \equiv -2(mod25) \end{cases}.$$

Bularni yechamiz:

$c_1)$ dan

$$\begin{cases} x \equiv 7 + 9t_1 \\ 7 + 9t_1 \equiv -5(mod25) \end{cases} \rightarrow \begin{cases} x \equiv 7 + 9t_1 \\ 9t_1 \equiv -12(mod25) \end{cases}$$

$$\rightarrow \begin{cases} x \equiv 7 + 9t_1 \\ 3t_1 \equiv -4(mod25) \end{cases} \rightarrow$$

$$\begin{cases} x \equiv 7 + 9t_1 \\ t_1 \equiv 7(mod25) \end{cases} \rightarrow x = 7 + 9(7 + 25t_2) = 70 + 225t_2.$$

$c_2)$ dan

$$\begin{cases} x \equiv 7 + 9t_1 \\ 7 + 9t_1 \equiv -1(mod25) \end{cases} \rightarrow \begin{cases} x \equiv 7 + 9t_1 \\ 9t_1 \equiv -8(mod25) \end{cases}$$

$$\rightarrow \begin{cases} x \equiv 7 + 9t_1 \\ 9t_1 \equiv 117(mod25) \end{cases} \rightarrow$$

$$\begin{cases} x \equiv 7 + 9t_1 \\ t_1 \equiv 13(mod25) \end{cases} \rightarrow x = 7 + 9(13 + 25t_2) = 124 + 225t_2.$$

$c_3)$ dan

$$\begin{cases} x \equiv 7 + 9t_1 \\ 7 + 9t_1 \equiv -2(mod25) \end{cases} \rightarrow \begin{cases} x \equiv 7 + 9t_1 \\ 9t_1 \equiv -9(mod25) \end{cases} \rightarrow \begin{cases} x \equiv 7 + 9t_1 \\ t_1 \equiv -1(mod25) \end{cases} \rightarrow$$

$$\rightarrow x = 7 + 9(-1 + 25t_2) = -2 + 225t_2.$$

Javob: $x \equiv 70; 124; 223(mod225)$.

7). $31x^4 + 57x^3 + 96x + 191 \equiv 0(mod225)$ taqqoslamani qaraymiz. Bu yerda $225 = 3^2 \cdot 5^2$ bo'lgani uchun bu taqqoslama

$$\begin{cases} f(x) \equiv 0(mod\ 3^2) \\ f(x) \equiv 0(mod\ 5^2) \end{cases} \quad (5)$$

ga, ya'ni

$$\begin{cases} 31x^4 + 57x^3 + 96x + 191 \equiv 0(mod3^2) \\ 31x^4 + 57x^3 + 96x + 191 \equiv 0(mod5^2) \end{cases}$$

ga teng kuchli. Birinchi taqqoslamani qaraymiz:

$31x^4 + 57x^3 + 96x + 191 \equiv 0(mod3) \to x^4 - 1 \equiv 0(mod3) \to (x^2-1)(x^2+1) \equiv 0(mod3) \to (x-1)(x+1)(x^2+1) \equiv 0(mod3)$ bo'lgani uchun bu taqqoslamaning yechimlari $x \equiv -1, 1(mod3)$ dan iborat.

$a_1)$ $x = -1 + 3t_1$ yechimni qaraymiz. Bundan foydalanib t_1 ga nisbatan $\frac{f(-1)}{3} + f'(-1)t_1 \equiv 0(mod3)$ taqqoslamani tuzib olamiz. Bunda $f(-1) = 69$, $f'(x) = 124x^2 + 171x + 96$, $f'(-1) = 143$ bo'lgani uchun$23 + 143t_1 \equiv 0(mod3) \to 2t_1 \equiv 1(mod3) \to t_1 \equiv 2(mod3) \to t_1 = -1 + 3t_2 \to x = -4 + 9t_2$.

$a_2)$ $x = 1 + 3t_1$ yechimni qaraymiz. Bundan foydalanib t_1 ga nisbatan $\frac{f(1)}{3} + f'(1)t_1 \equiv 0(mod3)$ taqqoslamani tuzib olamiz. Bunda $f(1) = 375$, $f'(x) = 124x^2 + 171x + 96$, $f'(1) = 391$ bo'lgani uchun$125 + 391t_1 \equiv 0(mod3) \to t_1 \equiv -2(mod3) \to t_1 \equiv 1(mod3) \to t_1 = 1 + 3t_2 \to x = 4 + 9t_2$.Endi (1) dagi ikkinchi taqqoslamani yechamiz. Buning uchun avvalo, $f(x) \equiv 0(\text{mod } 5)$ ni, ya'ni $31x^4 + 57x^3 + 96x + 191 \equiv 0(mod\ 5)$ ni qaraymiz. Bu taqqoslama $x^4 + 2x^3 + x + 1 \equiv 0(mod\ 5)$ ga teng kuchli. Bundan $x \equiv 1, 2(mod5)$ lar berilgan taqqoslamaning yechimi ekanligini topamiz.

$b_1)x \equiv 1(mod5) \to x = 1 + 5t_1$ ni qaraymiz. Bundan foydalanib t_1 ga nisbatan $\frac{f(1)}{5} + f'(1)t_1 \equiv 0(mod5)$ taqqoslamani tuzib olamiz. Bunda $f(1) = 375$, $f'(1) = 391$ bo'lgani uchun$75 + 391t_1 \equiv 0(mod5) \to t_1 \equiv 0(mod5) \to t_1 = 5t_2 \to x = 1 + 25t_2$.

$b_2)x \equiv 2(mod5) \to x = 2 + 5t_1$ ni qaraymiz. Bundan foydalanib t_1 ga nisbatan $\frac{f(2)}{5} + f'(2)t_1 \equiv 0(mod5)$ taqqoslamani tuzib olamiz. Bunda $f(2) = 1335$, $f'(1) = 1772$ bo'lgani uchun $267 + 1772t_1 \equiv 0(mod5) \to 2t_1 \equiv -2(mod5) \to t_1 = -1 + 5t_2 \to x = -3 + 25t_2$.

Bulardan quyidagi chiziqli tenglamalar sistemasiga kelamiz:

$$c_1)\begin{cases} x \equiv 5(mod9) \\ x \equiv 1(mod25) \end{cases}; c_2)\begin{cases} x \equiv 5(mod9) \\ x \equiv -3(mod25) \end{cases};$$

$$c_3)\begin{cases} x\equiv 4(mod9)\\ x\equiv 1(mod25)\end{cases}; c_4)\begin{cases} x\equiv 4(mod9)\\ x\equiv -3(mod25)\end{cases}.$$

Bularni yechamiz:

$c_1)$ dan

$$\begin{cases} x\equiv 5+9t_1\\ 5+9t_1\equiv 1(mod25)\end{cases}\to\begin{cases} x\equiv 5+9t_1\\ 9t_1\equiv -4(mod25)\end{cases}\to\begin{cases} x\equiv 5+9t_1\\ 9t_1\equiv 21(mod25)\end{cases}$$

$$\to$$

$$\begin{cases} x=5+9t_1\\ 3t_1\equiv 7(mod25)\end{cases}\to\begin{cases} x=5+9t_1\\ 3t_1\equiv -18(mod25)\end{cases}\to\begin{cases} x=5+9t_1\\ t_1\equiv -6(mod25)\end{cases}\to$$

$$x=-49+225t_2, t_2\in Z.$$

$c_2)$ dan

$$\begin{cases} 5+9t_1\equiv -3(mod25)\\ x=5+9t_1\end{cases}\to\begin{cases} 9t_1\equiv -8(mod25)\\ x=5+9t_1\end{cases}$$

$$\to\begin{cases} 9t_1\equiv -33(mod25)\\ x=5+9t_1\end{cases}\to$$

$$\begin{cases} 3t_1\equiv -11(mod25)\\ x=5+9t_1\end{cases}$$

$$\to\begin{cases} 3t_1\equiv -36(mod25)\\ x=5+9t_1\end{cases}\to\begin{cases} t_1\equiv -12(mod25)\\ x=5+9t_1\end{cases}\to$$

$$x=-103+225t_2, t_2\in Z.$$

$c_3)$ dan

$$\begin{cases} 4+9t_1\equiv 1(mod25)\\ x=4+9t_1\end{cases}\to\begin{cases} 9t_1\equiv -3(mod25)\\ x=4+9t_1\end{cases}\to\begin{cases} 3t_1\equiv -1(mod25)\\ x=4+9t_1\end{cases}$$

$$\begin{cases} t_1\equiv 8(mod25)\\ x=4+9(5+25t_2)=76+225t_2\end{cases}\to x=76+225t_2, t_2\in Z.$$

$c_4)$ dan

$$\begin{cases} 4+9t_1\equiv -3(mod25)\\ x=4+9t_1\end{cases}\to\begin{cases} 9t_1\equiv -7(mod25)\\ x=4+9t_1\end{cases}\to$$

$$\begin{cases} t_1\equiv 2(mod25)\\ x=4+9t_1\end{cases}\to\qquad x=22+225t_2, t_2\in Z.$$

Javob: $x\equiv -103,-49,22,76(mod225).$

8). $f(x)=2x^6-6x^4-7x^2-4\equiv 0(mod\ 441)$ taqqoslamani qaraymiz. Bu yerda $441=3^2\cdot 7^2$ bo'lgani uchun bu taqqoslama

$$\begin{cases} f(x)\equiv 0(mod\ 9)\\ f(x)\equiv 0(mod\ 49)\end{cases}\qquad(6)$$

ga teng kuchli. Buning birinchisidan $2x^6-6x^4-7x^2-4\equiv 0(mod\ 3^2)$ ni yechish uchun $-x^6-x^2-1\equiv 0(mod\ 3)\to -x^2+1\equiv$

$0(mod\ 3)$. Buni 3 moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm 1$, dagi chegirmalarni sinab ko'rish usuli bilan yechsak $x \equiv 1, -1(mod\ 3)$ larining yechimi ekanligini topamiz.

$a_1)$ $x \equiv 1(mod\ 3) \rightarrow x = 1 + 3t_1$ yechimni qaraymiz. 290-misoldagi (7) –formulaga asosan t_1 ga nisbatan $\frac{f(1)}{3} + f'(1)t_1 \equiv 0(mod 3)$ taqqoslamaga ega bo'lamiz. Bunda $f(1) = -15$ va $f'(x) = 12x^5 - 24x^3 - 14x$, $f'(1) = -26$ bo'lgani uchun $-5 - 26t_1 \equiv 0(mod 3) \rightarrow t_1 \equiv 2(mod 3) \rightarrow t_1 \equiv -1 + 3t_2 \rightarrow x = -2 + 9t_2, t_2 \in Z$.

$a_2)$ $x \equiv -1(mod\ 3) \rightarrow x = -1 + 3t_1$ yechimni qaraymiz. 290-misoldagi (7) –formulaga asosan t_1 ga nisbatan $\frac{f(-1)}{3} + f'(-1)t_1 \equiv 0(mod 3)$ taqqoslamaga ega bo'lamiz. Bunda $f(-1) = -15$ va $f'(x) = 12x^5 - 24x^3 - 14x$, $f'(-1) = 26$ bo'lgani uchun $-5 + 26t_1 \equiv 0(mod 3) \rightarrow 2t_1 \equiv 2(mod 3) \rightarrow t_1 \equiv 1 + 3t_2 \rightarrow x = 2 + 9t_2, t_2 \in Z$.

Endi (1) dagi ikkinchi taqqoslamani yechamiz. Buning uchun avvalo, $f(x) \equiv 0(\text{mod } 7)$ ni, ya'ni $2x^6 - 6x^4 - 7x^2 - 4 \equiv 0(mod\ 7)$' ni qaraymiz. Bu taqqoslama $2x^6 + x^4 + 3 \equiv 0(mod\ 7)$ ga teng kuchli. Buni 7 moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm 1, \pm 2, \pm 3$ larni qo'yib tanlash usuli bilan yechsak $x \equiv \pm 2(mod 7)$ lar berilgan taqqoslamaning yechimi ekanligini topamiz.

$b_1)$ $x \equiv 2(mod\ 7) \rightarrow x = 2 + 7t_1$ yechimni qaraymiz. 290-misoldagi (7) –formulaga asosan t_1 ga nisbatan $\frac{f(2)}{3} + f'(2)t_1 \equiv 0(mod 7)$ taqqoslamaga ega bo'lamiz. Bunda $f(2) = 0$ va $f'(12) = 164$ bo'lgani uchun $164t_1 \equiv 0(mod 7) \rightarrow t_1 \equiv 0(mod 7) \rightarrow t_1 \equiv 7t_2 \rightarrow x = 2 + 49t_2, t_2 \in Z$.

$b_2)$ $x \equiv -2(mod\ 7) \rightarrow x = -2 + 7t_1$ yechimni qaraymiz. 290-misoldagi (7) –formulaga asosan t_1 ga nisbatan $\frac{f(-2)}{3} + f'(-2)t_1 \equiv 0(mod 7)$ taqqoslamaga ega bo'lamiz. Bunda $f(-1) = 0$ va $f'(-1) = -164$ bo'lgani uchun $-164t_1 \equiv 0(mod 7) \rightarrow t_1 \equiv 0(mod 7) \rightarrow t_1 \equiv 7t_2 \rightarrow x = -2 + 49t_2, t_2 \in Z$.

Shunday qilib, quyidagi chiziqli taqqoslamalar sistemasini yechishga keldik:

$$c_1)\begin{cases} x \equiv -2\ (mod\ 9) \\ x \equiv 2\ (mod\ 49) \end{cases} c_2)\begin{cases} x \equiv -2\ (mod\ 9) \\ x \equiv -2\ (mod\ 49) \end{cases}$$

$$c_3)\begin{cases} x \equiv 2 \ (mod\ 9) \\ x \equiv 2 \ (mod\ \ 49) \end{cases} c_4)\begin{cases} x \equiv 2 \ (mod\ 9) \\ x \equiv -2 \ (mod\ \ 49) \end{cases}$$

$c_1)$ dan

$$\begin{cases} x = -2 + 9t_1 \\ -2 + 9t_1 \equiv 2 \ (mod\ 49) \end{cases} \to \begin{cases} 9t_1 \equiv 4 \ (mod\ 49) \\ x = -2 + 9t_1 \end{cases} \to$$

$$\begin{cases} 9t_1 \equiv -45 \ (mod\ 49) \\ x = -2 + 9t_1 \end{cases} \to \begin{cases} t_1 \equiv -5 \ (mod\ 49) \\ x = -2 + 9t_1 \end{cases} \to \begin{cases} t_1 \equiv -5 + 49t_2 \\ x = -47 + 441t_2 \end{cases} \to$$

$$x = -47 + 441t_2, t_2 \in Z.$$

$c_2)$ dan

$$\begin{cases} x = -2 + 9t_1 \\ -2 + 9t_1 \equiv -2 \ \ (mod\ \ 49) \end{cases} \to \begin{cases} 9t_1 \equiv 0 \ (mod\ 49) \\ t_1 \equiv 0 \ (mod\ 49) \end{cases} \to$$

$$\begin{cases} t_1 \equiv \ 49t_2 \\ x = -2 + 441t_2 \end{cases} \to$$

$$x = -2 + 441t_2, t_2 \in Z.$$

$c_3)$ dan

$$\begin{cases} x = 2 + 9t_1 \\ 2 + 9t_1 \equiv 2 \ (mod\ 49) \end{cases} \to \begin{cases} 9t_1 \equiv 0 \ (mod\ 49) \\ x = 2 + 9t_1 \end{cases} \to \begin{cases} t_1 \equiv 0 \ (mod\ 49) \\ x = 2 + 9t_1 \end{cases}$$

$$\to \begin{cases} t_1 = 49t_2 \\ x = 2 + 9t_1 \end{cases} x = 2 + 441t_2, t_2 \in Z.$$

$c_4)$ dan

$$\begin{cases} x = 2 + 9t_1 \\ 2 + 9t_1 \equiv -2 \ (mod\ 49) \end{cases} \to \begin{cases} 9t_1 \equiv -4 \ (mod\ 49) \\ x = 2 + 9t_1 \end{cases} \to$$

$$\begin{cases} 9t_1 \equiv \ 45 \ (mod\ 49) \\ x = 2 + 9t_1 \end{cases} \to \begin{cases} t_1 \equiv 5 \ (mod\ 49) \\ x = 2 + 9t_1 \end{cases} \to \begin{cases} t_1 \equiv 5 + 49t_2 \\ x = 2 + 9t_1 \end{cases} \to x =$$

$47 +$

$441t_2, t_2 \in Z$. **Javob:** $x \ \equiv \ -47, \ -2, \ 2, \ 47 \ (mod\ \ 441)$.

9). $2x^6 - 6x^4 - 7x^2 - 4 \equiv 0 \ (mod\ \ 1225)$taqqoslamani qaraymiz. Bu yerda $1225 = 5^2 \cdot 7^2$ bo'lgani uchun bu taqqoslama

$$\begin{cases} f(x) \equiv 0(mod\ 25) \\ f(x) \equiv 0(mod\ 49) \end{cases} \quad (7)$$

ga teng kuchli. (1) dagi 2-taqqoslamani 291.8) misolda yechgan edik. Uning yechimlari $x \equiv -2 \ (mod\ \ 49)$va $x \equiv 2 \ (mod\ 49)$ iborat edi. Shuning uchun ham (1) dagi 1-taqqoslama $2x^6 - 6x^4 - 7x^2 - 4 \equiv 0(mod\ 5^2)$ ni yechamiz. Buning uchun $2x^6 - 6x^4 - 7x^2 - 4 \equiv 0(mod\ 5) \to 2x^6 - x^4 - 2x^2 + 1 \equiv 0(mod\ 5) \to 2x^2 - 1 - 2x^2 + 1 \equiv 0(mod\ 5)$ni qaraymiz. Bu taqqoslama $2x^6 - x^4 - 2x^2 + 1 \equiv 0(mod\ 5) \to 2x^2 - 1 - 2x^2 + 1 \equiv 0(mod\ 5)$ ayniy taqqoslamaga teng kuchli bo‘lgani uchun uning yechimlari $x \equiv -1, 1, 2, -2 \ (mod\ 5)$ dan iborat. Endi bu yechimlardan foydalanib $2x^6 - 6x^4 - 7x^2 - 4 \equiv 0(mod\ 5^2)$ ning yechimini topishga harakat qilamiz.

$a_1)$ $x \equiv -1 \ (mod\ 5\ )$, $x = -1 + 5t_1$ni qaraymiz. 290-misoldagi (7) –formulaga asosan t_1 ga nisbatan $\frac{f(-1)}{5} + f'(-1)t_1 \equiv 0(mod5)$ taqqoslamaga ega bo‘lamiz. Bunda$f(-1) = -15$ va $f'(x) = 12x^5 - 24x^3 - 14x$, $f'(-1) = 26$ bo‘lgani uchun $-3 + 26t_1 \equiv 0(mod5) \to t_1 \equiv 3(mod5) \to t_1 \equiv 3 + 5t_2 \to x = 14 + 25t_2, t_2 \in Z$.

$a_2)$ $x \equiv 1(mod\ 5) \to x = 1 + 5t_1$ yechimni qaraymiz. 290-misoldagi (7) –formulaga asosan t_1 ga nisbatan $\frac{f(1)}{5} + f'(1)t_1 \equiv 0(mod5)$ taqqoslamaga ega bo‘lamiz. Bunda$f(1) = -15$ va $f'(-1) = -26$ bo‘lgani uchun$-3 - 26t_1 \equiv 0(mod5) \to t_1 \equiv -3(mod5) \to t_1 \equiv -3 + 5t_2 \to x = -14 + 25t_2, t_2 \in Z$.

$a_3)$ $x \equiv 2(mod\ 5) \to x = 2 + 5t_1$ yechimni qaraymiz. 290-misoldagi (7) -formulaga asosan t_1 ga nisbatan $\frac{f(2)}{5} + f'(2)t_1 \equiv 0(mod5)$ taqqoslamaga ega bo‘lamiz. Bunda $f(2) = 0$ va $f'(2) = 164$ bo‘lgani uchun $164t_1 \equiv 0(mod5) \to t_1 \equiv 0(mod5) \to t_1 \equiv 5t_2 \to x = 2 + 25t_2, t_2 \in Z$.

$a_4)$ $x \equiv -2(mod\ 5) \to x = -2 + 5t_1$ yechimni qaraymiz. 290-misoldagi (7) –formulaga asosan t_1 ga nisbatan $\frac{f(-2)}{5} + f'(-2)t_1 \equiv 0(mod5)$ taqqoslamaga ega bo‘lamiz. Bunda$f(-2) = 0$ va $f'(-2) = -164$ bo‘lgani uchun$164t_1 \equiv 0(mod5) \to t_1 \equiv 0(mod5) \to t_1 \equiv 5t_2 \to x = -2 + 25t_2, t_2 \in Z$. Shunday qilib, quyidagi chiziqli taqqoslamalar sistemasini yechishga keldik:

$$c_1)\begin{cases} x \equiv -2(mod\ 49) \\ x \equiv 14\ (mod\ 25) \end{cases}; c_2)\begin{cases} x \equiv -2(mod\ 49) \\ x \equiv -14\ (mod\ 25) \end{cases};$$

$$c_3)\begin{cases} x \equiv -2(mod\ 49) \\ x \equiv 2\ (mod\ 25) \end{cases}; \quad c_4)\left\{\begin{cases} x \equiv -2(mod\ 49) \\ x \equiv -2\ (mod\ 25) \end{cases}\right.;$$

$$c_5)\begin{cases} x \equiv 2(mod\ 49) \\ x \equiv 14\ (mod\ 25) \end{cases}; \quad c_6)\begin{cases} x \equiv 2(mod\ 49) \\ x \equiv -14\ (mod\ 25) \end{cases};$$

$$c_7)\begin{cases} x \equiv 2(mod\ 49) \\ x \equiv 2\ (mod\ 25) \end{cases}; c_8)\begin{cases} x \equiv 2(mod\ 49) \\ x \equiv -2\ (mod\ 25) \end{cases}.$$

$c_1)$ dan

$$\begin{cases} x = -2 + 49t_1 \\ -2 + 49t_1 \equiv 14 \ (mod\ 25) \end{cases} \rightarrow \begin{cases} 49t_1 \equiv 16 \ (mod\ 25) \\ x = -2 + 49t_1 \end{cases} \rightarrow$$

$$\rightarrow \begin{cases} t_1 \equiv 9 \ (mod\ 25) \\ x = -2 + 49t_1 \end{cases} \rightarrow x = -2 + 49(9 + 25t_2) = 439 +$$

$$1225t_2, t_2 \in Z.$$

c_2) dan

$$\begin{cases} x = -2 + 49t_1 \\ -2 + 49t_1 \equiv -14 \ (mod\ 25) \end{cases} \rightarrow \begin{cases} x = -2 + 49t_1 \\ 49t_1 \equiv -12 \ (mod\ 25) \end{cases} \rightarrow$$

$$\rightarrow \begin{cases} t_1 \equiv 12 \ (mod\ 25) \\ x = -2 + 49t_1 \end{cases} \rightarrow \quad x = -2 + 49 \cdot (12 + 25t_2) = 586 +$$

$1225t_2, t_2 \in Z.$

c_3) dan

$$\begin{cases} x = -2 + 49t_1 \\ -2 + 49t_1 \equiv 2 \ (mod\ 25) \end{cases} \rightarrow \quad \begin{cases} t_1 \equiv -4 \ (mod\ 25) \\ x = -2 + 49t_1 \end{cases} \rightarrow x = -2 +$$

$49(-4 + 25\, t_2) = -198 + 1225\, t_2,\ t_2 \in Z.$

c_4) dan

$$\begin{cases} x = -2 + 49t_1 \\ -2 + 49t_1 \equiv -2 \ (mod\ 25) \end{cases} \rightarrow \begin{cases} t_1 \equiv 0 \ (mod\ 25) \\ t_1 = 25t_2 \end{cases} \rightarrow$$

$$x = -2 + 1225t_2\, , t_2 \in Z.$$

c_5) dan

$$\begin{cases} x = 2 + 49t_1 \\ 2 + 49t_1 \equiv 14 \ (mod\ 25) \end{cases} \rightarrow \begin{cases} t_1 \equiv -12 \ (mod\ 25) \\ x = 2 + 49t_1 \end{cases} \rightarrow$$

$$x = -586 + 1225t_2\, , t_2 \in Z.$$

c_6) dan

$$\begin{cases} x = 2 + 49t_1 \\ 2 + 49t_1 \equiv -14 \ (mod\ 25) \end{cases} \rightarrow \begin{cases} t_1 \equiv 16 \ (mod\ 25) \\ x = 2 + 49t_1 \end{cases} \rightarrow$$

$$x = 786 + 1225t_2\, , t_2 \in Z.$$

c_7) dan

$$\begin{cases} x = 2 + 49t_1 \\ 2 + 49t_1 \equiv 2 \ (mod\ 25) \end{cases} \rightarrow \begin{cases} t_1 \equiv 0 \ (mod\ 25) \\ x = 2 + 49t_1 \end{cases} \rightarrow t_1 = 25t_2 \rightarrow$$

$$x = 2 + 1225t_2\, , t_2 \in Z.$$

c_8) dan

$$\begin{cases} x = 2 + 49t_1 \\ 2 + 49t_1 \equiv -2 \ (mod\ 25) \end{cases} \rightarrow \begin{cases} t_1 \equiv 4 \ (mod\ 25) \\ x = 2 + 49t_1 \end{cases} \rightarrow$$

$$\begin{cases} t_1 \equiv 4 + 25t_2 \\ x = 2 + 49t_1 \end{cases} \rightarrow$$

$$x = 198 + 1225t_2, t_2 \in Z.$$

Shunday qilib, berilgan taqqoslamaning yechimlari: $x \equiv -586, -198, -2, 2, 198, 439, 586, 786(mod\ 1225)$.

IV.6-§.

292.1). Avvalo, berilgan $2x^2 + 4x - 1 \equiv 0(mod5)$ taqqoslamani bosh hadining koeffitsiyenti 1 ga teng bo'lgan holga keltiramiz. $2a \equiv 1(mod5) \to a \equiv 3(mod5)$ bo'lgani uchun berilgan taqqoslamaning ikkala tomonini 3 ga ko'paytiramiz. U holda $6x^2 + 12x - 3 \equiv 0(mod5)$ tenglama hosil bo'ladi. Bundan $x^2 + 2x + 2 \equiv 0(mod5) \to (x+1)^2 + 1 \equiv 0(mod5) \to (x+1)^2 \equiv -1(mod5) \to (x+1)^2 \equiv 4(mod5) \to x + 1 \equiv 2(mod5)vax + 1 \equiv -2(mod5) \to x \equiv 1(mod5)vax \equiv -3(mod5)$ hosil bo'ladi.

Javob: $x \equiv -3, 1\ (mod5)$.

2). Avvalo, berilgan $3x^2 + 2x \equiv 1(mod7)$ taqqoslamani bosh hadining koeffitsiyenti 1 ga teng bo'lgan holga keltiramiz. $3a \equiv 1(mod7) \to a \equiv 5(mod7)$ bo'lgani uchun $3x^2 + 2x \equiv 1(mod7)$ taqqoslamaning ikkala tomonini 5 ga ko'paytiramiz. U holda $15x^2 + 10x - 5 \equiv 0(mod7) \to x^2 + 10x + 9 \equiv 0(mod7) \to (x+5)^2 - 16 \equiv 0(mod7) \to (x+5)^2 \equiv 16(mod7) \to x + 5 \equiv 4(mod7)$ va $x + 5 \equiv -4(mod7) \to x \equiv -1(mod7)$va $x \equiv -2(mod7)$ hosil bo'ladi.

Javob: $x \equiv -1, -2(mod7)$.

3). Avvalo, berilgan $2x^2 - 2x - 1 \equiv 0(mod7)$ taqqoslamani bosh hadining koeffitsiyenti 1 ga teng bo'lgan holga keltiramiz. $2a \equiv 1(mod7) \to a \equiv 4(mod7)$ bo'lgani uchun $2x^2 - 2x - 1 \equiv 0(mod7)$ taqqoslamaning ikkala tomonini 4 ga ko'paytiramiz. U holda $8x^2 - 8x - 4 \equiv 0(mod7)$, $x^2 - x + 3 \equiv 0(mod7) \to x^2 + 6x + 10 \equiv 0(mod7) \to (x+3)^2 + 1 \equiv 0(mod7) \to (x+3)^2 \equiv -1(mod7) \to (x+3)^2 \equiv 6(mod7)$. Bu taqqoslamaga 7 moduli bo'yicha chegirmalarning to'la sistemasidagi chegirmalarning to'la sistemasi $0, \pm1, \pm2, \pm3$ qo'yib tekshirsak taqqoslama yechimga ega emas.

Javob: taqqoslama yechimga ega emas.

4). Berilgan $3x^2 - x \equiv 0(mod5)$ taqqoslamani bosh hadining koeffitsiyenti 1 ga teng bo'lgan holga keltiramiz. $3a \equiv 1(mod5) \to a \equiv 2(mod5)$ bo'lgani uchun $3x^2 - x \equiv 0(mod5)$ taqqoslamaning ikkala tomonini 2 ga ko'paytiramiz. U holda $6x^2 - 2x \equiv 0(mod5) \to x^2 - 2x \equiv 0(mod5) \to (x-1)^2 - 1 \equiv 0(mod5) \to (x-1)^2 \equiv$

$1(mod5) \to x-1 \equiv 1(mod5)$ va $x-1 \equiv -1(mod5) \to x \equiv 2(mod5)$va $x \equiv 0(mod5)$ hosil bo'ladi. **Javob:** $x \equiv 0,2(mod5)$.

5). Berilgan $3x^2+7x+8 \equiv 0(mod17)$ taqqoslamani bosh hadining koeffitsiyenti 1 ga teng bo'lgan holga keltiramiz. $3a \equiv 1(mod17) \to a \equiv 6(mod17)$ bo'lgani uchun $3x^2+7x+8 \equiv 0(mod17)$ taqqoslamaning ikkala tomonini 6 ga ko'paytiramiz. U holda $18x^2+42x+48 \equiv 0(mod17) \to x^2+8x+48 \equiv 0(mod17) \to (x+4)^2+32 \equiv 0(mod17) \to (x+4)^2 \equiv 2(mod17) \to (x+4)^2 \equiv 36(mod17) \to x+4 \equiv 6(mod17)$va $x+4 \equiv -6(mod17) \to x \equiv 2(mod17)$va $x \equiv 7(mod17)$ hosil bo'ladi. **Javob**: $x \equiv 2{,}7(mod17)$.

6). Berilgan $3x^2+4x+7 \equiv 0(mod31)$ taqqoslamani $9x^2+12x+21 \equiv 0(mod31) \to (3x+2)^2+17 \equiv 0(mod31) \to (3x+2)^2 \equiv 14+31\cdot 5(mod31) \to (3x+2)^2 \equiv 169(mod31) \to 3x+2 \equiv 113(mod31)$va $3x+2 \equiv -13(mod31) \to 3x \equiv 11(mod31)$va $3x \equiv -15(mod31) \to x \equiv 14(mod31)$ va $x \equiv -5(mod31)$. **Javob:** $x - 5,14(mod31)$.

7). $4x^2-11x-3 \equiv 0(\text{mod}\,13)$taqqoslamani ikkihadli taqqoslama ko'rinishiga keltirib, keyin yeching.

$$4x^2-24x-16 \equiv 0(\text{mod}\,13) \to x^2-6x-4 \equiv 0(\text{mod}\,13) \to (x-3)^2 \equiv 0(\text{mod}\,13) \to x \equiv 3(\text{mod}\,13).$$

8).$3x^2+7x+8 \equiv 0(\text{mod}\,17)$ taqqoslamani ikkihadli taqqoslama ko'rinishiga keltirib, keyin yeching.

$$3x^2+24x-9 \equiv 0(\text{mod}\,17) \to x^2+8x-3 \equiv 0(\text{mod}\,17) \to (x+4)^2 \equiv 19(\text{mod}\,17) \to$$
$$(x+4)^2 \equiv 36(\text{mod}\,17) \to x+4 \equiv 6(\text{mod}\,17) \; va \; x+4 \equiv -6(\text{mod}\,17) \to x \equiv 6-4(\text{mod}\,17)$$
$$va \; x \equiv -10(\text{mod}\,17) \to x \equiv 2(\text{mod}\,17) \; va \; x \equiv 7(\text{mod}\,17).$$

293. 1).Berilgan kasr butun qiymat qabul qilishi uchun uning surati maxrajiga bo'limishi kerak, ya'ni $x^2+2x+7 \equiv 0(mod55)$ bajarilishi kerak. Bundan $(x+1)^2 \equiv -6(mod55) \to \begin{cases}(x+1)^2 \equiv -6(mod5)\\(x+1)^2 \equiv -6(mod11)\end{cases} \to$ $\begin{cases}(x+1)^2 \equiv 4(mod5)\\(x+1)^2 \equiv 16(mod11)\end{cases} \to \begin{cases}x+1 \equiv \pm 2(mod5)\\x+1 \equiv \pm 4(mod11)\end{cases}$ larga ega bo'lamiz. Keyingi sistemadagi birinchi taqqoslamaning yechimlari $x \equiv 1,2(mod5)$, ikkinchi taqqoslamaning yechimlari $x \equiv -5,3(mod11)$ dan iborat ekanligi kelib chiqadi. Bulardan quyidagi taqqoslamalar sistemalarini hosil qilamiz:

$$a_1)\begin{cases} x \equiv 1(mod5) \\ x \equiv 6(mod11) \end{cases};$$

$$a_2)\begin{cases} x \equiv 1(mod5) \\ x \equiv 3(mod11) \end{cases}; \quad a_3)\begin{cases} x \equiv 2(mod5) \\ x \equiv 6(mod11) \end{cases};$$

$$a_4)\begin{cases} x \equiv 2(mod5) \\ x \equiv 3(mod11) \end{cases}.$$

Bu sistemalarni yechmiz. U holda

$$a_1) \text{ dan}\begin{cases} x \equiv 1 + 5t_1 \\ 1 + 5t_1 \equiv 6(mod11) \end{cases} \to \begin{cases} 5t_1 \equiv 5(mod11) \\ x \equiv 1 + 5t_1 \end{cases} \to$$

$$\begin{cases} t_1 \equiv 1(mod11) \\ x \equiv 1 + 5t_1 \end{cases} \to$$

$$x \equiv 6 + 5t_2, \qquad t_2 \in Z.$$

$a_2)$ dan

$$\begin{cases} x \equiv 1 + 5t_1 \\ 1 + 5t_1 \equiv 3(mod11) \end{cases} \begin{cases} 5t_1 \equiv 2(mod11) \\ x \equiv 1 + 5t_1 \end{cases} \to \begin{cases} t_1 \equiv 7(mod11) \\ x \equiv 1 + 5t_1 \end{cases} \to$$

$x = 36 + 55t_2,\ t_2 \in Z.$

$$a_3) \quad \text{dan} \quad \begin{cases} x \equiv 2 + 5t_1 \\ 2 + 5t_1 \equiv 6(mod11) \end{cases} \to \begin{cases} 5t_1 \equiv 4(mod11) \\ x \equiv 2 + 5t_1 \end{cases} \to$$

$$\begin{cases} t_1 \equiv 3(mod11) \\ x \equiv 2 + 5t_1 \end{cases} \to$$

$x = 17 + 55t_2,\ t_2 \in Z.$

$$a_4) \text{ dan } \begin{cases} x \equiv 2 + 5t_1 \\ 2 + 5t_1 \equiv 3(mod11) \end{cases} \to$$

$$\begin{cases} 5t_1 \equiv 1(mod11) \\ x \equiv 2 + 5t_1 \end{cases} \to \begin{cases} t_1 \equiv -2(mod11) \\ x \equiv 2 + 5t_1 \end{cases} \to$$

$x = -8 + 55t_2,\ t_3 \in Z.$

Javob: $x = 6 + 55t,\ x = 17 + 55t,\ x = 36 + 55t,\ x = 47 + 55t, t \in Z.$

2).Berilgan kasr butun qiymat qabul qilishi uchun uning surati maxrajiga bo‘linishi kerak, ya’ni $x^2 + 3x + 1 \equiv 0(mod25)$ bajarilishi kerak. $x^2 + 3x + 1 \equiv 0(mod5)$ ni qaraymiz. Bu taqqoslamani 5 moduli bo‘yicha chegirmalarning to‘la sistemasi $0, \pm1, \pm2$ dagi sonlarni sinab ko‘rish usuli bilan yechsak, $x \equiv 1(mod5)$ uning yechimi ekanligini topamiz. Endi 5 moduldan 25 modulga o‘tamiz. Buning uchun 290-misoldagi singari ish tutamiz. U holda (7)-formulaga asosan $\frac{f(1)}{5} + f'(1)t_1 \equiv 0(mod25)$ ga ega bo‘lamiz. Bu yerda

$f(1) = 5, f'(1) = 2x + 3$ va $f'(1) = 5$ bo'lgani uchun $1 + 5t_1 \equiv 0(mod5) \rightarrow$

$5t_1 \equiv 4(mod5)$ ga ega bo'lamiz. Bunda$(5,25) = 5$, lekin 4 soni 5 ga bo'linmagani uchun bu taqqoslama yechimga ega emas, ya'ni berilgan ifoda butun qiymat qabul qiladigan x ning natural qiymatlari mavjud emas.

Javob: berilgan ifoda butun qiymat qabul qiladigan x ning natural qiymatlari mavjud emas.

3).Berilgan kasr butun qiymat qabul qilishi uchun uning surati maxrajiga bo'limishi kerak, ya'ni $x^2 + 3x + +5 \equiv 0(mod15)$ bajarilishi kerak. Bu taqqoslama ushbu

$$\begin{cases} x^2 + 3x + 5 \equiv 0(mod3) \\ x^2 + 3x + 5 \equiv 0(mod5) \end{cases}$$

taqqoslamalar sistemasiga teng kuchli. Bu sistemaning 1-taqqoslamasini yechamiz. U holda $x^2 + 3x + 5 \equiv 0(mod3) \rightarrow x^2 - 1 \equiv 0(mod3) \rightarrow x \equiv -1$ va$x \equiv 1(mod3)$ larni hosil qilamiz.

Endi 2- taqqoslamasini yechamiz:

$$x^2 + 3x + 5 \equiv 0(mod5 \rightarrow x^2 - 2x \equiv 0(mod5) \rightarrow x \equiv 0{,}2(mod5).$$

Bularga asoslanib quyidagi sistemalarni tuzib olamiz:

$$a_1) \begin{cases} x \equiv -1(mod3) \\ x \equiv 0(mod5) \end{cases}; a_2) \begin{cases} x \equiv -1(mod3) \\ x \equiv 2(mod5) \end{cases}; a_3) \begin{cases} x \equiv 1(mod3) \\ x \equiv 0(mod5) \end{cases}$$

$$a_4) \begin{cases} x \equiv 1(mod3) \\ x \equiv 2(mod5) \end{cases}.$$

Bu sistemalarni yechib yechimlarini topamiz:

$$a_1) \text{ dan} \begin{cases} x \equiv -1 + 3t_1 \\ -1 + 3t_1 \equiv 0(mod5) \end{cases} \rightarrow \begin{cases} 3t_1 \equiv 1(mod5 \\ x = -1 + 3t \end{cases} \rightarrow$$

$$\begin{cases} t_1 \equiv 2(mod5) \\ x = -1 + 3t_1 \end{cases} \rightarrow$$

$$x = 5 + 15t_2, t_2 \in Z.$$

$$a_2) \text{ dan} \begin{cases} x \equiv -1 + 3t_1 \\ -1 + 3t_1 \equiv 2(mod5) \end{cases} \rightarrow \begin{cases} 3t_1 \equiv 3(mod5 \\ x = -1 + 3t_1 \end{cases} \rightarrow$$

$$\begin{cases} t_1 \equiv 1(mod5) \\ x = -1 + 3t_1 \end{cases}$$

$x = 2 + 15t_2, t_2 \in Z.$

$a_3)$ dan $\begin{cases} x \equiv 1 + 3t_1 \\ 1 + 3t_1 \equiv 0(mod5) \end{cases} \begin{cases} 3t_1 \equiv -1(mod5) \\ x = 1 + 3t_1 \end{cases} \to$
$\begin{cases} t_1 \equiv -2(mod5) \\ x = 1 + 3t_1 \end{cases}$
$x = -5 + 15t_2, t_2 \in Z.$

$a_4)$ dan $\begin{cases} x \equiv 1 + 3t_1 \\ 1 + 3t_1 \equiv 2(mod5) \end{cases} \to \begin{cases} 3t_1 \equiv 1(mod5) \\ x = 1 + 3t_1 \end{cases} \to$
$\begin{cases} t_1 \equiv 2(mod5) \\ x = 1 + 3t_1 \end{cases}$
$x = 7 + 15t_2, \quad t_2 \in Z.$

Javob: $x = 2 + 15t_2$, $x = 5 + 15t_2$, $x = 7 + 15t_2$, $x = 10 + 15t_2, t_2 \in Z.$

294. $x^2 \equiv a(modp)$ taqqoslama yechimga ega bo'lishi uchun Eyler kriteryasiga asosan $a^{\frac{p-1}{2}} \equiv 1(modp)$ bajarilishi kerak. Bunda $p = 7$ da $a^3 \equiv 1(mod7)$ ga ega bo'lamiz 7 moduli bo'yicha 1,2,3,4,5,6 dan iborat. Bularni Eyler kriteryasiga qo'yib tekshirib ko'ramiz; $1^3 \equiv 1, 2^3 \equiv 1, 3^3 \equiv -1, 4^3 \equiv 1, \quad 5^3 \equiv -1, 6^3 \equiv -1$. Demak, 1,2,4 sonlari 7 modul bo'yicha kvadratik chegirma, qolganlari, ya'ni 3,5,6 lar esa kvadratik chegirma emas.

295. 1). $p = 11$ moduli bo'yicha kvadratik chegirmalar sinflarini aniqlash uchun Eyler kriteryasi $a^{\frac{p-1}{2}} \equiv 1(modp) \to a^5 \equiv 1(mod11)$ ning bajarilishini chegirmalarning keltirilgan sistemasi $1,2,3,4,5,6,7,8,9,10$ dagi chegirmalar uchun tekshirib ko'ramiz. U holda quyidagilarga ega bo'lamiz:

$1^5 \equiv 1(mod11),$

$2^5 \equiv -1(mod11), 3^5 \equiv 81 \cdot 3 \equiv 1(mod11), 4^5 \equiv 16 \cdot 16 \cdot 4 \equiv 5 \cdot 5 \cdot 4 \equiv 3 \cdot 4 \equiv 1(mod11), 5^5 \equiv 25 \cdot 25 \cdot 5 \equiv 3 \cdot 3 \cdot 5 \equiv 1(mod11),$

$6^5 \equiv 36 \cdot 36 \cdot 6 \equiv 3 \cdot 3 \cdot 6 \equiv -1(mod11)$, $7^5 \equiv 49 \cdot 49 \cdot 7 \equiv 5 \cdot 5 \cdot 7 \equiv -1(mod11)$, $8^5 \equiv 64 \cdot 64 \cdot 8 \equiv (-2) \cdot (-2) \cdot 8 \equiv -1(mod11)$,

$9^5 \equiv 81 \cdot 81 \cdot 9 \equiv 4 \cdot 4 \cdot (-2) \equiv 1(mod11)$. Bizga ma'lumki, $p > 2$ moduli bo'yicha chegirmalarning keltirilgan sistemasidagi chegirmalar yarmi kvadratik chegirma qolganlari esa kvadratik chegirma emas bo'ladi. Biz yuqorida $1,3,4,5,9$ larning $p = 11$ moduli bo'yicha kvadratik chegirmalar bo'lishini ko'rdik. Demak, $1 + 11k, 3 + 11k, 4 +$

$11k, 5+11k, 9+11k$ lar $p=11$ moduli bo'yicha kvadratik chegirmalar sinflari bo'ladi.

Javob: $1+11k, 3+11k, 4+11k, 5+11k, 9+11k, k \in Z$.

2). $p=13$ moduli bo'yicha kvadratik chegirmalar sinflarini aniqlash uchun Eyler kriteryasi $a^{\frac{p-1}{2}} \equiv 1(modp) \rightarrow a^6 \equiv 1(mod13)$ ning bajarilishini chegirmalarning keltirilgan sistemasi $1,2,3,4,5,6,7,8,9,10,$ $11,12$ dagi chegirmalar uchun tekshirib ko'ramiz. U holda quyidagilarga ega bo'lamiz:

$1^6 \equiv 1(mod13)$,

$2^6 \equiv -1(mod13), 3^6 \equiv 27\cdot 27 \equiv 1(mod13), 4^6 \equiv 16\cdot 16\cdot 16 \equiv 27 \equiv 1(mod13), 5^6 \equiv 25\cdot 25\cdot 25 \equiv -1(mod13)$,

$6^6 \equiv 36\cdot 36\cdot 36 \equiv -3\cdot(-3)\cdot(-3) \equiv -1(mod13)$, $7^6 \equiv 49\cdot 49\cdot 49 \equiv -3\cdot(-3)\cdot(-3) \equiv -1(mod13)$, $8^6 \equiv 64\cdot 64\cdot 64 \equiv (-1)\cdot(-1)\cdot(-1) \equiv -1(mod13), 9^6 \equiv 81\cdot 81\cdot 81 \equiv 3\cdot 3\cdot 3 \equiv 1(mod13)$, $10^6 \equiv 100\cdot 100\cdot 100 \equiv -3\cdot(-3)\cdot(-3) \equiv -1(mod13)$, $11^6 \equiv 121\cdot 121\cdot 121 \equiv 4\cdot 4\cdot 4 \equiv -1(mod13)$, $12^6 \equiv 144\cdot 144\cdot 144 \equiv 1(mod13)$. Bizga ma'lumki, $p>2$ moduli bo'yicha chegirmalarning keltirilgan sistemasidagi chegirmalar yarmi kvadratik chegirma qolganlari esa kvadratik chegirma emas bo'ladi. Biz yuqorida 1,3,4,9,10,12 larning $p=13$ moduli bo'yicha kvadratik chegirmalar bo'lishini ko'rdik. Demak, $1+13k, 3+13k, 4+13k, 9+13k, 10+13k, 12+13k$ lar $p=13$ moduli bo'yicha kvadratik chegirmalar sinflari bo'ladi.

Javob: $1+13k, 3+13k, 4+13k, 9+13k, 10+13k, 12+13k, k \in Z$.

3). $p=17$ moduli bo'yicha kvadratik chegirmalar sinflarini aniqlash uchun Eyler kriteryasi $a^{\frac{p-1}{2}} \equiv 1(modp) \rightarrow a^8 \equiv 1(mod17)$ ning bajarilishini chegirmalarning keltirilgan sistemasi $1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16$ dagi chegirmalar uchun tekshirib ko'ramiz. U holda quyidagilarga ega bo'lamiz:

$1^8 \equiv 1(mod17)$, $2^8 \equiv 2^4\cdot 2^4 \equiv 1(mod17), 3^8 \equiv 81\cdot 81 \equiv -4(-4) \equiv 1(mod17), 4^8 \equiv 16\cdot 16\cdot 16\cdot 16 \equiv 1(mod17), 5^8 \equiv 25\cdot 25\cdot 25\cdot 25 \equiv 8^2\cdot 8^2 \equiv 1(mod17), 6^8 \equiv 36^4 \equiv 2^4 \equiv -1(mod17)$, $7^8 \equiv 49^4 \equiv (-2)^4 \equiv -1(mod17)$, $8^8 \equiv 64^4 \equiv$

$(-4)^4 \equiv 1(mod17), 9^8 \equiv 81^4 \equiv (-4)^4 \equiv 1(mod17),\ 10^8 \equiv 100^4 \equiv (-2)^2 \equiv -1(mod17),\ 11^8 \equiv 121^4 \equiv 2^4 \equiv -1(mod17),\ 12^8 \equiv 144^4 \equiv 8^4 \equiv 64 \cdot 64 \equiv -1(mod17),\ 13^8 \equiv 169^4 \equiv (-1)^4 \equiv 1(mod17),\ 14^8 \equiv (-3)^8 \equiv 81^2 \equiv (-4)^2 \equiv -1(mod17),\ 15^8 \equiv (-2)^8 \equiv 2^4 \cdot 2^4 \equiv 1(mod17),\ 16^8 \equiv (-1)^8 \equiv 1(mod17)$. Bizga ma'lumki, $p > 2$ moduli bo'yicha chegirmalarning keltirilgan sistemasidagi chegirmalar yarmi kvadratik chegirma qolganlari esa kvadratik chegirma emas bo'ladi. Biz yuqorida 1,2,4,8,9,13,15,16 larning $p = 17$ moduli bo'yicha kvadratik chegirmalar bo'lishini ko'rdik. Demak, $1 + 17k, 2 + 17k, 4 + 17k, 9 + 17k, 9 + 17k, 13 + 17k, 15 + 17k, 16 + 17k$ lar $p = 17$ moduli bo'yicha kvadratik chegirmalar sinflari bo'ladi.

Javob: $1 + 17k, 2 + 17k, 4 + 17k, 9 + 17k, 9 + 17k, 13 + 17k, 15 + 17k,\ 16 + 17k\ k \in Z$.

296.1).7 moduli bo'yicha chegirmalarning keltirilgan sistemasini absolyut qiymati jihatidan eng kichik qilib olsak, $\pm1, \pm2, \pm3$ lardan iborat. Bularni berilgan taqqoslama $x^2 \equiv 2(mod7)$ ga qo'yib tekshirsak, $x \equiv \pm3(mod7)$ ning uni qanoatlantirishini ko'ramiz.

Javob: $x \equiv \pm3(mod7)$.

2).7 moduli bo'yicha chegirmalarning keltirilgan sistemasini absolyut qiymati jihatidan eng kichik qilib olsak, $\pm1, \pm2, \pm3$ lardan iborat. Bularni berilgan taqqoslama $x^2 \equiv 4(mod7)$ga qo'yib tekshirsak, $x \equiv \pm2(mod7)$ ning uni qanoatlantirishini ko'ramiz.

Javob: $x \equiv \pm2(mod7)$.

3).7 moduli bo'yicha chegirmalarning keltirilgan sistemasini absolyut qiymati jihatidan eng kichik qilib olsak, $\pm1, \pm2, \pm3$ lardan iborat. Bularni berilgan taqqoslama $x^2 \equiv 3(mod7)$ga qo'yib tekshirsak, ularning birortasi ham uni qanoatlantirmasligini ko'ramiz.

Javob: taqqoslama yechimga ega emas.

4).13 moduli bo'yicha chegirmalarning keltirilgan sistemasini absolyut qiymati jihatidan eng kichik qilib olsak, $\pm1, \pm2, \pm3, \pm4, \pm5, \pm6$ lardan iborat. Bularni berilgan taqqoslama $x^2 \equiv 3(mod13)$ga qo'yib tekshirsak, $x \equiv \pm4(mod7)$ ning uni qanoatlantirishini ko'ramiz.

Javob: $x \equiv \pm4(mod13)$.

5).11 moduli bo'yicha chegirmalarning keltirilgan sistemasini absolyut qiymati jihatidan eng kichik qilib olsak, $\pm1, \pm2, \pm3, \pm4, \pm5$

lardan iborat. Bularni berilgan taqqoslama $x^2 \equiv 4(mod11)$ga qo'yib tekshirsak, $x \equiv \pm 2(mod11)$ ning uni qanoatlantirishini ko'ramiz. **Javob:** $x \equiv \pm 2(mod11)$.

297. Lejandr simvolining qiymatini hisoblash uchun uning xossalaridan foydalanamiz.

1).$\left(\frac{63}{131}\right) = \left(\frac{3^2 \cdot 7}{131}\right)$ dan 4^0 –xossaga asosan $\left(\frac{3^2 \cdot 7}{131}\right) = \left(\frac{3^2}{131}\right) \cdot \left(\frac{7}{131}\right)$ ni hosil qilamiz. Ta'rifga ko'ra $\left(\frac{3^2}{131}\right) = \left(\frac{3}{131}\right)^2 = 1$, shuning uchun ham $\left(\frac{3^2 \cdot 7}{131}\right) = \left(\frac{7}{131}\right)$. Oxirgi tenglikning o'ng tomoniga kvadratik chegirmalarning o'zgalik qonuni 6^0 –xossani qo'llaymiz. U holda $\left(\frac{7}{131}\right) = (-1)^{\frac{131-1}{2}\frac{7-1}{2}}\left(\frac{131}{7}\right) = -\left(\frac{18 \cdot 7+5}{7}\right)$ hosil bo'ladi. Bu yerda 1^0 –xossadan foydalansak $\left(\frac{7}{131}\right) = -\left(\frac{5}{7}\right)$ ekanligi kelib chiqadi. Bu tenglikning o'ng tomonida yana bir marta 6^0 –xossadan foydalanamiz: $\left(\frac{7}{131}\right) = -\left(\frac{5}{7}\right) = -(-1)^{\frac{5-1}{2}\frac{7-1}{2}}\left(\frac{7}{5}\right) = -\left(\frac{5 \cdot 1+2}{5}\right) = -\left(\frac{2}{5}\right)$. Bunga 5^0 – xossani qo'llaymiz. U holda $-\left(\frac{2}{5}\right) = -(-1)^{\frac{5^2-1}{8}}$=1. Demak, $\left(\frac{63}{131}\right) = 1$.

Javob: 1.

2).$\left(\frac{35}{97}\right) = \left(\frac{5 \cdot 7}{97}\right)$ dan 4^0 –xossaga asosan $\left(\frac{5 \cdot 7}{97}\right) = \left(\frac{5}{97}\right) \cdot \left(\frac{7}{97}\right)$ ni hosil qilamiz. Oxirgi tenglikning o'ng tomonida har bir ko'paytuvchi uchun kvadratik chegirmalarning o'zgalik qonuni 6^0 –xossani qo'llaymiz. U holda $\left(\frac{5}{97}\right) \cdot \left(\frac{7}{97}\right) = (-1)^{\frac{97-1}{2}\frac{5-1}{2}} \cdot \left(\frac{97}{5}\right) \cdot (-1)^{\frac{97-1}{2}\frac{7-1}{2}} \cdot \left(\frac{97}{7}\right) = \left(\frac{97}{5}\right) \cdot \left(\frac{97}{7}\right) = \left(\frac{5 \cdot 19+2}{5}\right) \cdot \left(\frac{7 \cdot 13+6}{7}\right)$ hosil bo'ladi. Bu yerda 1^0 –xossadan foydalansak $\left(\frac{2}{5}\right) \cdot \left(\frac{6}{7}\right)$ ekanligi kelib chiqadi. Bu tenglikning o'ng tomonidagi birinchi ko'paytuvchiga 5^0 – xossani qo'llaymiz, ikkinchisini esa $\left(\frac{6}{7}\right) = \left(\frac{2}{7}\right)\left(\frac{3}{7}\right)$ deb yozish mumkin. Shuning uchun ham $\left(\frac{2}{5}\right) \cdot \left(\frac{6}{7}\right) = (-1)^{\frac{5^2-1}{8}} \cdot (-1)^{\frac{7^2-1}{8}} \cdot \left(\frac{3}{7}\right) = -1 \cdot 1 \cdot \left(\frac{3}{7}\right) = -\left(\frac{3}{7}\right)$ ga ega bo'lamiz. Bu tenglikning o'ng tomonida yana bir marta 6^0 –xossadan foydalanamiz: $-\left(\frac{3}{7}\right) = -(-1)^{\frac{3-1}{2}\frac{7-1}{2}}\left(\frac{7}{3}\right) = \left(\frac{3 \cdot 2+1}{3}\right) = \left(\frac{1}{5}\right) = 1$. Demak, $\left(\frac{35}{97}\right) = 1$. **Javob:** 1.

3). $\left(\frac{47}{73}\right)$ dan 6^0 –xossaga asosan $(-1)^{\frac{47-1}{2}\cdot\frac{73-1}{2}}\cdot\left(\frac{73}{47}\right)=\left(\frac{47\cdot1+26}{47}\right)$ ni hosil qilamiz. Bu yerda 1^0 –xossadan foydalansak $\left(\frac{47\cdot1+26}{47}\right)=\left(\frac{26}{47}\right)=$ $\left(\frac{2}{47}\right)\left(\frac{13}{47}\right)$ ekanligi kelib chiqadi. Bu tenglikning o‘ng tomonidagi birinchi ko‘paytuvchiga 5^0 – xossani, ikkinchisiga esa 6^0 –xossani qo‘llaymiz. $\left(\frac{2}{47}\right)\left(\frac{13}{47}\right)=(-1)^{\frac{47^2-1}{8}}\cdot(-1)^{\frac{13-1}{2}\cdot\frac{47-1}{2}}\left(\frac{47}{13}\right)=\left(\frac{47}{13}\right)=\left(\frac{13\cdot3+8}{13}\right)$ deb yozish mumkin. Bu yerda 1^0 –xossadan foydalansak $\left(\frac{8}{13}\right)=$ $\left(\frac{2^2\cdot2}{13}\right)=\left(\frac{2^2}{13}\right)\cdot\left(\frac{2}{13}\right)=\left(\frac{2}{13}\right)$. Shuning uchun ham $\left(\frac{2}{13}\right)=(-1)^{\frac{13^2-1}{8}}=$ -1 ga ega bo‘lamiz. Demak, $\left(\frac{47}{73}\right)=-1$. **Javob:** -1.

4). $\left(\frac{29}{383}\right)$ dan 6^0 –xossaga asosan $(-1)^{\frac{383-1}{2}\cdot\frac{29-1}{2}}\cdot\left(\frac{383}{29}\right)=\left(\frac{13\cdot29+6}{29}\right)$ ni hosil qilamiz. bo‘ladi. Bu yerda 1^0 –xossadan foydalansak $\left(\frac{13\cdot29+6}{29}\right)=\left(\frac{6}{29}\right)=\left(\frac{2}{29}\right)\cdot\left(\frac{3}{29}\right)$ ekanligi kelib chiqadi. Bu tenglikning o‘ng tomonidagi birinchi ko‘paytuvchiga 5^0 – xossani, ikkinchisiga esa 6^0 –xossani qo‘llaymiz. $\left(\frac{2}{29}\right)\cdot\left(\frac{3}{29}\right)=(-1)^{\frac{29^2-1}{8}}\cdot(-1)^{\frac{3-1}{2}\cdot\frac{29-1}{2}}\left(\frac{29}{3}\right)=$ $\left(\frac{29}{3}\right)=\left(\frac{9\cdot3+2}{3}\right)$ deb yozish mumkin. Bu yerda 1^0 –xossadan foydalansak $\left(\frac{9\cdot3+2}{3}\right)=\left(\frac{2}{3}\right)=(-1)^{\frac{3^2-1}{8}}=-1$. Demak, $\left(\frac{29}{383}\right)=-1$. **Javob:** -1.

5). $\left(\frac{241}{593}\right)$ dan 6^0 –xossaga asosan $(-1)^{\frac{593-1}{2}\cdot\frac{241-1}{2}}\cdot\left(\frac{593}{241}\right)=$ $\left(\frac{593}{241}\right)=\left(\frac{241\cdot2+111}{241}\right)=\left(\frac{111}{241}\right)=\left(\frac{37\cdot3}{241}\right)=\left(\frac{37}{241}\right)\cdot\left(\frac{3}{241}\right)$ ni hosil qilamiz. bo‘ladi. Bu tenglikning o‘ng tomonidagi ikkala ko‘paytuvchiga ham 6^0 –xossani qo‘llaymiz. U holda $\left(\frac{37}{241}\right)\cdot\left(\frac{3}{241}\right)=(-1)^{\frac{37-1}{2}\cdot\frac{241-1}{2}}\cdot\left(\frac{241}{37}\right)\cdot$ $(-1)^{\frac{3-1}{2}\cdot\frac{241-1}{2}}\cdot\left(\frac{241}{3}\right)=\left(\frac{241}{37}\right)\cdot\left(\frac{241}{3}\right)=\left(\frac{37\cdot6+19}{37}\right)\cdot\left(\frac{3\cdot80+1}{3}\right)$. Bu yerda 1^0 –xossadan foydalansak $\left(\frac{37\cdot6+19}{37}\right)\cdot\left(\frac{3\cdot80+1}{3}\right)=\left(\frac{19}{37}\right)\cdot\left(\frac{1}{3}\right)=\left(\frac{19}{37}\right)$. Endi bunga yana 6^0 –xossani qo‘llaymiz. U holda $\left(\frac{19}{37}\right)=(-1)^{\frac{37-1}{2}\cdot\frac{19-1}{2}}\cdot$ $\left(\frac{37}{19}\right)=\left(\frac{37}{19}\right)=\left(\frac{19\cdot1+18}{19}\right)$. Bunga 1^0 –xossani tatbiq etsak $\left(\frac{19\cdot1+18}{19}\right)=$

$\left(\frac{18}{19}\right)=\left(\frac{2\cdot3^2}{19}\right)=\left(\frac{2}{19}\right)\cdot\left(\frac{3^2}{19}\right)=\left(\frac{2}{19}\right)$. Endi oxirgi tenglikning o'ng tomoniga 5^0 – xossani qo'llab $\left(\frac{2}{19}\right)=(-1)^{\frac{19^2-1}{8}}=-1$. Demak, $\left(\frac{241}{593}\right)=-1$. **Javob:** -1.

6). Lejandr simvolining qiymatini hisoblash uchun uning xossalaridan foydalanamiz. Yuqoridagi misollarning ishlanishiga qarang.

$$\left(\frac{257}{571}\right)\overset{6^0}{\cong}(-1)^{\frac{571-1}{2}\cdot\frac{257-1}{2}}\cdot\left(\frac{571}{257}\right)=\left(\frac{571}{257}\right)=\left(\frac{257\cdot2+57}{257}\right)\overset{1^0}{\cong}\left(\frac{57}{257}\right)=$$

$$\left(\frac{3\cdot19}{257}\right)\overset{4^0}{\cong}\left(\frac{3}{257}\right)\cdot\left(\frac{19}{257}\right)\overset{6^0}{\cong}(-1)^{\frac{3-1}{2}\cdot\frac{257-1}{2}}\cdot\left(\frac{257}{3}\right)\cdot(-1)^{\frac{19-1}{2}\cdot\frac{257-1}{2}}\cdot\left(\frac{257}{19}\right)=$$

$$\left(\frac{257}{3}\right)\cdot\left(\frac{257}{19}\right)=\left(\frac{3\cdot85+2}{3}\right)\cdot\left(\frac{19\cdot13+10}{19}\right)\overset{1^0}{\cong}\left(\frac{2}{3}\right)\cdot\left(\frac{10}{19}\right)\overset{5^0}{\cong}(-1)^{\frac{3^2-1}{8}}\cdot\left(\frac{2\cdot5}{19}\right)=$$

$$-\left(\frac{2}{19}\right)\cdot\left(\frac{5}{19}\right)\overset{5^0,6^0}{\cong}-(-1)^{\frac{19^2-1}{8}}\cdot(-1)^{\frac{19-1}{2}\cdot\frac{5-1}{2}}\left(\frac{19}{5}\right)=\left(\frac{19}{5}\right)=$$

$$\left(\frac{5\cdot3+4}{5}\right)\overset{1^0}{\cong}\left(\frac{4}{5}\right)=\left(\frac{2^2}{5}\right)=1.$$ **Javob:** 1.

7). Lejandr simvolining qiymatini hisoblash uchun uning xossalaridan foydalanamiz. Yuqoridagi misollarning ishlanishiga qarang.

$$\left(\frac{251}{577}\right)\overset{6^0}{\cong}(-1)^{\frac{577-1}{2}\cdot\frac{251-1}{2}}\cdot\left(\frac{577}{251}\right)=\left(\frac{577}{251}\right)=\left(\frac{251\cdot2+75}{251}\right)\overset{1^0}{\cong}\left(\frac{75}{251}\right)=$$

$$\left(\frac{5^2\cdot3}{251}\right)$$

$$\overset{4^0}{\cong}\left(\frac{5^2}{251}\right)\cdot\left(\frac{3}{251}\right)\overset{6^0}{\cong}(-1)^{\frac{3-1}{2}\cdot\frac{251-1}{2}}\cdot\left(\frac{251}{3}\right)=-\left(\frac{251}{3}\right)=$$

$$-\left(\frac{3\cdot83+2}{3}\right)\overset{1^0}{\cong}-\left(\frac{2}{3}\right)\overset{5^0}{\cong}-(-1)^{\frac{3^2-1}{8}}=1.$$ **Javob:** 1.

8). Lejandr simvolining qiymatini hisoblash uchun uning xossalaridan foydalanamiz. Yuqoridagi misollarning ishlanishiga qarang.

$$\left(\frac{342}{677}\right)=\left(\frac{2\cdot3^2\cdot19}{677}\right)\overset{4^0}{\cong}\left(\frac{2}{677}\right)\cdot\left(\frac{3^2}{677}\right)\cdot\left(\frac{19}{677}\right)=\left(\frac{2}{677}\right)\cdot$$

$$\left(\frac{19}{677}\right)\overset{5^0,6^0}{\cong}(-1)^{\frac{677^2-1}{8}}\cdot(-1)^{\frac{677-1}{2}\cdot\frac{19-1}{2}}\cdot\left(\frac{677}{19}\right)=-\left(\frac{677}{19}\right)=$$

$-\left(\frac{19\cdot35+12}{19}\right)\overset{1^0}{\cong}-\left(\frac{12}{19}\right)=-\left(\frac{2^2\cdot3}{19}\right)\overset{4^0}{\cong}-\left(\frac{2^2}{19}\right)\cdot\left(\frac{3}{19}\right)=-\left(\frac{3}{19}\right)\overset{6^0}{\cong}-$
$(-1)^{\frac{3-1}{2}\frac{19-1}{2}}\cdot\left(\frac{19}{3}\right)=\left(\frac{19}{3}\right)=\left(\frac{3\cdot6+1}{3}\right)\overset{1^0}{\cong}\left(\frac{1}{3}\right)=1.$ **Javob:** 1.

298. 1). Lejandr simvolidan foydalanib berilgan $x^2\equiv6(mod7)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlashimiz kerak va yechimlari bo‘lsa uni topishimiz kerak. Avvalo, berilgan $x^2\equiv6(mod7)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlaymiz. Buning uchun $\left(\frac{6}{7}\right)$ ning qiymatini aniqlaymiz.

$\left(\frac{6}{7}\right)=\left(\frac{2}{7}\right)\cdot\left(\frac{3}{7}\right)=(-1)^{\frac{7^2-1}{8}}\cdot\left(\frac{3}{7}\right)=\left(\frac{3}{7}\right)=(-1)^{\frac{3-1}{2}\frac{7-1}{2}}\cdot\left(\frac{7}{3}\right)=$
$-\left(\frac{3\cdot2+1}{3}\right)=-\left(\frac{1}{3}\right)=-1.$ Demak, berilgan taqqoslama yechimga ega emas.

Javob: berilgan taqqoslama yechimga ega emas.

2). Lejandr simvolidan foydalanib berilgan $x^2\equiv3(mod11)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlashimiz kerak va yechimlari bo‘lsa uni topishimiz kerak. Avvalo, berilgan $x^2\equiv3(mod11)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlaymiz. Buning uchun $\left(\frac{6}{7}\right)$ ning qiymatini aniqlaymiz.

$$\left(\frac{3}{11}\right)=(-1)^{\frac{3-1}{2}\frac{11-1}{2}}\cdot\left(\frac{11}{3}\right)=-\left(\frac{3\cdot3+2}{3}\right)=-\left(\frac{2}{3}\right)=-(-1)^{\frac{3^2-1}{8}}=1.$$

Demak, berilgan taqqoslama 2 ta yechimga ega.

Berilgan taqqoslamaning yechimlarini topish uchun 11 moduli bo‘yicha chegirmalarning keltirilgan sistemasidagi chegirmalar $\pm1,\pm2,\pm3,\pm4,\pm5$ larni taqqoslamaga qo‘yib sinab ko‘rishimiz yoki taqqoslamalarning xossalaridan foydalanishimiz mumkin. Biz bu yerda birinchi yo‘ldan boramiz va berilgan taqqoslamaning yechimlari $x\equiv\pm5(mod11)$ ekanligini topamiz.

Javob: berilgan taqqoslama yechimga ega va uning yechimlari $x\equiv\pm5(mod11)$ dan iborat.

3). Lejandr simvolidan foydalanib berilgan $x^2\equiv12(mod13)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlashimiz kerak va yechimlari bo‘lsa uni topishimiz kerak. Avvalo, berilgan $x^2\equiv$

$12(mod13)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlaymiz. Buning uchun $\left(\frac{12}{13}\right)$ ning qiymatini aniqlaymiz.

$$\left(\frac{12}{13}\right) = \left(\frac{2^2 \cdot 3}{13}\right) = \left(\frac{2^2}{13}\right) \cdot \left(\frac{3}{13}\right) = \left(\frac{3}{13}\right) = (-1)^{\frac{3-1}{2}\frac{13-1}{2}} \cdot \left(\frac{13}{3}\right) =$$

$\left(\frac{3 \cdot 4+1}{3}\right) = \left(\frac{1}{3}\right) = 1$. Demak, berilgan taqqoslama 2 ta yechimga ega. Berilgan taqqoslamaning yechimlarini topish uchun taqqoslamalarning xossalaridan foydalanamiz. U holda $x^2 \equiv 12(mod13) \to x^2 \equiv 25(mod13) \to x \equiv \pm 5(mod13)$ ekanligini topamiz.

Javob: berilgan taqqoslama yechimga ega va uning yechimlari $x \equiv \pm 5(mod13)$ dan iborat.

4).Lejandr simvolidan foydalanib berilgan $x^2 \equiv 3(mod13)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlashimiz kerak va yechimlari bo'lsa uni topishimiz kerak. Avvalo, berilgan $x^2 \equiv 3(mod13)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlaymiz. Buning uchun $\left(\frac{12}{13}\right)$ ning qiymatini aniqlaymiz.

$\left(\frac{3}{13}\right) = (-1)^{\frac{3-1}{2}\frac{13-1}{2}} \cdot \left(\frac{13}{3}\right) = \left(\frac{3 \cdot 4+1}{3}\right) = \left(\frac{1}{3}\right) = 1$. Demak, berilgan taqqoslama 2 ta yechimga ega.Berilgan taqqoslamaning yechimlarini topish uchun taqqoslamalarning xossalaridan foydalanamiz. U holda $x^2 \equiv 3(mod13) \to x^2 \equiv 16(mod13) \to x \equiv \pm 4(mod13)$ ekanligini topamiz.

Javob: berilgan taqqoslama yechimga ega va uning yechimlari $x \equiv \pm 4(mod13)$ dan iborat.

5).Lejandr simvolidan foydalanib berilgan $x^2 \equiv 5(mod11)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlashimiz kerak va yechimlari bo'lsa uni topishimiz kerak. Avvalo, berilgan $x^2 \equiv 5(mod11)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlaymiz. Buning uchun $\left(\frac{5}{11}\right)$ ning qiymatini aniqlaymiz.

$\left(\frac{5}{11}\right) = (-1)^{\frac{5-1}{2}\frac{11-1}{2}} \cdot \left(\frac{11}{5}\right) = \left(\frac{5 \cdot 2+1}{5}\right) = \left(\frac{1}{5}\right) = 1$. Demak, berilgan taqqoslama 2 ta yechimga ega. Berilgan taqqoslamaning yechimlarini topish uchun taqqoslamalarning xossalaridan foydalanamiz. U holda $x^2 \equiv 5(mod11) \to x^2 \equiv 16(mod11) \to x \equiv \pm 4(mod11)$ ekanligini topamiz.

Javob: berilgan taqqoslama yechimga ega va uning yechimlari $x \equiv \pm 4(mod 11)$ dan iborat.

6). Lejandr simvolidan foydalanib berilgan $x^2 \equiv 13(mod 17)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlashimiz kerak va yechimlari bo'lsa uni topishimiz kerak. Avvalo, berilgan $x^2 \equiv 13(mod 17)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlaymiz. Buning uchun $\left(\frac{13}{17}\right)$ ning qiymatini aniqlaymiz. $\left(\frac{13}{17}\right) = (-1)^{\frac{13-1}{2}\frac{17-1}{2}} \cdot \left(\frac{17}{13}\right) = \left(\frac{13 \cdot 1+4}{13}\right) = \left(\frac{4}{13}\right) = \left(\frac{2^2}{13}\right) = 1.$ Demak, berilgan taqqoslama 2 ta yechimga ega.Berilgan taqqoslamaning yechimlarini topish uchun taqqoslamalarning xossalaridan foydalanamiz. U holda $x^2 \equiv 13(mod 17) \to x^2 \equiv 13 + 17 \cdot 3(mod 17) \to x^2 \equiv 64(mod 17) \to x \equiv \pm 8(mod 17)$ ekanligini topamiz.

Javob: berilgan taqqoslama yechimga ega va uning yechimlari $x \equiv \pm 8(mod 17)$ dan iborat.

7). Lejandr simvolidan foydalanib berilgan $x^2 \equiv 5(mod 17)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlashimiz kerak va yechimlari bo'lsa uni topishimiz kerak. Avvalo, berilgan $x^2 \equiv 5(mod 17)$ taqqoslamaning yechimga ega yoki ega emasligini aniqlaymiz. Buning uchun $\left(\frac{5}{17}\right)$ ning qiymatini aniqlaymiz.

$$\left(\frac{5}{17}\right) = (-1)^{\frac{5-1}{2}\frac{17-1}{2}} \cdot \left(\frac{17}{5}\right) = \left(\frac{5 \cdot 3+2}{5}\right) = \left(\frac{2}{5}\right) = (-1)^{\frac{5^2-1}{8}} = -1$$

Demak, berilgan taqqoslama yechimga ega emas.

Javob: berilgan taqqoslama yechimga ega emas.

299. 1). Berilgan taqqoslama $x^2 \equiv a(mod 5)$ taqqoslama yechimga a ning qiymatini topishimiz kerak. Bizga ma'lumki, $x^2 \equiv a(mod p)$ taqqoslama yechimga bo'lishi uchun a soni $a^{\frac{p-1}{2}} \equiv 1\ (mod p)$ shartni qanoatlantirishi kerak. Bundan $a^2 \equiv 1\ (mod 5) \to a^2 - 1 \equiv\ (mod 5) \to (a-1)(a+1) \equiv 0(mod 5) \to a - 1 \equiv 0(mod 5)$ yoki $a + 1 \equiv 0(mod 5) \to a \equiv \pm 1(mod 5)$.

Javob: $a = \pm 1 + 5t, t \in Z.$

2). Berilgan taqqoslama $x^2 \equiv a(mod 7)$ taqqoslama yechimga a ning qiymatini topishimiz kerak. Bizga ma'lumki, $x^2 \equiv a(mod p)$ taqqoslama yechimga bo'lishi uchun a soni $a^{\frac{p-1}{2}} \equiv 1\ (mod p)$ shartni qanoatlantirishi kerak. Bundan $a^3 \equiv 1\ (mod 7)$. Bu taaqoslamani

7 moduli bo'yicha chegirmalarning keltilgan sistemasi $\pm1, \pm2, \pm3$ larni taqqoslamaga qo'yib sinab ko'ramiz. U holda

$a \equiv -3, 1, 2(mod 7)$ larning berilgan taqqoslamani qanoatlantirishini ko'ramiz.

Javob: $a = -3 + 5t, a = 1 + 5t, a = 2 + 5t, t \in Z.$

3). Berilgan taqqoslama $x^2 \equiv a(mod 11)$ taqqoslama yechimga a ning qiymatini topishimiz kerak. Bizga ma'lumki, $x^2 \equiv a(modp)$ taqqoslama yechim bo'lishi uchun a soni $a^{\frac{p-1}{2}} \equiv 1\ (modp)$ shartni qanoatlantirishi kerak. Bundan $a^5 \equiv 1\ (mod 11)$. Bu taqoslamani 11 moduli bo'yicha chegirmalarning keltilgan sistemasi $\pm1, \pm2, \pm3, \pm4, \pm5$ larni taqqoslamaga qo'yib sinab ko'ramiz. U holda $a \equiv\ 1, 3, 4, 5, 9(mod 11)$ larning berilgan taqqoslamani qanoatlantirishini ko'ramiz.

Javob: $x = 1 + 11t, a = 3 + 11t, a = 4 + 11t, a = 5 + 11t, a = 9 + 11t, t \in Z.$

4).Berilgan taqqoslama $x^2 \equiv a(mod 13)$ taqqoslama yechimga a ning qiymatini topishimiz kerak. Bizga ma'lumki, $x^2 \equiv a(modp)$ taqqoslama yechim bo'lishi uchun a soni $a^{\frac{p-1}{2}} \equiv 1\ (modp)$ shartni qanoatlantirishi kerak. Bundan $a^6 \equiv 1\ (mod 13)$. Bu taqoslamani 13 moduli bo'yicha chegirmalarning keltilgan sistemasi $\pm1, \pm2, \pm3, \pm4, \pm5, \pm6$ larni taqqoslamaga qo'yib sinab ko'ramiz. U holda $a \equiv\ \pm1, \pm3, \pm4\ (mod 13)$ larning berilgan taqqoslamani qanoatlantirishini ko'ramiz. **Javob:** $a = 1 + 13t, a = 3 + 13t, a = 4 + 13t, a = 9 + 13t, a = 10 + 13t, a = 10 + 13t, t \in Z.$

5).Berilgan taqqoslama $x^2 \equiv a(mod 3)$ taqqoslama yechimga a ning qiymatini topishimiz kerak. Bizga ma'lumki, $x^2 \equiv a(modp)$ taqqoslama yechimga ega bo'lishi uchun a soni $a^{\frac{p-1}{2}} \equiv 1\ (modp)$ shartni qanoatlantirishi kerak. Bundan $a \equiv 1\ (mod 3)$. **Javob:** $a = 1 + 3t, t \in Z.$

300. $x^2 + 1 \equiv 0(modp)$ taqqoslama yechimga ega bo'lishi uchun $(-1)^{\frac{p-1}{2}} \equiv 1\ (modp)$ shart bajarilishi kerak. Agar $p = 4n + 1$ ko'rinishidagi tub son bo'lsa, u holda $(-1)^{2n} \equiv 1\ (modp)$ bajariladi va taqqoslama ikkita yechimga ega.

Endi agar berilgan taqqoslama $x^2 + 1 \equiv 0(modp)$ yechimga ega bo'lsa, $p = 4n + 1$ ko'rinishidagi tub son bo'lishini ko'rsatamiz. Butun sonlarni 4ga bo'lgandagi qoldiqlar bo'yicha yozsak: $4n, 4n + 1, 4n +$

$2, 4n+3$ ko'rinishlarda bo'ladi. p – tub son bo'lsa $p = 4n+1$ yoki $p = 4n+3$ ko'rinishda bo'lishi mumkin. Agar $p = 4n+3$ ko'rinishda bo'lsa, Eyler kriteriyasiga ko'ra $(-1)^{\frac{p-1}{2}} \equiv 1\,(modp) \to (-1)^{2n+1} \equiv 1\,(modp)$ bo'lishi kerak, lekin bu taqqoslama bu taqqoslama o'rinli emas. Shuning uchun ham $p = 4n+1$.

301. $a^2 + b^2 \equiv 0(modp)$ bo'lsa, $(a,b) = 1$ bo'lgani uchun $a \not\equiv 0(modp)$ va $b \not\equiv 0(modp)$ bo'lishi kerak. Faraz etaylik, x soni $bx \equiv 1(modp)$ taqqoslamaning yechimi bo'lsin. U holda $(bx)^2 \equiv 1(modp)$ va $(ax)^2 + (bx)^2 \equiv (ax)^2 + 1 \equiv 0(modp)$ bo'lishi kerak. 300-misolga asosan$(ax)^2 + 1 \equiv 0(modp)$taqqoslama faqat va faqat $p = 4n+1$ ko'rinishidagi tub son bo'lsagina o'rinli.

302. $x(x+1) \equiv 1(mod13)$ desak, $x^2 + x - 1 \equiv 0(mod13) \to \left(x+\frac{1}{2}\right)^2 - \frac{5}{4} \equiv 0(mod13) \to (2x+1)^2 \equiv 5(mod13)$. Bu taqqoslama yechimga ega emas. Chunki, Lejandr simvolining qiymati $\left(\frac{5}{13}\right) = (-1)^{\frac{13-1}{2}\frac{5-1}{2}} \cdot \left(\frac{13}{5}\right) = \left(\frac{5\cdot 2+3}{5}\right) = \left(\frac{3}{5}\right) = (-1)^{\frac{3-1}{2}\frac{5-1}{2}} \cdot \left(\frac{5}{3}\right) = \left(\frac{3\cdot 1+2}{3}\right) = \left(\frac{2}{3}\right) = (-1)^{\frac{9-1}{8}} = -1$ ga teng.

303. 302-misolga asosan berilgan $x(x+1) \equiv a(mod13)$ taqqoslamani $(2x+1)^2 \equiv 4a + 1(mod13)$ ko'rinishida yozish mumkin. Ma'lumki, $p > 2$ – moduli bo'yicha chegirmalarning to'la sistemasi $0, \pm 1, \pm 2, \dots, \pm\frac{p-1}{2}$ dagi chegirmalarning yarmi kvadratik chegirma, qolgan yarmi esa kvadratik chegirma emas bo'ladi. Kvadratk chegirmalar sifatida $0, 1, 2, \dots, \frac{p-1}{2}$ larning kvadratlarini olish mumkin. Shuning uchun ham $4a + 1 \equiv 0, 1, 4, 9, 3, 12, 10(mod13) \to 4a \equiv -1, 0, 3, 8, 2, 11, 9(mod13) \to a \equiv 3, 0, 4, 2, 7, 6, 12(mod13)$. Shunday qilib, $a = 13t,\ a = 2 + 13t, 3 + 13t, 4 + 13t, 6 + 13t, 7 + 13t, 12 + 13t, t \in Z$.

Javob: $a = 13t, a = 2 + 13t, 3 + 13t, 4 + 13t, 6 + 13t, 7 + 13t, 12 + 13t, t \in Z$.

304. Faraz etaylik, bunday tub sonlarning soni chekli bo'lib, ular $p_1, p_2, \dots, p_k$ lardan iborat bo'lsin. $N = (p_1 p_2 \cdots p_k)^2 + 1$ sonini qaraymiz. Bu son 300-misolga asosan faqat $4n+1$ ko'rinishidagi tub sonlarga bo'linadi. Lekin N soni $p_1, p_2, \dots, p_k$ ning birortasiga ham bo'linmaydi. Shuning uchun ham N ning o'zi tub son yoki u biror p_{k+1} tub bo'luvchiga ega. Demak, $4n+1$ ko'rinishidagi tub sonlar soni cheksiz ko'p.

305. 1).$4x^2-5y=6 \to 4x^2=6+5y \to 4x^2 \equiv 6(mod5) \to 2x^2 \equiv 3(mod5) \to 2x^2 \equiv 8(mod5) \to x^2 \equiv 4(mod5) \to x \equiv \pm 2(mod5) \to x=\pm 2+5t, t \in Z. x$ ning topilgan qiymatini berilgan tenglamaga qo'yib y ning qiymatini aniqlaymiz: $4(\pm 2+5t)^2-5y=6 \to 4(4 \pm 20t+25t^2)-5y=6 \to 5y=10 \pm 80t+100t^2 \to y=2 \pm 16t+20t^2$. Shunday qilib, izlanayotgan yechim $(\pm 2+5t, 2 \pm 16t+20t^2), t \in Z$.

Javob:$(\pm 2+5t, 2 \pm 16t+20t^2), t \in Z$.

2). $5x^2=11y+7 \to 5x^2 \equiv 7(mod11) \to x^2 \equiv 8(mod11)$.

Oxirgi taqqoslamada $\left(\frac{8}{11}\right)=\left(\frac{4\cdot 2}{11}\right)=\left(\frac{2}{11}\right)=(-1)^{\frac{121-1}{8}}=-1$ bo'lgani uchun u yechimga ega emas. Demak, berilgan egri chiziq butun koordinatali nuqtadan o'tmaydi. **Javob:** $\emptyset$.

3).$x^2-10x+5=11y \to (x-5)^2-20=11y \to (x-5)^2 \equiv 20(mod11) \to (x-5)^2 \equiv 9(mod11) \to x-5 \equiv \pm 3(mod11) \to x \equiv 5 \pm 3(mod11) \to x \equiv 2(mod11)$va $x \equiv 8(mod11) \to x=2+11t$ va $x=8+11t, t \in Z. x$ ning bu topilgan qiymatlariga mos y ning qiymatlarini aniqlaymiz. Avvalo, $x=2+11t$ ga mos y ning qiymatini aniqlaymiz Buning uchun x ning topilgan qiymatini berilgan tenglamaga olib borib qo'yamiz: $(2+11t)^2-10(2+11t)+5=11y \to 11y=121t^2-66t-11 \to y=11t^2-6t-1$.

Endi $x=8+11t$ ga mos y ning qiymatini aniqlaymiz:

$$(8+11t)^2-10(8+11t)+5=11y \to 11y = 121t^2+176t+64-80-$$

$-110t+5 \to 11y=121t^2+66t-11 \to y=11t^2+6t-1$.Demak, yechimlar $(2+11t, 11t^2-6t-1)$ va $(8+11t, 11t^2+6t-1), t \in Z$.

Javob: $(2+11t, 11t^2-6t-1)$ va $(8+11t, 11t^2+6t-1), t \in Z$.

4).$x^2-21x+110=13y \to x^2-21x+110 \equiv 0(mod13) \to x^2-8x+6 \equiv 0(mod13) \to (x-4)^2 \equiv 10(mod13) \to (x-4)^2 \equiv 36(mod13) \to x-4 \equiv \pm 6(mod13) \to x \equiv 4 \pm 6(mod13) \to x \equiv -2(mod13)$va $x \equiv 10(mod13) \to x=-2+13t$ va $x=10+13t, t \in Z$. x ning bu topilgan qiymatlariga mos y ning qiymatlarini aniqlaymiz. Avvalo, $x = -2 + 13t$ ga mos y ning qiymatini aniqlaymiz Buning uchun x ning topilgan qiymatini berilgan tenglamaga olib borib qo'yamiz: $(-2+13t)^2 - 21(-2+13t) + 110 = 13y \to 4 - 52t + 169t^2 + 42 - 273t + 110 = 13y \to 169t^2 - 325t + 156 = 13y \to y = 13t^2 - 25t + 12, t \in Z$. Endi $x = 10 + 13t$ ga mos y ning qiymatini aniqlaymiz: $(10+13t)^2 - 21(10+13t) + 110 = 13y \to 13y = 169t^2 + 260t + 100 - 210 - 273t + 110 \to 13y = 169t^2 - 13t \to y = 13t^2 - t, t \in Z$. Demak, yechimlar $(-2+13t, 13t^2 - 25t + 12)$ va $(10+13t, 13t^2 - t), t \in Z$.

Javob: $((-2+13t, 13t^2 - 25t + 12)$ va $(10+13t, 13t^2 - t), t \in Z$.

5). $15x^2 - 7y^2 = 9 \to 15x^2 \equiv 9(mod7) \to x^2 \equiv 9(mod7) \to x = \pm 3 + 7t$. x ning bu topilgan qiymatlariga mos y ning qiymatlarini aniqlaymiz. Avvalo, $x = -3 + 7t$ ga mos y ning qiymatini aniqlaymiz. Buning uchun x ning topilgan qiymatini berilgan tenglamaga olib borib qo'yamiz: $15(-3+7t)^2 - 7y^2 = 9 \to 15(9 - 42t + 49t^2) - 7y^2 = 9 \to 135 - 630t + 735t^2 - 7y^2 = 9 \to 126 - 630t + 735t^2 = 7y^2 \to y^2 = 105t^2 - 90t + 18$. Bunda oxirgi ifodaning o'ng tomonidagi uchhadning diskriminanti 540 ga teng va shuning uchun ham u to'liq kvadratni bermaydi, ya'ni y ning butun qiymatlari mavjud emas. Endi $x = 3 + 7t$ ga mos y ning qiymatini aniqlaymiz: : $15(3+7t)^2 - 7y^2 = 9 \to 15(9 + 42t + 49t^2) - 7y^2 = 9 \to 135 + 630t + 735t^2 - 7y^2 = 9 \to 126 + 630t + 735t^2 = 7y^2 \to y^2 = 105t^2 + 90t + 18$. Bunda ham oxirgi ifodaning o'ng tomonidagi uchhadning diskriminanti 540 ga teng va shuning uchun ham u to'liq kvadratni bermaydi, ya'ni y ning butun qiymatlari mavjud. Demak, berilgan tenglama butun sonlarda yechimga ega emas. **Javob:** berilgan tenglama yechimga ega emas.

306. 1).Lejandr simvolining ta'rifiga asosan $\left(\frac{5}{p}\right) = (-1)^{\frac{p-1}{2}\frac{5-1}{2}} \cdot \left(\frac{p}{5}\right) = \left(\frac{p}{5}\right)$ bo'lganidan, $a = 5$ soni p-tub moduli bo'yicha kvadratik chegirma bo'lishi uchun Eyler kriteriyasiga asosan $p^{\frac{5-1}{2}} \equiv 1(mod5) \to p^2 \equiv 1(mod5)$ ning bajarilishi zarur va yetarlidir. Bundan $p^2 \equiv 16(mod5) \to p \equiv \pm 4(mod5) \to p \equiv \pm 1(mod5)$ ni hosil qilamiz. Buni $p = \pm 1 + 5k$ ko'rinishida yozish mumkin.

Umuman, butun sonlarni 5 moduli bo'yicha 5 ta: $5k, 5k+1, 5k+2, 5k+3, 5k+4$ sinfga ajratish mumkin bo'lgani uchun, agar $p = 5k +$

1 yoki $p = 5k + 4$ ko'rinishdagi tub son bo'lsa, $a = 5$ soni p-tub moduli bo'yicha kvadratik chegirma, agar $p = 5k + 2$ yoki $p = 5k + 3$ ko'rinishdagi tub son bo'lsa, $a = 5$ soni p-tub moduli bo'yicha kvadratik chegirma emas bo'lar ekan.

Javob: $a = 5$ soni $p = 5k + 1$ va $p = 5k + 4$ ko'rinishdagi tub modullar bo'yicha kvadratik chegirma, $p = 5k + 2$ va $p = 5k + 3$ ko'rinishdagi tub modullar bo'yicha kvadratik chegirma emas bo'ladi.

2).Lejandr simvolining ta'rifiga asosan $\left(\frac{-3}{p}\right) = \left(\frac{-1\cdot 3}{p}\right) = \left(\frac{-1}{p}\right)\left(\frac{3}{p}\right) = (-1)^{\frac{p-1}{2}}(-1)^{\frac{p-1}{2}\frac{3-1}{2}} \cdot \left(\frac{p}{3}\right) = \left(\frac{p}{3}\right)$ bo'lganidan, $a = -3$ soni p-tub moduli bo'yicha kvadratik chegirma bo'lishi uchun Eyler kriteriyasiga asosan $p^{\frac{3-1}{2}} \equiv 1(mod3) \rightarrow p \equiv 1(mod3)$ ning bajarilishi zarur va yetarlidir. Buni $p = 1 + 3k$ ko'rinishida yozish mumkin.

Umuman, butun sonlarni 3 moduli bo'yicha 3 ta: $3k, 3k + 1, 3k + 2$ sinfga ajratish mumkin bo'lgani uchun, agar $p = 3k + 1$ ko'rinishdagi tub son bo'lsa, $a = -3$ soni p-tub moduli bo'yicha kvadratik chegirma, agar $p = 3k + 2$ yoki ko'rinishdagi tub son bo'lsa, $a = -3$ soni p-tub moduli bo'yicha kvadratik chegirma emas bo'lar ekan.

Javob: $a = -3$ soni $p = 3k + 1$ ko'rinishdagi tub modul bo'yicha kvadratik chegirma, $p = 3k + 2$ ko'rinishdagi tub modul bo'yicha kvadratik chegirma emas bo'ladi.

3).Lejandr simvolining ta'rifiga asosan $\left(\frac{3}{p}\right) = (-1)^{\frac{p-1}{2}\frac{3-1}{2}} \cdot \left(\frac{p}{3}\right) = (-1)^{\frac{p-1}{2}} \cdot \left(\frac{p}{3}\right)$ bo'lganidan, agar $p = 3k + 1$ ko'rinishida bo'lsa,

$$\left(\frac{3}{p}\right) = (-1)^{\frac{3k}{2}} \cdot \left(\frac{3k+1}{3}\right) = (-1)^{\frac{3k}{2}} \cdot \left(\frac{1}{3}\right) = (-1)^{\frac{3k}{2}} \qquad (*)$$

bo'ladi. Agar bunda $k = 4q$ bo'lsa, (*) dan $\left(\frac{3}{p}\right) = 1$ hosil bo'ladi, ya'ni 3 soni $p = 12q + 1$ ko'rinishdagi tub moduli bo'yicha kvadratik chegirma bo'ladi. 12 moduli bo'yicha barcha butun sonlarni 12 ta sinfga ajratish mumkin. Bulardan $12q + 1, 12q + 5, 12q + 7, 12q + 11$ sinflardagina tub sonlar bo'ladi. Agar $p = 12q + 5$ bo'lsa, u holda

$$\left(\frac{3}{p}\right) = (-1)^{\frac{12q+4}{2}} \cdot \left(\frac{12q+5}{3}\right) = \left(\frac{3\cdot(4q+1)+2}{3}\right) = \left(\frac{2}{3}\right) = (-1)^{\frac{9-1}{8}} = -1;$$

agarda $p = 12q + 7$ bo'lsa,

$\left(\frac{3}{p}\right) = (-1)^{\frac{12q+6}{2}} \cdot \left(\frac{12q+7}{3}\right) = -\left(\frac{3\cdot(4q+2)+1}{3}\right) = -\left(\frac{1}{3}\right) = -1;$

agarda $p = 12q + 11$ bo'lsa,

$\left(\frac{3}{p}\right) = (-1)^{\frac{12q+10}{2}} \cdot \left(\frac{12q+11}{3}\right) = -\left(\frac{3\cdot(4q+3)+2}{3}\right) = -\left(\frac{2}{3}\right) = 1$

larni hosil qilamiz. Shunday qilib, 3 soni $p = 12q + 1, p = 12q + 11$ ko'rinishdagi tub modullar bo'yicha kvadratik chegirma, $p = 12q + 5, p = 12q + 7$ ko'rinishdagi tub modullar bo'yicha kvadratik chegirma emas bo'lar ekan.

Javob: 3 soni $p = 12q + 1, p = 12q + 11$ ko'rinishdagi tub modullar bo'yicha kvadratik chegirma, $p = 12q + 5, p = 12q + 7$ ko'rinishdagi tub modullar bo'yicha kvadratik chegirma emas bo'ladi.

4). $a = 2$ ning p moduli bo'yicha kvadratik chegirma bo'lishi uchun Lejandr simvolining ta'rifiga asosan $\left(\frac{2}{p}\right) = (-1)^{\frac{p^2-1}{8}} = 1$ bajarilishi kerak. Buning uchun esa $\frac{p^2-1}{8} = 2q \to p^2 = 1 + 16q \to p^2 \equiv 1(mod16)$ ko'rinishida bo'lishi kerak. Oxirgi taqqoslamani $p = 16k + 1, p = 16k + 3, p = 16k + 5, p = 16k + 7, p = 16k + 9, p = 16k + 11, p = 16k + 13, p = 16k + 15$ lardan foydalanib tekshirsak $p = 16k + 1, p = 16k + 7, p = 16k + 9, p = 16k + 15$ uni qanoatlantiradi. Qolganlari qanoatlantirmaydi. Shuning uchun ham 2 soni $p = 16k + 1, p = 16k + 7, p = 16k + 9, p = 16k + 15$ modullar bo'yicha kvadratik chegirma, $p = 16k + 3, p = 16k + 5, p = 16k + 11, p = 16k + 13$modullar bo'yicha kvadratik chegirma emas bo'ladi. Bularni 8 moduli birlashtirib yozib olishimiz mumkin. U holda 2 soni $p = 8k + 1, p = 8k + 7$modullar bo'yicha kvadratik chegirma, $p = 8k + 3, p = 8k + 5,$ modullar bo'yicha kvadratik chegirma emas bo'ladi.

Javob: 2 soni $p = 8k + 1, p = 8k + 7$modullar bo'yicha kvadratik chegirma, $p = 8k + 3, p = 8k + 5,$ modullar bo'yicha kvadratik chegirma emas bo'ladi.

5).Lejandr simvolining ta'rifiga asosan$\left(\frac{-7}{p}\right) = \left(\frac{-1\cdot 7}{p}\right) = \left(\frac{-1}{p}\right)\left(\frac{7}{p}\right) = (-1)^{\frac{p-1}{2}} \cdot (-1)^{\frac{p-1}{2}\frac{7-1}{2}} \cdot \left(\frac{p}{7}\right) = \left(\frac{p}{7}\right)$ bo'lganidan, $a = -7$ soni p-tub moduli bo'yicha kvadratik chegirma bo'lishi uchun Eyler kriteriyasiga asosan

$p^{\frac{7-1}{2}} \equiv 1(mod7) \to p^3 \equiv 1(mod7)$ ning bajarilishi zarur va yetarlidir. Buni $p = 1 + 7k,\ p = 2 + 7k, p = 3 + 7k, p = 4 + 7k, p = 5 + 7k, p = 6 + 7k$ larni qo'yib tekshirsak, $p = 1 + 7k,\ p = 2 + 7k, p = 4 + 7k$ lar uni qanoatlantiradi, qolganlari esa qanoatlantirmaydi. Demak, $a = -7$ soni $p = 1 + 7k,\ p = 2 + 7k, p = 4 + 7k$ modullar bo'yicha kvadratik chegirma, $p = 3 + 7k,\ p = 5 + 7k, p = 6 + 7k$ modullari bo'yicha kvadratik chegirma emas bo'lar ekan.

Javob: $a = -7$ soni $p = 1 + 7k,\ p = 2 + 7k, p = 4 + 7k$ modullar bo'yicha kvadratik chegirma, $p = 3 + 7k,\ p = 5 + 7k, p = 6 + 7k$ modullari bo'yicha kvadratik chegirma emas bo'ladi.

307.1). Berilgan taqqoslamadan $x(x + 1) \equiv 1(modp) \to x^2 + x - 1 \equiv 0(modp) \to \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} - 1 \equiv 0(modp) \to (2x + 1)^2 \equiv 5(modp)$. Bu taqqoslama yechimga ega bo'lishi uchun $\left(\frac{5}{p}\right) = (-1)^{\frac{p-1}{2}\cdot\frac{5-1}{2}} \cdot \left(\frac{p}{5}\right) = \left(\frac{p}{5}\right) = 1$ bo'lishi kerak. $p = 1 + 5k,\ p = 2 + 5k, p = 3 + 5k, p = 4 + 5k$ larni qo'yib tekshirsak, $p = 1 + 5k,\ p = 4 + 5k$ lar uni qanoatlantiradi, qolganlari esa qanoatlantirmaydi.

Javob:$p = 1 + 5k,\ p = 4 + 5k$ modullar bo'yicha berilgan taqqoslama yechimga ega, $p = 2 + 5k, p = 3 + 5k$ modullar bo'yicha taqqoslama yechimga ega.

2). Berilgan taqqoslamadan $x(x - 1) \equiv 2(modp) \to x^2 - x - 2 \equiv 0(modp) \to \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 2 \equiv 0(modp) \to (2x - 1)^2 \equiv 9(modp)$. Bu yerda $\left(\frac{9}{p}\right) = 1$ bo'lgani uchun. Ixtiyoriy $p > 2$ tub modul uchun berilgan taqqoslama yechimga ega bo'ladi.

Javob: Ixtiyoriy $p > 2$ modul bo'yicha berilgan taqqoslama yechimga ega.

3). Berilgan taqqoslamadan $x(x - 1) \equiv 3(modp) \to x^2 - x - 3 \equiv 0(modp) \to \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 3 \equiv 0(modp) \to (2x - 1)^2 \equiv 13(modp)$.

Bu taqqoslama yechimga ega bo'lishi uchun $\left(\frac{13}{p}\right) = (-1)^{\frac{p-1}{2}\cdot\frac{13-1}{2}} \cdot \left(\frac{p}{13}\right) = \left(\frac{p}{13}\right) = 1$ bo'lishi kerak. $p = 1 + 13k,\ p = 3 + 13k,\ p = 4 +$

$13k,\ p = 9 + 13k, p = 10 + 13k, p = 12 + 13k$ va $p = 13$ lar uni qanoatlantiradi, qolganlari esa qanoatlantirmaydi.

Javob: $p = 1 + 13k,\ p = 3 + 13k,\ p = 4 + 13k,\ p = 9 + 13k, p = 10 + 13k,\ p = 12 + 13k$ **va** $p = 13$ modullar bo'yicha taqqoslama yechimga ega. $p = 2 + 13k,\ p = 5 + 13k,\ p = 6 + 13k,\ p = 7 + 13k, p = 8 + 13k,\ p = 11 + 13k$ modullar bo'yicha berilgan taqqoslama yechimga ega emas.

308.1). Agar $x^2 \equiv 13(mod p)$ yoki $x^2 \equiv 17(mod p)$ lardan birortasi o'rinli bo'lsa, berilgan taqqoslama $(x^2 - 13)(x^2 - 17)(x^2 - 221) \equiv 0(mod p)$ yechimga ega bo'ladi. Agar ularning ikkalasi ham yechimga ega bo'lmasa, $\left(\frac{13}{p}\right) = \left(\frac{17}{p}\right) = -1$ bajarilishi kerak. Bundan $\left(\frac{13}{p}\right) \cdot \left(\frac{17}{p}\right) = \left(\frac{221}{p}\right) = 1$ kelib chiqadi. Bu esa $x^2 \equiv 221(mod p)$ bajariladi degani. Demak, berilgan taqqoslama ixtiyoriy $p > 2$ tub modul bo'yicha o'rinli.

2). Agar $x^2 \equiv 3(mod p)$, yoki $x^2 \equiv 5(mod p)$, yoki $x^2 \equiv 7(mod p)$, yoki $x^2 \equiv 11(mod p)$ lardan birortasi o'rinli bo'lsa, berilgan taqqoslama$(x^2 - 3)(x^2 - 5)(x^2 - 7)(x^2 - 11)(x^2 - 1155) \equiv 0(mod p)$ yechimga ega bo'ladi. Agar ularning to'rtalasi ham yechimga ega bo'lmasa, $\left(\frac{3}{p}\right) = \left(\frac{5}{p}\right) = \left(\frac{7}{p}\right) = \left(\frac{11}{p}\right) = -1$ bajarilishi kerak. Bundan $\left(\frac{3}{p}\right) \cdot \left(\frac{5}{p}\right) \cdot \left(\frac{7}{p}\right) \cdot \left(\frac{11}{p}\right) = 1$ kelib chiqadi. Bu esa $x^2 \equiv 1155(mod p)$ bajariladi degani. Demak, berilgan taqqoslama ixtiyoriy $p > 2$ tub modul bo'yicha o'rinli.

V.1-§.

309.1). Buning uchun a sonining m moduli bo'yicha tegishli bo'lgan daraja ko'rsatkichi $\varphi(m)$ ning bo'luvchilari orasida bo'lishidan (2-natijadan) foydalanamiz. Bu misolda $a = 2, m = 7$ va $\varphi(7) = 6,$ bo'lib 6 ning bo'luvchilari: 1,2,3,6 lardan iborat. Shuning uchun ham 2 ning ana shu darajalarini tekshiramiz. U holda $2^1 \equiv 2,\ 2^2 \equiv 4,\ 2^3 \equiv 1(mod 7)$lardan 2 sonining 7 moduli bo'yicha tegishli bo'lgan daraja ko'rsatkichi $\delta = P_7(2) = 3$ ga teng degan xulosaga kelamiz.

Javob: $P_7(2) = 3.$

2).1-misoldagi singari mulohaza yuritamiz. Bu misolda $a = 3, m = 7$ va $\varphi(7) = 6,$ bo'lib 6 ning bo'luvchilari: 1,2,3,6 lardan iborat. Shuning uchun ham 3 ning ana shu darajalarini tekshiramiz. U holda $3^1 \equiv$

$3, \ 3^2 \equiv 2, \ 3^3 \equiv 6, \ 3^6 \equiv (3^3)^2 \equiv 1(mod7)$lardan 3 sonining 7 moduli bo'yicha tegishli bo'lgan daraja ko'rsatkichi $\delta = P_7(3) = 6$ ga teng degan xulosaga kelamiz. **Javob**:$P_7(3) = 6$.

3). 1va 2-misollardagi singari mulohaza yuritamiz. Bu misolda $a = 5, m = 7$ va $\varphi(7) = 6$, bo'lib 6 ning bo'luvchilari: 1,2,3,6 lardan iborat. Shuning uchun ham 3 ning ana shu darajalarini tekshiramiz. U holda $5^1 \equiv 5, \ 5^2 \equiv 4, \ 5^3 \equiv 6, \ 5^6 \equiv (5^3)^2 \equiv 1(mod7)$lardan 5 sonining 7 moduli bo'yicha tegishli bo'lgan daraja ko'rsatkichi $\delta = P_7(5) = 6$ ga teng degan xulosaga kelamiz.

Javob: $P_7(5) = 6$.

Shunday qilib bitta m moduli bo'yicha bir nechta boshlang'ich ildizlar bo'lishi mumkin ekan.

310. 1).Tanlash usuli bilan m moduli bo'yicha 2 dan $m - 1$ gacha sonlar orasidan m bilan o'zaro tublari tegishli bo'lgan daraja ko'rsatkichlarini topishimiz kerak.Bu misolda $m = 5$ bo'lgani uchun 2 dan 4 gacha sonlar orasidan 5 bilan o'zaro tublari: 2, 3, 4 lardan iborat. Bu sonlarning $m = 5$ moduli bo'yicha tegishli bo'lgan daraja ko'rsatkichlarini aniqlaymiz. Buning uchun 309-misollardagi singari mulohaza yuritamiz. $\varphi(5) = 4$, bo'lib 4 ning bo'luvchilari: 1,2,4 lardan iborat.U holda $2^1 \equiv 2, \ 2^2 \equiv 4, \ 2^4 \equiv 1(mod5); \ 3^1 \equiv 3, \ 3^2 \equiv 4, \ 3^4 \equiv 1(mod5); \ 4^1 \equiv 4, \ 4^2 \equiv 1(mod5)$lardan 2 va 3 sonlari 5 moduli bo'yicha 4 daraja ko'rsatkichiga, 4 soni esa 2 daraja ko'rsatkichiga tegishli ekan degan xulosaga kelamiz.

Javob: $P_5(2) = P_5(3) = 4, \ P_5(4) = 2$.

2). Bu misolda $m = 7$ bo'lgani uchun 2 dan 6 gacha sonlar orasidan 7 bilan o'zaro tublari: 2, 3, 4, 5, 6 lardan iborat. Bu sonlarning $m = 7$ moduli bo'yicha tegishli bo'lgan daraja ko'rsatkichlarini aniqlaymiz. Buning uchun 1-misoldagi singari mulohaza yuritamiz. 309-misolda $P_7(2) = 3, P_7(3) = P_7(5) = 6$ ekanliklarini aniqlagan edik. Shuning uchun 4, 6 sonlarining $m = 7$ moduli bo'yicha tegishli bo'lgan daraja ko'rsatkichlarini aniqlaymiz. $\varphi(7) = 6$, bo'lib 6 ning bo'luvchilari: 1,2,3,6 lardan iborat. U holda $4 = 2^2$ bo'lib $(2,3) = 1$ bo'lgani uchun $P_7(4) = 3$. $6^1 \equiv -1, \ 6^2 \equiv 1(mod7)$dan 6 soni 7 moduli bo'yicha 2 daraja ko'rsatkichiga tegishli ekan degan xulosaga kelamiz.

Javob: $P_7(2) = P_7(4) = 3, \ P_7(3) = P_7(5) = 6, P_7(6) = 2$.

3). Bu misolda $m = 8$ bo'lgani uchun 2 dan 7 gacha sonlar orasidan 8 bilan o'zaro tublari: 3, 5, 7 lardan iborat. Bu sonlarning $m = 8$moduli bo'yicha tegishli bo'lgan daraja ko'rsatkichlarini aniqlaymiz. Buning uchun 1,2-misollardagi singari mulohaza yuritamiz. $\varphi(8) = 4$, bo'lib 4 ning bo'luvchilari: 1,2,4 lardan iborat.U holda $3^1 \equiv 3$, $3^2 \equiv 1(mod8)$; $5^1 \equiv 5$, $5^2 \equiv 1(mod5)$; $7^1 \equiv -1$, $7^2 \equiv 1(mod8)$lardan qaralayotgan sonlarning barchasi 8 moduli bo'yicha 2 daraja ko'rsatkichiga tegishli ekan degan xulosaga kelamiz.

Javob: $P_8(3) = P_8(5) = P_8(7) = 2$.

4). Bu misolda $m = 10$ bo'lgani uchun 2 dan 9 gacha sonlar orasidan 10 bilan o'zaro tublari: 3, 7, 9 lardan iborat. Bu sonlarning $m = 10$ moduli bo'yicha tegishli bo'lgan daraja ko'rsatkichlarini aniqlaymiz. Buning uchun 1,2, 3-misollardagi singari mulohaza yuritamiz. $\varphi(10) = 4$, bo'lib 4 ning bo'luvchilari: 1,2,4 lardan iborat. U holda $3^1 \equiv 3$, $3^2 \equiv -1, 3^4 \equiv 1(mod10)$; $7^1 \equiv 7$, $7^2 \equiv -1, 7^4 \equiv 1(mod10)$; $9^1 \equiv -1$, $9^2 \equiv 1(mod10)$lardan qaralayotgan 3 va 7 sonlari 10 moduli bo'yicha 4 daraja ko'rsatkichiga, 9 soni esa 2 daraja ko'rsatkichiga tegishli ekan degan xulosaga kelamiz.

Javob: $P_{10}(3) = P_{10}(7) = 4, P_{10}(9) = 2$.

5). Bu misolda $m = 11$ bo'lgani uchun 2 dan 10 gacha sonlar orasidan 11 bilan o'zaro tublari: 2, 3, 4, 5, 6, 7, 8, 9, 10 lardan iborat. Bu sonlarning $m = 11$ moduli bo'yicha tegishli bo'lgan daraja ko'rsatkichlarini aniqlaymiz. Buning uchun 1,2, 3-misollardagi singari mulohaza yuritamiz. $\varphi(11) = 10$, bo'lib 10 ning bo'luvchilari: 1,2,5, 10 lardan iborat. U holda $2^1 \equiv 2$, $2^2 \equiv 4, 2^5 \equiv -1, 2^{10} \equiv 1(mod11)$; $3^1 \equiv 3$, $3^2 \equiv -2, 3^5 \equiv 1(mod11)$; $4^1 \equiv 4$, $4^2 \equiv 5, 4^5 \equiv 1(mod11)$; $5^1 \equiv 5$, $5^2 \equiv 3, 5^5 \equiv 1(mod11)$; $6^1 \equiv 6, 6^2 \equiv 3, 6^5 \equiv -1, 6^{10} \equiv 1(mod11)$; $7^1 \equiv 7$, $7^2 \equiv 5$, $7^5 \equiv -1$, $7^{10} \equiv 1(mod11)$; $8^1 \equiv 8$, $8^2 \equiv -2$, $8^5 \equiv -1$, $8^{10} \equiv 1(mod11)$; $9^1 \equiv -2$, $9^2 \equiv 4$, $9^5 \equiv 1(mod11)$;

$10^1 \equiv -1, 10^2 \equiv 1(mod11)$ lardan qaralayotgan 2,6,7 va 8 sonlari 11 moduli bo'yicha 10 daraja ko'rsatkichiga, 3, 4, 5, 9 sonlari 5 daraja ko'rsatkichiga, 10 soni esa 2 daraja ko'rsatkichiga tegishli ekan degan xulosaga kelamiz. **Javob**: $P_{11}(2) = P_{11}(6) = P_{11}(7) = P_{11}(8) = 10$, $P_{11}(3) = P_{11}(4) = P_{11}(5) = P_{11}(9) = 5, P_{11}(10) = 2$.

6). Bu misolda $m = 9$ bo'lgani uchun 2 dan 8 gacha sonlar orasidan 9 bilan o'zaro tublari: 2, 4, 5, 7, 8 lardan iborat. Bu sonlarning $m = 9$ moduli bo'yicha tegishli bo'lgan daraja ko'rsatkichlarini aniqlaymiz. Buning uchun yuqoridagi misollardagi singari mulohaza yuritamiz. $\varphi(9) = 6$, bo'lib 6 ning bo'luvchilari: 1,2,3,6 lardan iborat. U holda $2^1 \equiv 2,\ 2^2 \equiv 4, 2^3 \equiv -1, 2^6 \equiv 1(mod9); 4^1 \equiv 4,\ 4^2 \equiv -2, 4^3 \equiv 1, (mod9); 5^1 \equiv 5,\ 5^2 \equiv -2, 5^3 \equiv -1, 5^6 \equiv 1(mod9); 7^1 \equiv -2,\ 7^2 \equiv 4, 7^3 \equiv 1(mod9);\ 8^1 \equiv -1,\ 8^2 \equiv 1(mod9)$; lardan qaralayotgan 2va 5 sonlari 9 moduli bo'yicha 6 daraja ko'rsatkichiga, 4, 7 sonlari 3 daraja ko'rsatkichiga, 8 soni esa 2 daraja ko'rsatkichiga tegishli ekan degan xulosaga kelamiz.

Javob:$P_9(2) = P_9(5) = 6,\ P_9(4) = P_9(7) = 3, P_9(8) = 2.$

311. Ta'rifga ko'ra $(m-1)^\delta \equiv 1(modm)$ shartni qanoatlantiruvchi eng kichik $\delta > 0$ natural sonni topish kerak. Bu taqqoslama $(-1)^\delta \equiv 1(modm$ ga teng kuchli. Bundan, agar $m = 2$ bo'lsa, $\delta = 1$ va agar $m \geq 3$ bo'lsa, $\delta = 2$ kelib chiqadi.

Javob: $P_m(m-1) = \begin{cases} 1, \text{agar } m = 2 \text{ bo'lsa}, \\ 2, \text{agar } m \geq 3 \text{ bo'lsa}. \end{cases}$

312. 1). 7 moduli bo'yicha barcha boshlang'ich ildizlarni topish uchun shu modul bo'yicha chegirmalarning keltirilgan sistemasi 2,3,4,5,6 lar orasidan $\varphi(7) = 6$daraja ko'rsatkichiga tegishlilarini ajratib olamiz. $\varphi(7) = 6$ ning bo'luvchilari 2,3 bo'lgani uchun $g^2 \not\equiv 1(mod7)$, $g^3 \not\equiv 1(mod7)$ shartlarni qanoatlantiruvchilari ajratib olishimiz kerak. 2,3,4,5,6 larni g ning o'rniga qo'yib tekshirib ko'ramiz: $2^2 \not\equiv 1(mod7),\ 2^3 \equiv 1(mod7);\ 3^2 \not\equiv 1(mod7),\ 3^3 \not\equiv 1(mod7);\ 4^2 \not\equiv 1(mod7),\ 4^3 \equiv 1(mod7);\ 5^2 \not\equiv 1(mod7),$
$5^3 \not\equiv 1(mod7);$

$6^2 \equiv 1(mod7)$. Demak, 7 moduli bo'yicha barcha boshlang'ich ildizlar 3,5 lardan iborat bo'lar ekan. Ularning soni $\varphi(\varphi(p)) = \varphi(p-1) = \varphi(6) = 2$ ta.

Javob: 3,5.

2). 11 moduli bo'yicha barcha boshlang'ich ildizlarni topish uchun shu modul bo'yicha chegirmalarning keltirilgan sistemasi 2,3,4,5,6,7,8,9,10 lar orasidan $\varphi(11) = 10$daraja ko'rsatkichiga tegishlilarini ajratib olamiz. $\varphi(11) = 10$ ning bo'luvchilari 2,5 bo'lgani uchun $g^2 \not\equiv 1(mod7)$, $g^5 \not\equiv 1(mod11)$ shartlarni qanoatlantiruvchilarini ajratib olishimiz kerak. $2,3,4,5,6,7,8,9,10$ larni g ning o'rniga qo'yib tekshirib ko'ramiz: $2^2 \not\equiv 1(mod11)$, $2^5 \not\equiv 1(mod11)$; $3^2 \not\equiv 1(mod11)$, $3^5 \equiv 1(mod11)$; $4^2 \not\equiv 1(mod11)$, $4^5 \equiv 1(mod11)$; $5^2 \not\equiv 1(mod11)$, $5^5 \equiv 1(mod11)$; $6^2 \not\equiv 1(mod11)$, $6^5 \not\equiv 1(mod11)$; $7^2 \not\equiv 1(mod11)$, $7^5 \not\equiv 1(mod11)$; $8^2 \not\equiv 1(mod11)$, $8^5 \not\equiv 1(mod11)$; $9^2 \not\equiv 1(mod11)$, $9^5 \equiv 1(mod11)$; $10^2 \equiv 1(mod11)$. Demak, 11 moduli bo'yicha barcha boshlang'ich ildizlar 2,6,7,8 lardan iborat bo'lar ekan. Ularning soni $\varphi(\varphi(p)) = \varphi(p-1) = \varphi(10) = 4$ ta.

Javob: 2,6,7,8.

3). 13 moduli bo'yicha barcha boshlang'ich ildizlarni topish uchun shu modul bo'yicha chegirmalarning keltirilgan sistemasi $2,3,4,5,6,7,8,9,10,11,12$ lar orasidan $\varphi(13) = 12$daraja ko'rsatkichiga tegishlilarini ajratib olamiz. $\varphi(13) = 12 = 2^2 \cdot 3$ ning tub bo'luvchilari 2,3 bo'lgani uchun $g^4 \not\equiv 1(mod13)$, $g^6 \not\equiv 1(mod13)$ shartlarni qanoatlantiruvchilarini ajratib olishimiz kerak. $2,3,4,5,6,7,8,9,10,11,12$ larni g ning o'rniga qo'yib tekshirib ko'ramiz: $2^4 \not\equiv 1(mod13)$, $2^6 \not\equiv 1(mod13)$. Demak, 13 moduli bo'yicha eng kichik boshlang'ich ildiz 2 ekan. Boshlang'ich ildizlarni aniqlashning ikkinchi bir usuli bu agar p moduli bo'yicha boshlang'ich ildizlardan birortasi (yaxshisi eng kichigi) g ma'lum bo'lsa, qolgan barchasini $g^k\ (modp)$ ning eng kichik musbat chegirmasi sifatida aniqlash mumkin. Bunda $(k, p-1) = 1\ va\ 1 < k < p-1$. Qolgan boshlang'ich ildizlarni topish uchun ana shu tasdiqdan foydalanamiz. Bizda $g = 2$ va $2^k\,(mod13)$ ni qaraymiz. Bunda $(k, 12) = 1$ va$1 < k < 12$ bajarilishi kerak. Bundan $k = 5,7,11$ ekanligini topamiz. U holda $g^k\,(mod13)$ larni eng kichik musbat chegirma ko'rinishida yozib $2^5 \equiv 6\,(mod13)$; $2^7 \equiv 11(mod13)$; $2^{11} \equiv 7\,(mod13)$ larni hosil qilamiz. Shunday qilib, 13 moduli bo'yicha barcha boshlang'ich ildizlar $2,6,7,11$ lardan iborat bo'lar ekan. Ularning soni $\varphi(\varphi(p)) = \varphi(p-1) = \varphi(12) = 4$ ta. **Javob**: 2,6,7,11.

4). 17 moduli bo'yicha barcha boshlang'ich ildizlarni topish uchun shu modul bo'yicha chegirmalarning keltirilgan sistemasi $2,3,4,5,6,7,8,9,10,11,12,13,14,15,16$ lar orasidan $\varphi(17) = 16$daraja ko'rsatkichiga tegishlilarini ajratib olamiz. $\varphi(17) = 16 = 2^4$ ning tub bo'luvchilari 2 bo'lgani uchun $g^8 \not\equiv 1(mod 17)$ shartlarni qanoatlantiruvchilarini ajratib olishimiz kerak. $2,3,4, \dots ,16$ larni g ning o'rniga qo'yib tekshirib ko'ramiz: $2^8 \equiv 1(mod17)$; $3^8 \not\equiv 1(mod17), 4^8 \equiv 1(mod17)$; $5^8 \equiv (5^2)^4 \equiv 8^4 \equiv 64^2 \equiv (-4)^2 \equiv -1 \not\equiv 1(mod17)$; $6^8 \equiv 2^4 \equiv -1 \not\equiv 1(mod17)$; $7^8 \equiv (-2)^4 \equiv -1 \not\equiv 1(mod17)$; $8^8 \equiv (-4)^4 \equiv 1(mod17)$; $9^8 \equiv (-4)^4 \equiv 1(mod17)$; $10^8 \equiv (-2)^4 \equiv -1 \not\equiv 1(mod17)$; $11^8 \equiv 2^4 \equiv -1 \not\equiv 1(mod17)$; $12^8 \equiv (-5)^8 \equiv -1 \not\equiv 1(mod17)$; $13^8 \equiv (-4)^8 \equiv 1(mod17)$; $14^8 \equiv (-3)^8 \not\equiv 1(mod17)$; $15^8 \equiv (-2)^8 \equiv 1(mod17)$; $16^8 \equiv (-1)^8 \equiv 1(mod17)$ larni hosil qilamiz. Shunday qilib, 17 moduli bo'yicha barcha boshlang'ich ildizlar $3,5,6,7,10,11,12,14,$ lardan iborat bo'lar ekan. Ularning soni $\varphi(\varphi(p)) = \varphi(p-1) = \varphi(16) = 8$ ta.

Javob: $3,5,6,7,10,11,12,14$.

313. 1).p – tub moduli bo'yicha barcha boshlang'ich ildizlar soni $\varphi(\varphi(p)) = \varphi(p-1)$ ga teng. Bizning misolimizda $p = 19$ bo'lgani uchun $\varphi(19-1) = \varphi(18) = 6$, ya'ni 19 moduli bo'yicha barcha boshlang'ich ildizlar soni 6 ga teng. Endi 19 moduli bo'yicha eng kichik boshlang'ich ildizni topamiz. Buning uchun 19 moduli bo'yicha chegirmalarning keltirilgan sistemasi $2,3,4, \dots ,18$ lar orasidan $\varphi(19) = 18$daraja ko'rsatkichiga tegishli eng kichik sonni topishimiz kerak. $\varphi(19) = 18 = 2 \cdot 3^2$ning tub bo'luvchilari 2 va 3 bo'lgani uchun $g^6 \not\equiv 1(mod19)$, $g^9 \not\equiv 1(mod19)$shartlarni qanoatlantiruvchi eng kichik son g topishimiz kerak.

$$2^6 \equiv -4 \not\equiv 1(mod17), \quad 2^9 \equiv -32 \equiv 6 \not\equiv 1(mod19).$$

Bulardan 2 sonining 19 moduli bo'yicha eng kichik boshlang'ich ildiz ekanligi kelib chiqadi. **Javob**: 6 va 2.

2).Bizda $p = 23$ bo'lgani uchun $\varphi(23-1) = \varphi(22) = 10$, ya'ni 23 moduli bo'yicha barcha boshlang'ich ildizlar soni 10 ga teng. Endi 23 moduli bo'yicha eng kichik boshlang'ich ildizni topamiz. Buning uchun 23 moduli bo'yicha chegirmalarning keltirilgan sistemasi $2,3,4, \dots ,22$ lar orasidan $\varphi(23) = 22$daraja ko'rsatkichiga tegishli eng kichik sonni topishimiz kerak. $\varphi(23) = 22 = 2 \cdot 11$ning tub bo'luvchilari 2 va 3 bo'lgani uchun $g^2 \not\equiv 1(mod23), g^{11} \not\equiv$

$1(mod 23)$ shartlarni qanoatlantiruvchi eng kichik son g topishimiz kerak.

$2^2 \equiv 4 \not\equiv 1(mod 23)$, $2^{11} \equiv (2^5)^2 \cdot 2 \equiv 81 \cdot 2 \equiv -22 \equiv 1(mod 23)$;
$3^2 \equiv 9 \not\equiv 1(mod 23)$, $3^{11} \equiv (3^5)^2 \cdot 3 \equiv 13^2 \cdot 3 \equiv 8 \cdot 3 \equiv 1(mod 23)$;
$4^2 \equiv -7 \not\equiv 1(mod 23)$, $4^{11} \equiv (4^3)^3 \cdot 4^2 \equiv (-5)^3 \cdot 16 \equiv -125 \cdot 16 \equiv -10 \cdot 16 \equiv 1(mod 23)$;
$5^2 \equiv 2 \not\equiv 1(mod 23), 5^{11} \equiv (5^2)^5 \cdot 5 \equiv 2^5 \cdot 5 \equiv 45 \equiv -1 \not\equiv 1(mod 23)$;

Bulardan 5 sonining 23 moduli bo'yicha eng kichik boshlang'ich ildiz ekanligi kelib chiqadi. **Javob**: 10 va 2.

3). Bizda $p = 31$ bo'lgani uchun $\varphi(31-1) = \varphi(30) = 8$, ya'ni 31 moduli bo'yicha barcha boshlang'ich ildizlar soni 8 ga teng. Endi 31 moduli bo'yicha eng kichik boshlang'ich ildizni topamiz. Buning uchun 31 moduli bo'yicha chegirmalarning keltirilgan sistemasi $2,3,4,\dots,30$ lar orasidan $\varphi(31) = 30$ daraja ko'rsatkichiga tegishli eng kichik sonni topishimiz kerak. $\varphi(31) = 30 = 2 \cdot 3 \cdot 5$ning tub bo'luvchilari 2 ,3 va 5 bo'lgani uchun $g^6 \not\equiv 1(mod 31), g^{10} \not\equiv 1(mod 31)$, $g^{15} \not\equiv 1(mod 31)$ shartlarni qanoatlantiruvchi eng kichik son g topishimiz kerak.

$2^6 \equiv 2 \not\equiv 1(mod 31), 2^{10} \equiv (2^5)^2 \equiv 1(mod 31); 3^6 \equiv (3^3)^2 \equiv (-4)^2 \not\equiv 1(mod 31)$, $3^{10} \equiv (3^5)^2 \equiv (-5)^2 \equiv 25 \equiv 1 \not\equiv 1(mod 31)$;, $3^{15} \equiv (3^5)^5 \equiv (-5)^3 \equiv -125 \equiv -1 \not\equiv 1(mod 31)$. Bulardan 3 sonining 31 moduli bo'yicha eng kichik boilang'ich ildiz ekanligi kelib chiqadi. **Javob**: 8 va 3.

4). Bizda $p = 37$ bo'lgani uchun $\varphi(37-1) = \varphi(36) = 12$, ya'ni37 moduli bo'yicha barcha boshlang'ich ildizlarsoni 12 ga teng. Endi 37 moduli bo'yicha eng kichik boshlang'ich ildizni topamiz. Buning uchun 37 moduli bo'yicha chegirmalarning keltirilgan sistemasi $2,3,4,\dots,36$ lar orasidan $\varphi(37) = 36$daraja ko'rsatkichiga tegishli eng kichik sonni topishimiz kerak. $\varphi(37) = 36 = 2^2 \cdot 3^2$ning tub bo'luvchilari 2 va 3 bo'lgani uchun $g^{12} \not\equiv 1(mod 37)$, $g^{18} \not\equiv 1(mod 37)$ shartlarni qanoatlantiruvchi eng kichik son g topishimiz kerak.

$2^{12} \equiv (2^6)^2 \equiv (-10)^2 \equiv -11 \not\equiv 1(mod 37), 2^{18} \equiv (2^6)^3 \equiv (-10)^3 \equiv (37 \cdot 27 + 1) \equiv -1 \not\equiv 1(mod 37)$. Bulardan 2 sonining 37 moduli bo'yicha eng kichik boslang'ich ildiz ekanligi kelib chiqadi. **Javob**: 12 va 2.

5). Bizda $p = 43$ bo'lgani uchun $\varphi(43 - 1) = \varphi(42) = 12$, ya'ni43 moduli bo'yicha barcha boshlang'ich ildizlarsoni 12 ga teng. Endi 43 moduli bo'yicha eng kichik boshlang'ich ildizni topamiz. Buning uchun 43 moduli bo'yicha chegirmalarning keltirilgan sistemasi 2,3,4, ..., 42 lar orasidan $\varphi(43) = 42$daraja ko'rsatkichiga tegishli eng kichik sonni topishimiz kerak. $\varphi(43) = 42 = 2 \cdot 3 \cdot 7$ ning tub bo'luvchilari $2, 3$va 7 bo'lgani uchun $g^6 \not\equiv 1(mod43)$, $g^{14} \not\equiv 1(mod43)$, $g^{21} \not\equiv 1(mod43)$ shartlarni qanoatlantiruvchi eng kichik son g topishimiz kerak. $2^6 \equiv 64 \equiv 21 \not\equiv 1(mod43), 2^{14} \equiv (2^7)^2 \equiv (-1)^2 \equiv 1(mod43)$; $3^6 \equiv 3^4 \cdot 3^2 \equiv -5 \cdot 9 \equiv -2 \not\equiv 1(mod43), 3^{14} \equiv (3^6)^2 \cdot 3^2 \equiv (-2)^2 \cdot 9 \equiv 36 \not\equiv 1(mod43)$, $3^{21} \equiv (3^7)^3 \equiv (-6)^3 \equiv -216 \equiv -1 \not\equiv 1(mod43)$. Bulardan 3 sonining 43 moduli bo'yicha eng kichik boslang'ich ildiz ekanligi kelib chiqadi.

Javob: 12 va 3.

6). Bizda $p = 53$ bo'lgani uchun $\varphi(53 - 1) = \varphi(52) = 24$, ya'ni53 moduli bo'yicha barcha boshlang'ich ildizlarsoni 24 ga teng. Endi 53 moduli bo'yicha eng kichik boshlang'ich ildizni topamiz. Buning uchun 53 moduli bo'yicha chegirmalarning keltirilgan sistemasi 2,3,4, ..., 52 lar orasidan $\varphi(53) = 52$daraja ko'rsatkichiga tegishli eng kichik sonni topishimiz kerak. $\varphi(53) = 52 = 2^2 \cdot 13$ ning tub bo'luvchilari 2 va 13 bo'lgani uchun $g^4 \not\equiv 1(mod53)$, $g^{26} \not\equiv 1(mod53)$ shartlarni qanoatlantiruvchi eng kichik son g topishimiz kerak.

$2^4 \not\equiv 1(mod53), 2^{26} \equiv (2^7)^3 \cdot 2^5 \equiv (22)^3 \cdot (-21) \equiv -11^3 \cdot 8 \cdot 21 \equiv -121 \cdot 11 \cdot 168 \equiv -15 \cdot 11 \cdot 8 \equiv -6 \cdot 8 \equiv 5 \not\equiv 1(mod53)$; Bulardan 2 sonining 53 moduli bo'yicha eng kichik boslang'ich ildiz ekanligi kelib chiqadi.

Javob: 24 va 2.

314.1). $p = 19$ moduli bo'yicha eng kichik boslang'ich ildiz 313.1)-misolga asosan $g = 2$ ga teng.Boshlang'ich ildizlarni aniqlashning usuli bu agar p moduli bo'yicha boshlang'ich ildizlardan birortasi (yaxshisi eng kichigi) $g$ma'lum bo'lsa qolgan barchasini $g^k\ (modp)$ ning eng kichik musbat chegirmasi sifatida aniqlash mumkin. Bunda $(k, p - 1) = 1\ va\ 1 < k < p - 1$. Bizning misolimizda $p = 19, g = 2$ bo'lgani uchun $2^k\ (mod19)$ ning $(k, 18) = 1$ va $1 < k < 18$ shartlarda eng kichik musbat chegirmasinianiqlaymiz. Bundan $k =$

$5, 7, 11, 13, 17$ va $2^5 \equiv 13(mod19)$; $2^7 \equiv 13 \cdot 4 \equiv 14(mod19)$; $2^{11} \equiv 14 \cdot 16 \equiv -5 \cdot (-3) \equiv 15(mod19)$; $2^{13} \equiv 15 \cdot 4 \equiv 4 \cdot (-4) \equiv 3(mod19)$; $2^{17} \equiv 3 \cdot 16 \equiv 10(mod19)$. Demak, $2, 3, 10,13, 14, 15$ sonlari 19 moduli bo'yicha boshlang'ich ildiz bo'ladi.
Javob:$2, 3, 10,13, 14, 15$.

2). $p = 23$ moduli bo'yicha eng kichik boslang'ich ildiz 313.2)-misolga asosan $g = 5$ ga teng.Bizning misolimizda $p = 23, g = 5$ bo'lgani uchun $5^k(mod23)$ ning $(k, 22) = 1$ va $1 < k < 22$ shartlarda eng kichik musbat chegirmasinianiqlaymiz. Bundan $k = 3, 5, 7, 9, 13, 15, 17,19, 21$ va $5^3 \equiv 125 \equiv 10(mod23)$; $5^5 \equiv 10 \cdot 25 \equiv 10 \cdot 2 \equiv 20\ (mod23)$; $5^7 \equiv -3 \cdot 2 \equiv 17(mod23)$; $5^9 \equiv -6 \cdot 2 \equiv 11(mod23)$; $5^{13} \equiv 5^9 \cdot 5^4 \equiv 11 \cdot 4 \equiv 21(mod23)$; $5^{15} \equiv -2 \cdot 2 \equiv 19(mod23)$; $5^{17} \equiv -4 \cdot 2 \equiv 15(mod23)$;
$5^{19} \equiv -8 \cdot 2 \equiv 7(mod23)$; $5^{21} \equiv 7 \cdot 2 \equiv 14(mod23)$.

Demak, $5, 7, 10,13, 14, 15, 17, 19, 20, 21$ sonlari 23 moduli bo'yicha boshlang'ich ildiz bo'ladi.

Javob:$5, 7, 10,13, 14, 15, 17, 19, 20, 21$.

3). $p = 31$ moduli bo'yicha eng kichik boslang'ich ildiz 313.3)-misolga asosan $g = 3$ ga teng. Bizning misolimizda$p = 31$, $g = 3$ bo'lgani uchun $3^k(mod31)$ ning $(k, 30) = 1$ va $1 < k < 30$ shartlarda eng kichik musbat chegirmasinianiqlaymiz. Bundan $k = 7, 11, 13, 17,19, 23, 29$ va $3^7 \equiv 3^3 \cdot 3^3 \cdot 3 \equiv (-4)^2 \cdot 3 \equiv 17(mod31)$; $3^{11} \equiv 3^7 \cdot 3^4 \equiv 17 \cdot 19 \equiv 323 \equiv 13(mod31)$; $3^{13} \equiv 13 \cdot 9 \equiv 24(mod31)$; $3^{17} \equiv -7 \cdot 19 \equiv -133 \equiv 22(mod31)$; $3^{19} \equiv 22 \cdot 9 \equiv -81 \equiv 12(mod31)$; $3^{23} \equiv 12 \cdot 81 \equiv 12 \cdot 19 \equiv 228 \equiv 11(mod31)$; $3^{29} \equiv 3^{23} \cdot 3^2 \cdot 3^4 \equiv 11 \cdot 9 \cdot 19 \equiv 6 \cdot 19 \equiv 114 \equiv 21(mod31)$. Demak, $3, 11,12, 13,17, 21, 22, 24$ sonlari 31 moduli bo'yicha boshlang'ich ildiz bo'ladi.

Javob:$3, 11,12, 13,17, 21, 22, 24$.

315. 6 moduli bo'yicha $\varphi(\varphi(6)) = \varphi(2) = 1$ ta boshlang'ich ildizlar sinfi mavjud. U $1 < x < 6$, $(x, 6) = 1$ shartni qanoatlantirishi kerak. Bu shartni qanoatlantiruvchi bitta 5 soni mavjud va $5^1 \equiv 5(mod6)$; $5^2 \equiv 25 \equiv 1(mod6)$ bo'lgani uchun 6 moduli bo'yicha 1 ta boshlang'ich ildizlar sinfi mavjud va u $x \equiv 5(mod6)$ dan iborat.

Javob: $x \equiv 5(mod6)$.

316. 312.2)-misolga asosan $g = 2$ soni $p = 11$ moduli bo'yicha boshlang'ich ildiz. 2^0 – xossaga asosan $2, 2^2, \dots, 2^{10}$ sonlari $p = 11$ moduli bo'yicha chegermalarning keltirilgan sistemasini tashkil etadi.

317. $p > 2 -$ tub soni $2^{2^n} + 1, (n = 1,2,\dots)$sonining tub bo'luvchisi bo'lsa, $2^{2^n} + 1 \equiv 0(modp)$ bajarilishi kerak, bundan $2^{2^n} \equiv -1(modp)$. Buning ikkala tomonini kvadratga ko'tarsak $2^{2^{n+1}} \equiv 1(modp)$ hosil bo'ladi. Bundan esa 2 soni p moduli bo'yicha 2^{n+1} ko'rsatkichiga tegishli ekanligi kelib chiqadi. U holda 2^{n+1}soni $\varphi(p) = p - 1$ ning bo'luvchisi bo'lishi kerak, ya'ni $p - 1 \equiv 0(mod2^{n+1}) \rightarrow p \equiv 1(mod2^{n+1}) \rightarrow p = k \cdot 2^{n+1} + 1$.

318. Ma'lumki agar $a, (a, m) = 1$ soni m moduli bo'yicha $\delta > 0$ ko'rsatkichga tegishli bo'lsa, δ soni $a^\delta \equiv 1(modm)$ shartni qanoatlantiruvchi eng kichik musbat son bo'lib $\varphi(m)$ ning bo'luvchisi bo'lishi kerak. Endi $a > 1$ sonining $a^m - 1$ moduli bo'yicha qanday ko'rsatkichga tegishli ekanligini aniqlaylik. Tushunarliki, $a^m \equiv 1(mod(a^m - 1))$ bajariladi. $a > 1$ bo'lgani uchun $1 < k < m$ bo'lsa, $a^k \not\equiv 1(mod(a^m - 1))$ bo'ladi. Shuning uchun ham $P_{a^m - 1}(a) = m$ va m soni $\varphi(a^m - 1)$ ning bo'luvchisi bo'lishi kerak. Demak, $\varphi(a^m - 1) \equiv 0(modm)$ bajariladi.

319. $m = 8$ moduli bo'yicha chegirmalarning keltirilgan sistemasi 1,3,5,7 sonlari orasida 1 boshqalarining $\varphi(8) = 4$ ko'rsatkichiga tegishlilari yo'q ekanligini ko'rsatish yetarli.$3^1 \equiv 3(mod8)$, $3^2 \equiv 1(mod8)$; $5^1 \equiv 5(mod8)$, $5^2 \equiv 1(mod8)$; $7 \equiv 7(mod8), 7^2 \equiv 1(mod8)$. Bundan ko'rinadiki, bu sonlarning barchasi 2 ko'rsatkichiga tegishli.

320.1).Bu yerda$(5,9) = 1$ va $\varphi(9) = 6$ bo'lgani uchun ham $5^2 \equiv 7(mod9)$, $5^3 \equiv 8(mod9)$ lardan $5^6 \equiv 1(mod9)$ ekanligi kelib chiqadi, ya'ni 5 soni 9 moduli bo'yicha boshlang'ich ildiz bo'ladi. Shuning uchun ham $5^0 \equiv 1, 5^1 \equiv 5, 5^2 \equiv 7, 5^3 \equiv 8, 5^4 \equiv 4, 5^5 \equiv 2$ sonlari 9 moduli bo'yicha chegirmalarning keltirilgan sistemasini tashkil qiladi. Demak,berilgan taqqoslama b ning $(b, 9) = 1$ shartni qanoatlantiruvchi barcha qiymatlarida yechimga ega.

Javob: b ning $(b, 9) = 1$ shartni qanoatlantiruvchi barcha qiymatlari.

2).Bu yerda $(4,9) = 1$ va $\varphi(9) = 6$ bo'lgani uchun ham $4^2 \equiv -2(mod9)$, $4^3 \equiv 1(mod9)$, ya'ni 4 soni 9 moduli bo'yicha 3 ko'rsatkichiga tegishli. Shuning uchun ham $4^0 \equiv 1, 4^1 \equiv 4, 4^2 \equiv 7$ sonlari 9 moduli bo'yichahar xil sinflarga tegishli bo'ladi. Demak, berilgan taqqoslama b ning $(b, 9) = 1$ shartni qanoatlantiruvchi $b \equiv 1, 4, 7 (mod 9)$ qiymatlarida yechimga ega.

Javob: $b \equiv 1, 4, 7 (mod 9)$ qiymatlari.

3). Bu yerda b ning $(b, m) = 1$ va $b \leq m$ shartni qanoatlantiruvchi qiymatlari soni $\varphi(m)$ta bo'lib ulardan $a^x \equiv b (mod m)$ taqqoslama yechimga ega bo'ladigan b larning soni $P_m(a)$ga teng. b ning jami qiymatlari soni $\varphi(m)$dan berilgan taqqoslama yechimga ega bo'ladigan b larning soni $P_m(a)$ni ayirsak berilgan taqqoslama yechimga ega bo'lmaydigan b larning soni $\varphi(m) - P_m(a)$ ga ega bo'lamiz.

Javob: $\varphi(m) - P_m(a)$.

V.2-§.

321. 1).2 asosga ko'ra 29 moduli boyicha indekslar jadvalini tuzish talab etilmoqda.$g = 2$ soni 29 moduli bo'yicha boshlang'ich ildiz bo'ladi (tekshirib ko'ring). Shuning uchun ham 29 moduli bo'yicha chegirmalarning keltirilgan sismasidagi sonlar $2^0, 2^1, 2^2, \ldots, 2^{27}$ ni eng kichik manfiy bo'lmagan chegirmalar ko'rinishida yozib olamiz. $2^0 \equiv 1, 2^1 \equiv 2, 2^2 \equiv 4, 2^3 \equiv 8, 2^4 \equiv 16, 2^5 \equiv 3,\ 2^6 \equiv 6, 2^7 \equiv 12,\ 2^8 \equiv 24,\ 2^9 \equiv 19,\ 2^{10} \equiv 9, 2^{11} \equiv 18, 2^{12} \equiv 7,\ 2^{13} \equiv 14,\ 2^{14} \equiv 28,\ 2^{15} \equiv 27,\ 2^{16} \equiv 25,\ 2^{17} \equiv 21,\ 2^{18} \equiv 13, 2^{19} \equiv 26,\ 2^{20} \equiv 23, 2^{21} \equiv 17, 2^{22} \equiv 5,\ 2^{23} \equiv 10, 2^{24} \equiv 20,\ 2^{25} \equiv 11,\ 2^{26} \equiv 22,\ 2^{27} \equiv 15 (mod 29)$. Bu aniqlangan qiymatlarni quyidagi jadval ko'rinishida yozish mumkin:

N	0	1	2	3	4	5	6	7	8	9
0		0	1	5	2	22	6	12	3	10
1	23	25	7	18	13	27	4	21	11	9
2	24	17	26	20	8	16	19	15	14	

2). 5 asosga ko'ra 23 moduli boyicha indekslar jadvalini tuzish talab etilmoqda.$g = 5$ soni 23 moduli bo'yicha boshlang'ich ildiz bo'ladi (tekshirib ko'ring). Shuning uchun ham 23 moduli bo'yicha chegirmalarning keltirilgan sismasidagi sonlar $5^0, 5^1, 5^2, \ldots, 5^{21}$ ni eng kichik manfiy bo'lmagan chegirmalar ko'rinishida yozib olamiz. $5^0 \equiv$

$1, 5^1 \equiv 5, 5^2 \equiv 2, 5^3 \equiv 10, 5^4 \equiv 4,\ 5^5 \equiv 20,\ 5^6 \equiv 8, 5^7 \equiv 17,\ 5^8 \equiv 16,$ $5^9 \equiv 11,\ 5^{10} \equiv 9, 5^{11} \equiv 22,\ 5^{12} \equiv 18,\ 5^{13} \equiv 21,\ 5^{14} \equiv 13,\ 5^{15} \equiv 19,$ $5^{16} \equiv 3,\ 5^{17} \equiv 15,\ 5^{18} \equiv 6, 5^{19} \equiv 7,\ 5^{20} \equiv 12,\ 5^{21} \equiv 14(mod23)$. Bu aniqlangan qiymatlarni quyidagi jadval ko'rinishida yozish mumkin:

N	0	1	2	3	4	5	6	7	8	9
0		0	2	16	4	1	18	19	6	10
1	3	9	20	14	21	17	8	7	12	15
2	5	13	11							

322. 11 moduli boyicha indekslar jadvalini tuzish talab etilmoqda. Buning uchun avvalo shu modul bo'yicha birorta boshlang'ich ildizni aniqlab olishimiz kerak. 312.2)-misolda $g = 2$ soni 11 moduli bo'yicha boshlang'ich ildiz bo'lishi ko'rsatilgan edi. Shuning uchun ham 11 moduli bo'yicha chegirmalarning keltirilgan sismasidagi sonlar $2^0, 2^1, 2^2, \ldots, 2^9$ ni eng kichik manfiy bo'lmagan chegirmalar ko'rinishida yozib olamiz. $2^0 \equiv 1, 2^1 \equiv 2, 2^2 \equiv 4, 2^3 \equiv 8, 2^4 \equiv 5, 2^5 \equiv 10,\ 2^6 \equiv 9, 2^7 \equiv 7,\ 2^8 \equiv 3,\ 2^9 \equiv 6(mod11)$. Bu aniqlangan qiymatlarni quyidagi jadval ko'rinishida yozish mumkin:

N	0	1	2	3	4	5	6	7	8	9
0		0	1	8	2	4	9	7	3	6
1	5									

323. 1).$5^\delta \equiv 1(mod7)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind5 \equiv ind1(mod6)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind5$ ni 7 moduli bo'yicha indekslar jadvalidan topamiz: $ind5 = 5$. Bulardan $5\delta \equiv 0(mod6) \to \delta \equiv 0(mod6) \to \delta = 6t$. Bundan δ ning eng kichik musbat qiymati $\delta = 6$. **Javob**: $\delta = 6$.

2). $5^\delta \equiv 1(mod11)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind5 \equiv ind1(mod10)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind5$ ni 11 moduli bo'yicha indekslar jadvalidan topamiz: $ind5 = 4$. Bulardan $4\delta \equiv 0(mod10) \to 2\delta \equiv 0(mod5) \to \delta \equiv 0(mod5) \to \delta \equiv 0{,}5(mod10) \to \delta = 10t$ va $\delta = 5 + 10t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 5$.

Javob: $\delta = 5$.

3). $8^{\delta} \equiv 1(mod13)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind8 \equiv ind1(mod12)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind8$ ni 13 moduli bo'yicha indekslar jadvalidan topamiz: $ind8 = 3$. Bulardan $3\delta \equiv 0(mod12) \to \delta \equiv 0(mod4) \to \delta \equiv 0,4,8(mod12)$
$\to \delta = 12t,\ \delta = 4 + 12t$ va $\delta = 8 + 12t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 4$.

Javob: $\delta = 4$.

4).$12^{\delta} \equiv 1(mod17)$taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind12 \equiv ind1(mod16)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind12$ ni 17 moduli bo'yicha indekslar jadvalidan topamiz: $ind12 = 13$. Bulardan $13\delta \equiv 0(mod16) \to \delta \equiv 0(mod16) \to \delta = 16t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 16$.

Javob: $\delta = 16$.

5). $24^{\delta} \equiv 1(mod31)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind24 \equiv ind1(mod30)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind24$ ni 31 moduli bo'yicha indekslar jadvalidan topamiz: $ind24 = 13$. Bulardan $13\delta \equiv 0(mod30) \to \delta \equiv 0(mod30) \to \delta = 30t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 30$.

Javob: $\delta = 30$.

6).$10^{\delta} \equiv 1(mod13)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind10 \equiv ind1(mod12)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind10$ ni 13 moduli bo'yicha indekslar jadvalidan topamiz: $ind10 = 10$. Bulardan $10\delta \equiv 0(mod12) \to 5\delta \equiv 0(mod6) \to \delta \equiv 0(mod6) \to \delta \equiv 0, 6(mod12) \to \delta = 12t, \delta = 6 + 12t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 6$.

Javob: $\delta = 6$.

7).$27^{\delta} \equiv 1(mod17)$ ni $10^{\delta} \equiv 1(mod17)$ ko'rinishda yozib olib, ikkala tomonini indekslaymiz. U holda $\delta ind10 \equiv ind1(mod16)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind10$ ni 17 moduli bo'yicha indekslar jadvalidan topamiz: $ind10 = 3$. Bulardan $3\delta \equiv 0(mod16) \to \delta \equiv 0(mod16) \to \delta = 16t,\ t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 16$. **Javob**: $\delta = 16$.

8).$18^{\delta} \equiv 1(mod11)$ ni $7^{\delta} \equiv 1(mod11)$ ko'rinishda yozib olib, ikkala tomonini indekslaymiz. U holda $\delta ind7 \equiv ind1(mod10)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind7$ ni 11 moduli bo'yicha indekslar jadvalidan topamiz: $ind7 = 7$. Bulardan $7\delta \equiv 0(mod10) \to \delta \equiv$

$0(mod10) \to \delta = 10t,\ t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 10$. **Javob:** $\delta = 10$.

9). $23^\delta \equiv 1(mod41)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind23 \equiv ind1(mod40)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind23$ ni 41 moduli bo'yicha indekslar jadvalidan topamiz: $ind23 = 36$. Bulardan $36\delta \equiv 0(mod40) \to 9\delta \equiv 0(mod10) \to \delta \equiv 0(mod10)$ $\to \delta \equiv 0, 10, 20, 30(mod40) \to \delta = 40t, \delta = 10 + 40t, \delta = 20 + 40t,$ $\delta = 30 + 40t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 10$. **Javob:** $\delta = 10$.

324. 1). $p = 5$ bo'lgani uchun 2 dan 4 gacha bo'lgan 2,3,4 sonlarning tegishli bo'lgan daraja ko'rsatkichini aniqlashimiz kerak. Buning uchun $2^\delta \equiv 1(mod5)$, $3^\delta \equiv 1(mod5)$, $4^\delta \equiv 1(mod5)$ taqqoslamalarning har birini yechib ularni qanoatlantiruvchi eng kichik $\delta > 0$ ni aniqlashimiz kerak. $2^\delta \equiv 1(mod5)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind2 \equiv ind1(mod4)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind2$ ni 5 moduli bo'yicha indekslar jadvalidan topamiz: $ind2 = 1$.Shuning uchun ham $\delta \equiv 0(mod4) \to \delta = 4t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 4$.

$3^\delta \equiv 1(mod5)$taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind3 \equiv ind1(mod4)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind3$ ni 5 moduli bo'yicha indekslar jadvalidan topamiz: $ind3 = 3$. Shuning uchun ham $3\delta \equiv 0(mod4) \to \delta = 4t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 4$.

$4^\delta \equiv 1(mod5)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind4 \equiv ind1(mod4)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind4$ ni 5 moduli bo'yicha indekslar jadvalidan topamiz: $ind4 = 2$.Shuning uchun ham $2\delta \equiv 0(mod4) \to \delta \equiv 0(mod2) \to \delta \equiv 0,2\ (mod4) \to \delta = 4t,\ 2 + 4t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 2$. **Javob:** 4, 4, 2.

2). $p = 7$ bo'lgani uchun 2 dan 6 gacha bo'lgan 2,3,4,5,6 sonlarning tegishli bo'lgan daraja ko'rsatkichini aniqlashimiz kerak. Buning uchun $2^\delta \equiv 1(mod7)$, $3^\delta \equiv 1(mod7)$, $4^\delta \equiv 1(mod7)$, $5^\delta \equiv 1(mod7)$, $6^\delta \equiv 1(mod7)$ taqqoslamalarning har birini yechib ularni qanoatlantiruvchi eng kichik $\delta > 0$ ni aniqlashimiz kerak. $2^\delta \equiv 1(mod7)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind2 \equiv ind1(mod6)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$va $ind2$ ni 7 moduli bo'yicha indekslar jadvalidan topamiz: $ind2 = 2$.Shuning uchun ham$2\delta \equiv 0(mod6) \to \delta \equiv 0(mod3) \to \delta \equiv 0,3(mod6) \to \delta = 6t, \delta = 3 + 6t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 3$.

$3^{\delta} \equiv 1(mod7)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind3 \equiv ind1(mod6)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind3$ ni 7 moduli bo'yicha indekslar jadvalidan topamiz: $ind3 = 1$. Shuning uchun ham $\delta \equiv 0(mod6) \to \delta = 6t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 6$.

$4^{\delta} \equiv 1(mod7)$taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind4 \equiv ind1(mod6)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind4$ ni 7 moduli bo'yicha indekslar jadvalidan topamiz: $ind4 = 4$.Shuning uchun ham $4\delta \equiv 0(mod6) \to 2\delta \equiv 0(mod3) \to \delta \equiv 0(mod3) \to \delta \equiv 0,3\ (mod6) \to \delta = 6t,\ 3 + 6t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 3$.

$5^{\delta} \equiv 1(mod7)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind5 \equiv ind1(mod6)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind5$ ni 7 moduli bo'yicha indekslar jadvalidan topamiz: $ind5 = 5$. Shuning uchun ham $5\delta \equiv 0(mod6) \to \delta \equiv 0(mod6) \to \delta = 6t,\ t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 6$.

$6^{\delta} \equiv 1(mod7)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind6 \equiv ind1(mod6)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind6$ni 7 moduli bo'yicha indekslar jadvalidan topamiz: $ind6 = 3$. Shuning uchun ham $3\delta \equiv 0(mod6) \to \delta \equiv 0(mod2) \to \delta \equiv 0,2,4\ (mod6) \to \delta = 6t,\ 2 + 6t, 4 + 6t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 2$. **Javob**: 3, 6, 3,6,2.

3). $p = 11$ bo'lgani uchun 2 dan 10 gacha bo'lgan 2,3,4,5,6,7,8,9,10 sonlarning tegishli bo'lgan daraja ko'rsatkichini aniqlashimiz kerak. Buning uchun $2^{\delta} \equiv 1(mod11)$, $3^{\delta} \equiv 1(mod11)$, $4^{\delta} \equiv 1(mod11)$, $5^{\delta} \equiv 1(mod11)$, $6^{\delta} \equiv 1(mod11), 7^{\delta} \equiv 1(mod11), 8^{\delta} \equiv 1(mod11), 9^{\delta} \equiv 1(mod11), 10^{\delta} \equiv 1(mod11)$ taqqoslamalarning har birini yechib ularni qanoatlantiruvchi eng kichik $\delta > 0$ larni aniqlashimiz kerak. $2^{\delta} \equiv 1(mod11)$taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind2 \equiv ind1(mod10)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind2$ ni 11 moduli bo'yicha indekslar jadvalidan topamiz: $ind2 = 1$.Shuning uchun ham$\delta \equiv 0(mod10) \to \delta = 10t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 10$.

$3^{\delta} \equiv 1(mod11)$taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind3 \equiv ind1(mod10)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind3$ ni 11 moduli bo'yicha indekslar jadvalidan topamiz: $ind3 = 8$.Shuning uchun ham $8\delta \equiv 0(mod10) \to 4\delta \equiv 0(mod5) \to \delta \equiv 0(mod5) \to \delta \equiv 0{,}5(mod10) \to \delta = 10t, 5 + 10t,\ t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 5$.

$4^{\delta} \equiv 1(mod11)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind4 \equiv ind1(mod10)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind4$ ni 11 moduli bo'yicha indekslar jadvalidan topamiz: $ind4 = 2$.Shuning uchun ham $2\delta \equiv 0(mod10) \to \delta \equiv 0(mod5) \to \delta \equiv 0{,}5\ (mod10) \to \delta = 10t,\ 5 + 10t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 5$.

$5^{\delta} \equiv 1(mod11)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind5 \equiv ind1(mod10)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind5$ ni 11 moduli bo'yicha indekslar jadvalidan topamiz: $ind5 = 4$. Shuning uchun ham $4\delta \equiv 0(mod10) \to 2\delta \equiv 0(mod5) \to \delta \equiv 0(mod5) \to \delta \equiv 0{,}5(mod10) \to \delta = 10t, 5 + 10t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 5$.

$6^{\delta} \equiv 1(mod11)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind6 \equiv ind1(mod10)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind6$ni 11 moduli bo'yicha indekslar jadvalidan topamiz: $ind6 = 9$. Shuning uchun ham $9\delta \equiv 0(mod10) \to \delta \equiv 0(mod10) \to \delta = 10t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 10$.

$7^{\delta} \equiv 1(mod11)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind7 \equiv ind1(mod10)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind7$ni 11 moduli bo'yicha indekslar jadvalidan topamiz: $ind7 = 7$. Shuning uchun ham$7\delta \equiv 0(mod10) \to \delta \equiv 0(mod10) \to \delta = 10t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 10$.

$8^{\delta} \equiv 1(mod11)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind8 \equiv ind1(mod10)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind8$ ni 11 moduli bo'yicha indekslar jadvalidan topamiz: $ind8 = 3$. Shuning uchun ham $3\delta \equiv 0(mod10) \to \delta \equiv 0(mod10) \to \delta = 10t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 10$.

$9^{\delta} \equiv 1(mod11)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind9 \equiv ind1(mod10)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind9$ ni 11 moduli bo'yicha indekslar jadvalidan topamiz: $ind9 = 6$.

Shuning uchun ham $6\delta \equiv 0(mod 10) \rightarrow 3\delta \equiv 0(mod 5) \rightarrow \delta \equiv 0(mod 5) \rightarrow \delta \equiv 0{,}5(mod 10) \rightarrow \delta = 10t, 5 + 10t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 5$.

$10^{\delta} \equiv 1(mod 11)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind 10 \equiv ind 1(mod 10)$ ga ega bo'lamiz. Bu yerda $ind 1 = 0$ va $ind 10$ni 11 moduli bo'yicha indekslar jadvalidan topamiz: $ind 10 = 5$.Shuning uchun ham $5\delta \equiv 0(mod 10) \rightarrow \delta \equiv 0(mod 2) \rightarrow \delta \equiv 0{,}2{,}4{,}6{,}8(mod 10) \rightarrow \delta = 10t, 2 + 10t, 4 + 10t, 6 + 10t, 8 + 10t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 2$.

Javob: 10, 5, 5,5,10,10,10,5,2.

325. p moduli bo'yicha a sonining boshlang'ich ildiz bo'lishi uchun u $\delta = \varphi(p) = p - 1$ ko'satkichiga tegishli bo'lishi kerak. Indekslashdan foydalanib $\delta > 0$ *ni aniqlash uchun* 324- misoldagi singari mulohaza yuritamiz. Misolda $p = 59$ va $\varphi(59) = 58$.

1).$2^{\delta} \equiv 1(mod 59)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind 2 \equiv ind 1(mod 58)$ ga ega bo'lamiz. Bu yerda $ind 1 = 0$ va $ind 2$ ni 59 moduli bo'yicha indekslar jadvalidan topamiz: $ind 2 = 1$.Shuning uchun ham $\delta \equiv 0(mod 58) \rightarrow \delta = 58t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 58$ va demak, 2 soni 59 moduli bo'yicha boshlang'ich ildiz bo'ladi.

Javob: bo'ladi.

2).$3^{\delta} \equiv 1(mod 59)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind 3 \equiv ind 1(mod 58)$ ga ega bo'lamiz. Bu yerda $ind 1 = 0$ va $ind 3$ ni 59 moduli bo'yicha indekslar jadvalidan topamiz: $ind 3 = 50$. Shuning uchun ham $50\delta \equiv 0(mod 58) \rightarrow 25\delta \equiv 0(mod 29) \rightarrow \delta \equiv 0(mod 29) \rightarrow \delta \equiv 0, 29(mod 58)$, $\delta = 58t, 29 + 58t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 29$ va demak, 3 soni 59 moduli bo'yicha boshlang'ich ildiz bo'lmaydi.

Javob: bo'lmaydi.

3). $6^{\delta} \equiv 1(mod 59)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind 6 \equiv ind 1(mod 58)$ ga ega bo'lamiz. Bu yerda $ind 1 = 0$ va $ind 6$ ni 59 moduli bo'yicha indekslar jadvalidan topamiz: $ind 6 = 51$.Shuning uchun ham $51\delta \equiv 0(mod 58) \rightarrow \delta \equiv 0(mod 58) \rightarrow \delta = 58t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 58$ va demak, 6 soni 59 moduli bo'yicha boshlang'ich ildiz bo'ladi. **Javob:** bo'ladi.

4). $8^{\delta} \equiv 1(mod59)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind8 \equiv ind1(mod58)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind8$ni 59 moduli bo'yicha indekslar jadvalidan topamiz: $ind8 = 3$.Shuning uchun ham $3\delta \equiv 0(mod58) \rightarrow \delta \equiv 0(mod58) \rightarrow \delta = 58t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 58$ va demak, 8 soni 59 moduli bo'yicha boshlang'ich ildiz bo'ladi. **Javob:** bo'ladi.

5).$12^{\delta} \equiv 1(mod59)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind12 \equiv ind1(mod58)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind12$ ni 59 moduli bo'yicha indekslar jadvalidan topamiz: $ind12 = 52$. Shuning uchun ham $52\delta \equiv 0(mod58) \rightarrow 26\delta \equiv 0(mod29) \rightarrow \delta \equiv 0(mod29) \rightarrow \delta \equiv 0, 29(mod58)$, $\delta = 58t, 29 + 58t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 29$ va demak, 12 soni 59 moduli bo'yicha boshlang'ich ildiz bo'lmaydi. **Javob:** bo'lmaydi.

6).$13^{\delta} \equiv 1(mod59)$taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind13 \equiv ind1(mod58)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind13$ni 59 moduli bo'yicha indekslar jadvalidan topamiz: $ind13 = 45$.Shuning uchun ham $45\delta \equiv 0(mod58) \rightarrow \delta \equiv 0(mod58) \rightarrow \delta = 58t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 58$ va demak, 13 soni 59 moduli bo'yicha boshlang'ich ildiz bo'ladi.

Javob: bo'ladi.

7).$14^{\delta} \equiv 1(mod59)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind14 \equiv ind1(mod58)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind14$ni 59 moduli bo'yicha indekslar jadvalidan topamiz: $ind14 = 19$.Shuning uchun ham $19\delta \equiv 0(mod58) \rightarrow \delta \equiv 0(mod58) \rightarrow \delta = 58t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 58$ va demak, 14 soni 59 moduli bo'yicha boshlang'ich ildiz bo'ladi.

Javob: bo'ladi.

8).$19^{\delta} \equiv 1(mod59)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $\delta ind19 \equiv ind1(mod58)$ ga ega bo'lamiz. Bu yerda $ind1 = 0$ va $ind19$ ni 59 moduli bo'yicha indekslar jadvalidan topamiz: $ind19 = 38$.Shuning uchun ham$38\delta \equiv 0(mod58) \rightarrow 19\delta \equiv 0(mod29) \rightarrow \delta \equiv 0(mod29) \rightarrow \delta \equiv 0{,}29(mod58) \rightarrow \delta = 58t, 29 + 58t, t \in Z$. Bundan δ ning eng kichik musbat qiymati $\delta = 29$ va demak, 19 soni 59 moduli bo'yicha boshlang'ich ildiz bo'lmaydi. **Javob:** bo'lmaydi.

326. p moduli bo'yicha berilgan a sonining boshlang'ich ildiz bo'lishi uchun u $\delta = \varphi(p) = p - 1$ ko'satkichiga tegishli bo'lishi kerak.

Buning bajarilishi uchun $a^{\delta} \equiv 1(modp) \to \delta inda \equiv 0(modp-1)$ bajarilishi kerak. Agar bu yerda $(inda, p-1) = 1$ (*) bo'lsa, u holda $\delta = p-1$ bo'lishi kelib chiqadi. Demak, biz p moduli bo'yicha indekslar jadvalidan (*) shartni qanoatlantiruvchi a larni ajratib olsak, ular p moduli bo'yicha boshlang'ich ildiz bo'ladi. Chegirmalarning keltirilgan sistemasidagi qolgan a lar p moduli bo'yicha boshlang'ich ildiz bo'lmaydi.

1). Bu misolda $p = 17$ va $\varphi(17) = 16$ bo'lgani uchun 17 moduli bo'yicha chegirmalarning keltirilgan sistemasidagi $2,3,4,\dots,16$ sonlarning indekslarini ilovadan qarab (*) shartni, ya'ni $(inda, 16) = 1$ ni qanoatlantiruvchilarini ajratib olamiz. Qaralayotgan sonlarning indekslari mos ravishda 14, 1, 12, 5, 15, 11, 3, 7, 13, 4, 9, 6, 8 lardan iborat. Bular orasida 16 bilan o'zaro tublari 1, 5, 15, 11, 3, 7, 13, 9 lar va bu indekslarga mos sonlar $3,5,6,7,10,11,12,14$ bo'lib ular 17 moduli bo'yicha boshlang'ich ildiz bo'ladi. Chegirmalarning keltirilgan sistemasidagi qolgan $2,4,8,9,13,15,16$ lar p moduli bo'yicha boshlang'ich ildiz bo'lmaydi. **Javob:** $3,5,6,7,10,11,12,14$.

2). Bu misolda $p = 19$ va $\varphi(19) = 18$ bo'lgani uchun 19 moduli bo'yicha chegirmalarning keltirilgan sistemasidagi $2,3,4,\dots,16,17,18$ sonlarning indekslarini ilovadan qarab (*) shartni, ya'ni $(inda, 18) = 1$ ni qanoatlantiruvchilarini ajratib olamiz. Qaralayotgan sonlarning indekslari mos ravishda 1,13, 2, 16, 14, 6, 3, 8, 17,12, 15, 5, 7, 11, 4, 10, 9 lardan iborat. Bular orasida 18 bilan o'zaro tublari 1, 13, 17, 5, 7, 11 lar va bu indekslarga mos sonlar $2,3,10,13,14,15$ bo'lib ular 19 moduli bo'yicha boshlang'ich ildiz bo'ladi. Chegirmalarning keltirilgan sistemasidagi qolgan$4,5,6,7,8,9,11,12,16,17,18$ lar 19 moduli bo'yicha boshlang'ich ildiz bo'lmaydi.

Javob: $2,3,10,13,14,15$.

3).Bu misolda $p = 23$ va $\varphi(23) = 22$ bo'lgani uchun 23 moduli bo'yicha chegirmalarning keltirilgan sistemasidagi $2,3,4,\dots,22$ sonlarning indekslarini ilovadan qarab (*) shartni, ya'ni $(inda, 22) = 1$ ni qanoatlantiruvchilarini ajratib olamiz. Qaralayotgan sonlarning indekslari mos ravishda 2, 16, 4, 1, 18, 19, 6, 10, 3, 9, 20, 14, 21, 17, 8, 7, 12, 15, 5, 13,11 lardan iborat. Bular orasida 22 bilan o'zaro tublari 1, 19, 3, 9, 21, 17, 7, 15, 5, 13lar va bu indekslarga mos sonlar $5,7,10,11,14,15,17,19,$

$20, 21$ bo'lib ular 23 moduli bo'yicha boshlang'ich ildiz bo'ladi. Chegirmalarning keltirilgan sistemasidagi qolgan $2, 3, 4, 6, 8, 9, 12, 13,$ $16, 18, 22$ lar 23 moduli bo'yicha boshlang'ich ildiz bo'lmaydi.

Javob: $5, 7, 10, 11, 14, 15, 17, 19, 20, 21.$

327. 1). $7x \equiv 23(mod17)$ ni $7x \equiv 6(mod17)$ ko'rinishda yozib olib uning ikkala tomonini indekslab quyidagi taqqoslamani hosil qilamiz: $ind7 + indx \equiv ind6(mod16)$. Bu yerdagi $ind7, ind6$ larning qiymatlarini 17 moduli bo'yicha indekslar jadvalidan $ind7 = 11$, $ind6 = 15$ larni topib yuqoridagi taqqoslamaga qo'yamiz, u holda : $11 + indx \equiv 15(mod16) \rightarrow indx \equiv 4(mod16)$ ga ega bo'lamiz. Endi 17 moduli bo'yicha anti indekslar jadvalidan indeksi 4 ga teng bo'lgan sonni topamiz va berilgan taqqoslamaning yechimi $x \equiv 13(mod17)$ ni hosil qilamiz. **Javob:** $x \equiv 13(mod17)$.

2). $39x \equiv 84(mod97)$ ning ikkala tomonini indekslab quyidagi taqqoslamani hosil qilamiz: $ind39 + indx \equiv ind84(mod96)$. Bu yerdagi $ind39, ind84$ larning qiymatlarini 97 moduli bo'yicha indekslar jadvalidan $ind39 = 95, ind84 = 73$ larni topib yuqoridagi taqqoslamaga qo'yamiz, u holda: $95 + indx \equiv 73(mod96) \rightarrow indx \equiv -22(mod96) \rightarrow indx \equiv 74(mod97)$ ga ega bo'lamiz. Endi 97 moduli bo'yicha anti indekslar jadvalidan indeksi 74 ga teng bo'lgan sonni topamiz va berilgan taqqoslamaning yechimi $x \equiv 32(mod97)$ ni hosil qilamiz. **Javob:** $x \equiv 32(mod97)$.

3). $125x \equiv 7(mod79)$ ni $46x \equiv 7(mod79)$ ko'rinishda yozib olib uning ikkala tomonini indekslab quyidagi taqqoslamani hosil qilamiz: $ind46 + indx \equiv ind7(mod78)$. Bu yerdagi $ind46$, $ind7$ larning qiymatlarini 79 moduli bo'yicha indekslar jadvalidan $ind46 = 30, ind7 = 53$ larni topib yuqoridagi taqqoslamaga qo'yamiz, u holda: $30 + indx \equiv 53(mod78) \rightarrow indx \equiv 23(mod78)$ ga ega bo'lamiz. Endi 79 moduli bo'yicha anti indekslar jadvalidan indeksi 23 ga teng bo'lgan sonni topamiz va berilgan taqqoslamaning yechimi $x \equiv 74(mod79)$ ni hosil qilamiz. **Javob:** $x \equiv 74(mod79)$.

4). $37x \equiv 25(mod89)$ ning ikkala tomonini indekslab quyidagi taqqoslamani hosil qilamiz: $ind37 + indx \equiv ind25(mod88)$. Bu yerdagi $ind37, ind25$ larning qiymatlarini 89 moduli bo'yicha indekslar jadvalidan $ind37 = 11, ind25 = 52$ larni topib yuqoridagi taqqoslamaga qo'yamiz, u holda: $11 + indx \equiv 52(mod88) \rightarrow indx \equiv 41(mod88)$

ga ega bo'lamiz. Endi 89 moduli bo'yicha anti indekslar jadvalidan indeksi 41 ga teng bo'lgan sonni topamiz va berilgan taqqoslamaning yechimi $x \equiv 56(mod89)$ ni hosil qilamiz.

Javob: $x \equiv 56(mod89)$.

5). $4x \equiv 13(mod37)$ning ikkala tomonini indekslab quyidagi taqqoslamani hosil qilamiz: $ind4 + indx \equiv ind13(mod36)$. Bu yerdagi $ind4, ind13$ larning qiymatlarini 37 moduli bo'yicha indekslar jadvalidan $ind4 = 2$, $ind13 = 11$ larni topib yuqoridagi taqqoslamaga qo'yamiz, u holda: $2 + indx \equiv 11(mod36) \rightarrow indx \equiv 9(mod36)$ ga ega bo'lamiz. Endi 37 moduli bo'yicha anti indekslar jadvalidan indeksi 9 ga teng bo'lgan sonni topamiz va berilgan taqqoslamaning yechimi $x \equiv 31(mod37)$ ni hosil qilamiz. **Javob:** $x \equiv 31(mod37)$.

6). $37x \equiv 5(mod221)$ ni qaraymiz. Bu yerda $221 = 13 \cdot 17$ bo'lgani uchun berilgan taqqoslama quyidagi taqqoslamalar sistemasi

$$\begin{cases} 37x \equiv 5(mod13) \\ 37x \equiv 5(mod17) \end{cases} \rightarrow \begin{cases} 11x \equiv 5(mod13) \\ 3x \equiv 5(mod17) \end{cases}$$

ga teng kuchli. Bu sistemadagi har bir taqqoslamani indekslab va indekslar jadvalidan foydalanib quyidagini hosil qilamiz:

$$\begin{cases} ind11 + indx \equiv ind5(mod13) \\ ind3 + indx \equiv ind5(mod17) \end{cases} \rightarrow \begin{cases} 7 + indx \equiv 9(mod13) \\ 1 + indx \equiv 5(mod17) \end{cases} \rightarrow \begin{cases} indx \equiv 2(mod13) \\ indx \equiv 4(mod17) \end{cases}.$$

Endi anti indekslar jadvallaridan foydalanib

$$\begin{cases} x \equiv 4(mod13) \\ x \equiv 13(mod17) \end{cases} \rightarrow \begin{cases} x = 4 + 13t, t \in Z \\ x \equiv 13(mod17) \end{cases} \rightarrow \begin{cases} x = 4 + 13t, t \in Z \\ 4 + 13t \equiv 13(mod17) \end{cases} \rightarrow$$

$$\begin{cases} x = 4 + 13t, t \in Z \\ 13t \equiv 9(mod17) \end{cases} \rightarrow \begin{cases} x = 4 + 13t, t \in Z \\ 13t \equiv 26(mod17) \end{cases} \rightarrow \begin{cases} x = 4 + 13t, t \in Z \\ t \equiv 2(mod17) \end{cases} \rightarrow$$

$$\begin{cases} x = 4 + 13t, t \in Z \\ t = 2 + 17t_1 \end{cases} \rightarrow x = 4 + 13(2 + 17t_1) = 30 + 221t_1, t_1 \in Z.$$

Javob: $x \equiv 30(mod221)$.

7). $47x \equiv 13(mod667)$ ni qaraymiz. Bu yerda $667 = 23 \cdot 29$ bo'lgani uchun berilgan taqqoslama quyidagi taqqoslamalar sistemasi

$$\begin{cases} 47x \equiv 13(mod23) \\ 47x \equiv 13(mod29) \end{cases} \rightarrow \begin{cases} x \equiv 13(mod23) \\ 18x \equiv 13(mod29) \end{cases}$$

ga teng kuchli. Bu sistemadagi ikkinchi taqqoslamani indekslab va indekslar jadvalidan foydalanib quyidagini hosil qilamiz:

$$\begin{cases} x \equiv 13(mod23) \\ ind18 + indx \equiv ind13(mod28) \end{cases} \rightarrow \begin{cases} x \equiv 13(mod23) \\ 11 + indx \equiv 18(mod28) \end{cases} \rightarrow$$

$$\begin{cases} x \equiv 13(mod23) \\ indx \equiv 7(mod28) \end{cases}.$$

Endi anti indekslar jadvallaridan foydalanib sistema yechsak

$$\begin{cases} x \equiv 13(mod23) \\ x \equiv 12(mod29) \end{cases} \to \begin{cases} x = 13 + 23t, t \in Z \\ 13 + 23t \equiv 12(mod29) \end{cases} \to \begin{cases} x = 13 + 23t, t \in Z \\ 23t \equiv -1(mod29) \end{cases}$$

$$\to$$

$$\begin{cases} x = 13 + 23t, t \in Z \\ -6t \equiv 28(mod29) \end{cases} \to \begin{cases} x = 13 + 23t, t \in Z \\ -3t \equiv 14(mod29) \end{cases} \to \begin{cases} x = 13 + 23t, t \in Z \\ -3t \equiv -15(mod29) \end{cases}$$

$$\to$$

$$\begin{cases} x = 13 + 23t, t \in Z \\ t \equiv 5(mod29) \end{cases} \to \begin{cases} x = 13 + 23t, t \in Z \\ t \equiv 5 + 29t_1 \end{cases} \to x = 128 + 667t_1, t_1 \in Z.$$

ega ega bo'lamiz. **Javob:** $x \equiv 128(mod667)$.

8). $228x \equiv 317(mod1517)$ni qaraymiz. Bu yerda $1517 = 37 \cdot 41$ bo'lgani uchun berilgan taqqoslama quyidagi taqqoslamalar sistemasi

$$\begin{cases} 228x \equiv 317(mod37) \\ 228x \equiv 317(mod41) \end{cases} \to \begin{cases} 6x \equiv 21(mod37) \\ 23x \equiv 30(mod41) \end{cases}$$

ga teng kuchli. Bu sistemadagi har ikkala taqqoslamani indekslab va indekslar jadvalidan foydalanib quyidagini hosil qilamiz:

$$\begin{cases} ind6 + indx \equiv ind21(mod36) \\ ind23 + indx \equiv ind30(mod40) \end{cases} \to \begin{cases} 27 + indx \equiv 22(mod36) \\ 36 + indx \equiv 23(mod40) \end{cases}$$

$$\to \begin{cases} indx \equiv 31(mod36) \\ indx \equiv 27(mod40) \end{cases}.$$

Endi anti indekslar jadvallaridan foydalanib

$$\begin{cases} x \equiv 22(mod37) \\ x \equiv 12(mod41) \end{cases} \to \begin{cases} x = 22 + 37t, t \in Z \\ 22 + 37t \equiv 12(mod41) \end{cases}$$

$$\to \begin{cases} x = 22 + 37t, t \in Z \\ 37t \equiv -10(mod41) \end{cases} \to$$

$$\begin{cases} x = 22 + 37t, t \in Z \\ -4t \equiv -10(mod41) \end{cases} \to \begin{cases} x = 22 + 37t, t \in Z \\ -2t \equiv -5(mod41) \end{cases}$$

$$\to \begin{cases} x = 22 + 37t, t \in Z \\ -2t \equiv -46(mod41) \end{cases} \to$$

$$\begin{cases} x = 22 + 37t, t \in Z \\ t \equiv 23(mod41) \end{cases} \to \begin{cases} x = 22 + 37t, t \in Z \\ t \equiv 23 + 41t_1 \end{cases} \to x = 873 + 1517t_1, t_1 \in Z.$$

Javob: $x \equiv 873(mod1517)$.

328. 1). Berilgan $2^x \equiv 7(mod67)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $x \cdot ind2 \equiv ind7\ (mod66)$ hosil bo'ladi. Bu yerdagi $ind2, ind7$ larning qiymatlarini 67 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind2 = 1, ind7 = 23$ bo'lgani uchun $x \equiv 23\ (mod66)$ taqqoslamaga kelamiz.

Javob: $x = 23 + 66t, t \in Z.$

2). Berilgan $13^x \equiv 12(mod47)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $x \cdot ind13 \equiv ind12\ (mod46)$ hosil bo'ladi. Bu yerdagi $ind13, ind12$ larning qiymatlarini 46 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind13 = 11, ind12 = 10$ bo'lgani uchun $11x \equiv 10\ (mod46)$ taqqoslamaga kelamiz. Buni yechsak $-35x \equiv 10\ (mod46) \to -7x \equiv 2(mod46) \to -7x \equiv 2 + 3 \cdot 46(mod46) \to -7x \equiv 140(mod46) \to$

$x \equiv -20(mod46) \to x \equiv 26(mod46).$

Javob: $x = 26 + 46t, t \in Z.$

3). Berilgan $16^x \equiv 11(mod53)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $x \cdot ind16 \equiv ind11\ (mod52)$ hosil bo'ladi. Bu yerdagi $ind16, ind11$ larning qiymatlarini 52 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind16 = 4, ind11 = 6$ bo'lgani uchun $4x \equiv 6\ (mod52) \to 2x \equiv 3\ (mod26)$taqqoslamaga kelamiz. Bu taqqoslamada $(2,26) = 2$, lekin 3 soni ikkiga bo'linmaydi. Shuning uchun ham bu taqqoslama va demak, berilgan taqqoslama ham yechimga ega emas. **Javob:** yechimga ega emas.

4). Berilgan $52^x \equiv 38(mod61)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $x \cdot ind52 \equiv ind38\ (mod60)$ hosil bo'ladi. Bu yerdagi $ind52, ind58$ larning qiymatlarini 61 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind52 = 42, ind38 = 27$ bo'lgani uchun $42x \equiv 27\ (mod60) \to 14x \equiv 9\ (mod10)$ taqqoslamaga kelamiz. Bu taqqoslamada $(14,10) = 2$, lekin 9 soni ikkiga bo'linmaydi. Shuning uchun ham bu taqqoslama va demak, berilgan taqqoslama ham yechimga ega emas.

Javob: yechimga ega emas.

5). Berilgan $12^x \equiv 17(mod31)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $x \cdot ind12 \equiv ind17(mod30)$ hosil bo'ladi. Bu yerdagi $ind12, ind17$ larning qiymatlarini 31 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind12 = 19, ind17 = 7$ bo'lgani uchun $19x \equiv 7\ (mod30) \to 19x \equiv 7 + 8 \cdot 30\ (mod30) \to x \equiv 13\ (mod30)$.

Javob: $x = 13 + 30t, t \in Z$.

6). Berilgan $20^x \equiv 21(mod41)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $x \cdot ind20 \equiv ind21(mod40)$ hosil bo'ladi. Bu yerdagi $ind20, ind21$ larning qiymatlarini 41 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind20 = 34, ind21 = 14$ bo'lgani uchun $34x \equiv 14\ (mod40) \to 17x \equiv 7(mod20) \to -3x \equiv 27(mod20)$ $\to x - 9\ (mod20) \to x \equiv 11{,}31(mod40)$.

Javob: $x = 11 + 40t,\ x = 31 + 40t, t \in Z$.

329. 1). Berilgan $37x^{15} \equiv 62(mod73)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $ind37 + 15indx \equiv ind62(mod72)$ hosil bo'ladi. Bu yerdagi $ind37, ind62$ larning qiymatlarini 73 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind37 = 64, ind62 = 19$ bo'lgani uchun $64 + 15indx \equiv 19\ (mod72) \to 15indx \equiv -45\ (mod72) \to 5indx \equiv -15\ (mod24) \to indx \equiv 21(mod24) \to indx \equiv 21, 45, 69(mod72)$.

Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 17,\ 63,\ 66\ (mod73)$ larni hosil bo'ladi.

Javob: $x = 17 + 73t,\ x = 63 + 73t, x = 66 + 73t,\ t \in Z$.

2). Berilgan $5x^4 \equiv 3(mod11)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $ind5 + 4indx \equiv ind3(mod10)$ hosil bo'ladi. Bu yerdagi $ind5, ind3$ larning qiymatlarini 11 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind5 = 4, ind3 = 8$ bo'lgani uchun $4 + 4indx \equiv 8\ (mod10) \to 4indx \equiv 4\ (mod10) \to 2indx \equiv 2\ (mod5) \to indx \equiv 1(mod5) \to indx \equiv 1{,}6(mod10)$. Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 2, 9\ (mod11)$ larni hosil bo'ladi.

Javob: $x = 2 + 11t,\ x = 9 + 11t,\ t \in Z$.

3). Berilgan $2x^8 \equiv 5(mod13)$taqqoslamaning ikkala tomonini indekslaymiz. U holda $ind2 + 8indx \equiv ind5(mod12)$ hosil bo'ladi. Bu yerdagi $ind2, ind5$ larning qiymatlarini 13 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind2 = 1, ind5 = 9$ bo'lgani uchun $1 + 8indx \equiv 9(mod12) \to 8indx \equiv 8\ (mod12) \to 2indx \equiv 2\ (mod3) \to indx \equiv 1(mod3) \to indx \equiv 1{,}4{,}7{,}10(mod12)$. Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 2, 3, 11, 10\ (mod 13)$ larni hosil bo'ladi.

Javob: $x = 2 + 13t,\ x = 3 + 13t, x = 10 + 13t, x = 11 + 13t, t \in Z.$

4). Berilgan $2x^3 \equiv 17(mod 41)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $ind2 + 3indx \equiv ind17(mod 40)$ hosil bo'ladi. Bu yerdagi $ind2,\ ind17$ larning qiymatlarini 41 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind2 = 26, ind17 = 33$ bo'lgani uchun $26 + 3indx \equiv 33(mod 40) \rightarrow 3indx \equiv 7\ (mod 40) \rightarrow 3indx \equiv -33\ (mod 40) \rightarrow indx \equiv -11(mod 40) \rightarrow indx \equiv 29(mod 40)$. Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 22\ (mod 41)$ larni hosil bo'ladi.

Javob: $x = 22 + 41t,\ t \in Z.$

5). Berilgan$27x^5 \equiv 25(mod 31)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $ind27 + 5indx \equiv ind25(mod 30)$ hosil bo'ladi. Bu yerdagi $ind27, ind25$ larning qiymatlarini 31 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind27 = 3, ind25 = 10$ bo'lgani uchun $3 + 5indx \equiv 10(mod 30) \rightarrow 5indx \equiv 7\ (mod 30)$. Bu yerda $(5,30) = 5$, lekin 7 soni 5 ga bo'linmaydi. Shuning uchun ham oxirgi taqqoslama va demak, berilgan taqqoslama ham yechimga ega emas. **Javob:** taqqoslama yechimga ega emas.

6). Berilgan $11x^3 \equiv 6(mod 79)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $ind11 + 3indx \equiv ind6(mod 78)$ hosil bo'ladi. Bu yerdagi $ind11, ind6$ larning qiymatlarini 79 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind11 = 68,\ ind6 = 5$ bo'lgani uchun $68 + 3indx \equiv 5(mod 78) \rightarrow 3indx \equiv -63\ (mod 78) \rightarrow 3indx \equiv 15\ (mod 78) \rightarrow indx \equiv 5(mod 26) \rightarrow indx \equiv 5{,}31{,}57(mod 78)$. Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 6, 59, 14\ (mod 79)$ larni hosil bo'ladi.

Javob: $x = 6 + 79t,\ x = 14 + 79t, x = 59 + 79t,\ t \in Z.$

7). Berilgan $23x^3 \equiv 15(mod 73)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $ind23 + 3indx \equiv ind15(mod 73)$ hosil bo'ladi. Bu yerdagi $ind23,\ ind15$ larning qiymatlarini 73 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind23 = 46,\ ind15 = 7$ bo'lgani uchun $46 + 3indx \equiv 7(mod 72) \rightarrow 3indx \equiv -39\ (mod 72)$

$\rightarrow indx \equiv -13\ (mod24) \rightarrow indx \equiv 11(mod24) \rightarrow indx \equiv$ $11{,}35{,}59(mod72)$. Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 31, 29, 13\ (mod73)$ larni hosil bo'ladi.

Javob: $x = 13 + 73t,\ x = 29 + 73t, x = 31 + 73t,\ t \in Z.$

8).Berilgan $8x^{26} \equiv 37(mod41)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $ind8 + 26indx \equiv ind37(mod40)$ hosil bo'ladi. Bu yerdagi $ind8,\ ind37$ larning qiymatlarini 40 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind8 = 38,\ ind37 = 32$ bo'lgani uchun $38 + 26indx \equiv 32(mod40) \rightarrow 26indx \equiv -6\ (mod40) \rightarrow 13indx \equiv -3\ (mod20) \rightarrow -7indx \equiv -63(mod20) \rightarrow indx \equiv 9(mod20) \rightarrow indx \equiv 9{,}29(mod40).$

Endi anti indekslar jadvallaridan foydalanib x ni topamiz.

U holda$x \equiv 19, 22(mod41)$ larni hosil bo'ladi.

Javob: $x = 19 + 41t,\ x = 22 + 41t,\ t \in Z.$

9).Berilgan $37x^8 \equiv 59(mod61)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $ind37 + 8indx \equiv ind59(mod60)$ hosil bo'ladi. Bu yerdagi $ind37,\ ind59$ larning qiymatlarini 61 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind37 = 39,\ ind59 = 31$ bo'lgani uchun $39 + 8indx \equiv 31(mod60) \rightarrow 8indx \equiv -8\ (mod60) \rightarrow 2indx \equiv -2\ (mod15) \rightarrow indx \equiv 14(mod15) \rightarrow indx \equiv 14{,}29{,}44{,}59(mod60)$. Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 36, 30, 25, 31\ (mod61)$ larni hosil bo'ladi.

Javob: $x = 25 + 61t,\ x = 30 + 61t, x = 31 + 61t, x = 36 + 61t,\ t \in Z.$

10). Berilgan $18x^8 \equiv 6(mod13)$ni $5x^8 \equiv 6(mod13)$ ko'rinishda yozib olamiz va uning ikkala tomonini indekslaymiz. U holda $ind5 + 8indx \equiv ind6(mod12)$ hosil bo'ladi. Bu yerdagi $ind5,\ ind6$ larning qiymatlarini 13 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind5 = 9,\ ind6 = 5$ bo'lgani uchun $9 + 8indx \equiv 5(mod12) \rightarrow 8indx \equiv -4\ (mod12) \rightarrow 2indx \equiv -1\ (mod3) \rightarrow 2indx \equiv 2(mod3) \rightarrow indx \equiv 1(mod3) \rightarrow indx \equiv 1{,}4{,}7{,}10(mod12).$ Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 2, 3, 10, 11\ (mod13)$ larni hosil bo'ladi.

Javob: $x = 2 + 13t,\ x = 3 + 13t, x = 10 + 13t, x = 11 + 13t,\ t \in Z.$

330.1). Berilgan $x^{12} \equiv 37(mod41)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $12indx \equiv ind37(mod40)$ hosil bo'ladi.

Bu yerdagi $ind37$ ning qiymatlarini41 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind37 = 32$ bo'lgani uchun $12indx \equiv 32(mod40) \to 3indx \equiv 8(mod10) \to indx \equiv 6\ (mod10) \to indx \equiv 6, 16, 26, 36(mod40)$. Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 39, 18, 2, 23\ (mod40)$ larni hosil bo'ladi.

Javob: $x = 2 + 41t,\ x = 18 + 41t, x = 23 + 41t, x = 39 + 41t,$ $t \in Z$.

2). Berilgan $x^{55} \equiv 17(mod97)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $55indx \equiv ind17(mod96)$ hosil bo'ladi. Bu yerdagi $ind17$ ning qiymatlarini 97 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind17 = 89$ bo'lgani uchun $55indx \equiv 89(mod96) \to 55indx \equiv 185(mod96) \to 11indx \equiv 37\ (mod96) \to 11indx \equiv 37 + 5 \cdot 96\ (mod96) \to 11indx \equiv 517\ (mod96) \to indx \equiv 47(mod96)$. Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 58(mod97)$ ni hosil bo'ladi.

Javob: $x = 58 + 97t, t \in Z$.

3).Berilgan $x^{35} \equiv 17(mod67)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $35indx \equiv ind17(mod66)$ hosil bo'ladi. Bu yerdagi $ind17$ ning qiymatlarini 67 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind17 = 64$ bo'lgani uchun $35indx \equiv 64(mod66) \to 35indx \equiv 64 + 66 \cdot 16(mod66) \to 35indx \equiv 1120(mod66) \to indx \equiv 32(mod66)$. Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 33(mod67)$ ni hosil bo'ladi. **Javob:** $x = 33 + 67t, t \in Z$.

4). Berilgan $x^{30} \equiv 46(mod73)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $30indx \equiv ind46(mod72)$ hosil bo'ladi. Bu yerdagi $ind46$ ning qiymatlarini 73 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind46 = 54$ bo'lgani uchun $30indx \equiv 54(mod72) \to 5indx \equiv 9(mod12) \to 5indx \equiv 9 + 12 \cdot 3(mod12) \to indx \equiv 9(mod12) \to indx \equiv 9{,}21{,}33{,}45{,}57{,}69(mod72)$. Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 10, 17, 7, 63, 56, 66(mod73)$ ni hosil bo'ladi.

Javob: $x = 7 + 73t, x = 10 + 73t,\ x = 17 + 73t,\ x = 56 + 73t,$ $x = 63 + 73t,\ x = 66 + 73t, t \in Z$.

5). Berilgan $x^8 \equiv 23(mod41)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $8indx \equiv ind23(mod40)$ hosil bo'ladi. Bu yerdagi $ind23$ ning qiymatlarini 41 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind23 = 36$ bo'lgani uchun $8indx \equiv 36(mod40) \to 2indx \equiv 9(mod10)$. Bu yerda $(2,10) = 2$, lekin 9 soni 2 ga bo'linmaydi. Shuning uchun ham oxirgi taqqoslama va demak, berilgan taqqoslama ham yechimga ega emas.

Javob: taqqoslama yechimga ega emas.

6). Berilgan $x^5 \equiv 74(mod71)$ ni $x^5 \equiv 3(mod71)$ ko'rinishida yozib olamiz va uning ikkala tomonini indekslaymiz. U holda $5indx \equiv ind3(mod70)$ hosil bo'ladi. Bu yerdagi $ind3$ ning qiymatlarini 71 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind3 = 39$ bo'lgani uchun $5indx \equiv 39(mod70)$. Bu yerda $(5,70) = 5$, lekin 39 soni 5 ga bo'linmaydi. Shuning uchun ham oxirgi taqqoslama va demak, berilgan taqqoslama ham yechimga ega emas.

Javob: taqqoslama yechimga ega emas.

7). Berilgan $x^{27} \equiv 39(mod43)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $27indx \equiv ind39(mod42)$ hosil bo'ladi. Bu yerdagi $ind39$ ning qiymatlarini 43 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind39 = 33$ bo'lgani uchun $27indx \equiv 33(mod42) \to 9indx \equiv 11(mod14) \to 9indx \equiv 11 + 14 \cdot 5(mod14) \to indx \equiv 9(mod14) \to indx \equiv 9,23,37(mod42)$. Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 32,34,20(mod43)$ ni hosil bo'ladi.

Javob: $x = 20 + 43t, x = 32 + 43t, \ x = 34 + 43t, t \in Z$.

8). Berilgan $x^8 \equiv 29(mod13)$ ni $x^8 \equiv 3(mod13)$ ko'rinishida yozib olamiz va uning ikkala tomonini indekslaymiz. U holda $8indx \equiv ind3(mod12)$ hosil bo'ladi. Bu yerdagi $ind3$ ning qiymatlarini 13 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind3 = 4$ bo'lgani uchun $8indx \equiv 4(mod12) \to 2indx \equiv 1(mod3) \to 2indx \equiv 4(mod3) \to indx \equiv 2(mod3) \to indx \equiv 2,5,8,11(mod12)$. Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda$x \equiv 4,6,9,7(mod13)$ ni hosil bo'ladi.

Javob:$x = 4 + 13t, x = 6 + 13t, \ x = 7 + 13t, x = 9 + 13t, t \in Z$.

9). Berilgan $x^2 \equiv 59(mod67)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $2indx \equiv ind59(mod66)$ hosil bo'ladi. Bu yerdagi $ind59$ ning qiymatlarini 67 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind59 = 36$ bo'lgani uchun $2indx \equiv 36(mod66) \to indx \equiv 18(mod33) \to indx \equiv 18{,}51(mod66)$. Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 40{,}27(mod67)$ ni hosil bo'ladi.

Javob: $x \equiv \pm 27(mod67)$.

10). Berilgan $x^2 \equiv 59(mod83)$ taqqoslamaning ikkala tomonini indekslaymiz. U holda $2indx \equiv ind59(mod82)$ hosil bo'ladi. Bu yerdagi $ind59$ ning qiymatlarini 83 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind59 = 34$ bo'lgani uchun $2indx \equiv 34(mod82) \to indx \equiv 17(mod41) \to indx \equiv 17{,}58(mod82)$. Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 15{,}68(mod83)$ ni hosil bo'ladi.

Javob: $x \equiv \pm 15(mod83)$.

11). Berilgan $x^2 \equiv 32(mod43)$ ning ikkala tomonini indekslaymiz. U holda $2indx \equiv ind32(mod42)$ hosil bo'ladi. Bu yerdagi $ind32$ ning qiymatlarini 43 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind32 = 9$ bo'lgani uchun $2indx \equiv 9(mod42)$. Bu yerda $(2{,}42) = 2$, lekin 9 soni 2 ga bo'linmaydi. Shuning uchun ham oxirgi taqqoslama va demak, berilgan taqqoslama ham yechimga ega emas.

Javob: taqqoslama yechimga ega emas.

12).Berilgan taqqoslama $x^2 \equiv -17(mod53)$ni $x^2 \equiv 36(mod53)$ ko'rinishda yozib olib, uning ikkala tomonini indekslaymiz. U holda $2indx \equiv ind36(mod52)$ hosil bo'ladi. Bu yerdagi $ind36$ ning qiymatlarini 53 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind36 = 36$ bo'lgani uchun$2indx \equiv 36(mod52) \to indx \equiv 18(mod26) \to indx \equiv 18{,}44(mod52)$. Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 6{,}47(mod53)$ ni hosil bo'ladi.

Javob: $x \equiv \pm 6(mod53)$.

13). Berilgan taqqoslama $x^2 \equiv -28(mod67)$ ni $x^2 \equiv 39(mod67)$ ko'rinishda yozib olib, uning ikkala tomonini indekslaymiz. U holda $2indx \equiv ind39(mod66)$ hosil bo'ladi. Bu yerdagi $ind39$ ning qiymatlarini 67 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind39 = 58$ bo'lgani uchun $2indx \equiv 58(mod66) \to indx \equiv$

$29(mod33) \to indx \equiv 29{,}62(mod66)$. Endi anti indekslar jadvallaridan foydalanib x ni topamiz. U holda $x \equiv 46{,}21(mod67)$ ni hosil bo'ladi. **Javob:** $x \equiv \pm 21(mod67)$.

14). Berilgan taqqoslama $x^2 \equiv 56(mod41)$ ni $x^2 \equiv 15(mod41)$ ko'rinishda yozib olib, uning ikkala tomonini indekslaymiz. U holda $2indx \equiv ind15(mod40)$ hosil bo'ladi. Bu yerdagi $ind15$ ning qiymatlarini 41 moduli bo'yicha indekslar jadvalidan topib olib kelib qo'yamiz. $ind15 = 37$ bo'lgani uchun $2indx \equiv 37(mod40)$. Bu yerda $(2,40) = 2$, lekin 37 soni 2 ga bo'linmaydi. Shuning uchun ham oxirgi taqqoslama va demak, berilgan taqqoslama ham yechimga ega emas. **Javob:** taqqoslama yechimga ega emas.

331. Eyler kriteriyasiga asosan a sonining p – tub modul bo'yicha kvadratik chegirma bo'lishi uchun $a^{\frac{p-1}{2}} \equiv 1(modp)$ $(*)$ shart bajarilishi kerak.

1). $p = 23$ da $(*)$ dan $a^{11} \equiv 1(mod23)$ kelib chiqadi. Berilgan sonlar $15, 16, 17, 18, 19, 20$ orasidan shu shartni ajratib olish uchun oxirgi taqqoslamani indekslardan foydalanib yechamiz. U holda quyidagiga ega bo'lamiz: $11inda \equiv 0(mod22) \to inda \equiv 0(mod2)$. Bu yerdan $inda$ ning juft son bo'lishi kerak ekanligi kelib chiqadi. $p =$ 23moduli bo'yicha indekslar jadvalidan berilgan sonlar $15, 16, 17, 18, 19, 20$ orasidan indekslari juft son bo'lganlarini ajratib olamiz. $ind15 = 17, ind16 = 8, ind17 = 7, ind18 = 12, ind19 = 15, ind20 = 5$ bo'lgani uchun qaralayotgan shartni qanoatlantiruvchilari 16 va 18 bo'ladi. Shuning uchun ham 16 va 18 sonlari 23 moduli bo'yicha kvadratik chegirma bo'ladi. **Javob:** 16 va 18.

2). $p = 29$ da $(*)$ dan $a^{14} \equiv 1(mod29)$ kelib chiqadi. Berilgan sonlar $15, 16, 17, 18, 19, 20$ orasidan shu shartni ajratib olish uchun oxirgi taqqoslamani indekslardan foydalanib yechamiz. U holda quyidagiga ega bo'lamiz: $14inda \equiv 0(mod28) \to inda \equiv 0(mod2)$. Bu yerdan $inda$ ning juft son bo'lishi kerak ekanligi kelib chiqadi. $p =$ 29 moduli bo'yicha indekslar jadvalidan berilgan sonlar $15, 16, 17, 18, 19, 20$ orasidan indekslari juft son bo'lganlarini ajratib olamiz. $ind15 = 27, ind16 = 4, ind17 = 21, ind18 = 11, ind19 = 9, ind20 = 24$ bo'lgani uchun qaralayotgan shartni qanoatlantiruvchilari 16 va 20 bo'ladi. Shuning uchun ham 16 va 20 sonlari 29 moduli bo'yicha kvadratik chegirma bo'ladi. **Javob:** 16 va 20.

3). $p = 41$ da (*) dan $a^{20} \equiv 1(mod41)$ kelib chiqadi. Berilgan sonlar $15, 16, 17, 18, 19, 20$ orasidan shu shartni ajratib olish uchun oxirgi taqqoslamani indekslardan foydalanib yechamiz. U holda quyidagiga ega bo'lamiz: $20inda \equiv 0(mod40) \rightarrow inda \equiv 0(mod2)$. Bu yerdan $inda$ ning juft son bo'lishi kerak ekanligi kelib chiqadi. $p = 41$ moduli bo'yicha indekslar jadvalidan berilgan sonlar $15, 16, 17, 18, 19, 20$ orasidan indekslari juft son bo'lganlarini ajratib olamiz. $ind15 = 37, ind16 = 24, ind17 = 33, ind18 = 16, ind19 = 9, ind20 = 34$ bo'lgani uchun qaralayotgan shartni qanoatlantiruvchilari 16,18va 20 bo'ladi. Shuning uchun ham 16,18 va 20 sonlari 41moduli bo'yicha kvadratik chegirma bo'ladi. Javob: 16,18 va 20.

4). $p = 73$ da (*) dan $a^{36} \equiv 1(mod73)$ kelib chiqadi. Berilgan sonlar $15, 16, 17, 18, 19, 20$ orasidan shu shartni ajratib olish uchun oxirgi taqqoslamani indekslardan foydalanib yechamiz. U holda quyidagiga ega bo'lamiz: $36inda \equiv 0(mod72) \rightarrow inda \equiv 0(mod2)$. Bu yerdan $inda$ ning juft son bo'lishi kerak ekanligi kelib chiqadi. $p = 73$moduli bo'yicha indekslar jadvalidan berilgan sonlar $15, 16, 17, 18, 19, 20$ orasidan indekslari juft son bo'lganlarini ajratib olamiz. $ind15 = 7, ind16 = 32, ind17 = 21, ind18 = 20, ind19 = 62, ind20 = 17$ bo'lgani uchun qaralayotgan shartni qanoatlantiruvchilari 16,18va 19 bo'ladi. Shuning uchun ham 16,18 va 19 sonlari 73moduli bo'yicha kvadratik chegirma bo'ladi. **Javob:** 16,18 va 19.

5). $p = 97$ da (*) dan $a^{48} \equiv 1(mod97)$ kelib chiqadi. Berilgan sonlar $15, 16, 17, 18, 19, 20$ orasidan shu shartni ajratib olish uchun oxirgi taqqoslamani indekslardan foydalanib yechamiz. U holda quyidagiga ega bo'lamiz: $48inda \equiv 0(mod96) \rightarrow inda \equiv 0(mod2)$. Bu yerdan $inda$ ning juft son bo'lishi kerak ekanligi kelib chiqadi. $p = 97$moduli bo'yicha indekslar jadvalidan berilgan sonlar $15, 16, 17, 18, 19, 20$ orasidan indekslari juft son bo'lganlarini ajratib olamiz. $ind15 = 71, ind16 = 40, ind17 = 89, ind18 = 78, ind19 = 81, ind20 = 69$ bo'lgani uchun qaralayotgan shartni qanoatlantiruvchilari 16 va 18 bo'ladi. Shuning uchun ham 16 va18 sonlari 97moduli bo'yicha kvadratik chegirma bo'ladi. **Javob:** 16 va 18.

332. Berilgan modul bo'yicha indekslarninga_1 asosga ko'ra sistemasidan ikkinchi bir a_2 ko'ra sistemasiga o'tish formulasini keltirib chiqarish talab etilsin. Ma'lumki, ${a_2}^{ind_{a_2}b} \equiv b(modp)$. Buning ikkala tomonini a_1 asosga ko'ra indekslaymiz. U holda $ind_{a_2}b \cdot ind_{a_1}a_2 \equiv ind_{a_1}b(mod p-1)$. Bundan

$$ind_{a_2}b \equiv (ind_{a_1}a_2)^{\varphi(p-1)-1} ind_{a_1}b(mod p-1).$$

Misol uchun $ind_2 7 = 7(mod 11)$ dan $ind_8 7 \equiv 7 \cdot (ind_2 8)^{\varphi(10)-1}(mod 10) \equiv 7 \cdot (ind_2 8)^3(mod 10) \equiv 7 \cdot 3^3(mod 10) \equiv 9(mod 10)$. Demak, $ind_8 7 = 9\ (mod 11)$.

Javob: $ind_{a_2}b \equiv (ind_{a_1}a_2)^{\varphi(p-1)-1} ind_{a_1}b(mod p-1)$.

333. 1). a ning qanday butun qiymatlarida $3a^2 - 5 \vdots 17$ munosabat o'rinli ekanligini aniqlashimiz kerak. Bu munosabat $3a^2 \equiv 5(mod 17)$ taqqoslamaga teng kuchli. Bundan $ind3 + 2inda \equiv ind5(mod 16)$. Bu yerda $ind3 = 1, ind5 = 5$ bo'lgani uchun $1 + 2inda \equiv 5(mod 16) \to 2inda \equiv 4(mod 16) \to inda \equiv 2(mod 8) \to inda \equiv 2, 10(mod 16) \to a \equiv 9, 8(mod 17)$.

Javob: $a \equiv \pm 8(mod 17)$.

2). a ning qanday butun qiymatlarida $7a^2 + 13 \vdots 23$munosabat o'rinli ekanligini aniqlashimiz kerak. Bu munosabat $7a^2 \equiv -13(mod 23) \to 7a^2 \equiv 10(mod 23)$ taqqoslamaga teng kuchli. Bundan $ind7 + 2inda \equiv ind10(mod 22)$. Bu yerda $ind7 = 19, ind10 = 3$ bo'lgani uchun $19 + 2inda \equiv 3(mod 22) \to 2inda \equiv -16(mod 22) \to 2inda \equiv 6(mod 22) \to inda \equiv 3(mod 11) \to inda \equiv 3{,}14(mod 22) \to a \equiv 10, 13(mod 23)$. **Javob:** $a \equiv \pm 10(mod 23)$.

3).a ning qanday butun qiymatlarida $13a^2 - 11 \vdots 29$ munosabat o'rinli ekanligini aniqlashimiz kerak. Bu munosabat $13a^2 \equiv 11(mod 29)$ taqqoslamaga teng kuchli. Bundan $ind13 + 2inda \equiv ind11(mod 28)$. Bu yerda $ind13 = 18, ind11 = 25$ bo'lgani uchun $18 + 2inda \equiv 25(mod 28) \to 2inda \equiv 7(mod 28)$. Bu yerda (2,28) = 2, lekin 25 soni 2 ga bo'linmaydi. Shuning uchun ham oxirgi taqqoslama yechimga ega emas. Demak, a ning $13a^2 - 11 \vdots 29$ ifoda o'rinli bo'lgan butun qiymatlari mavjud emas.

Javob: bunday qiymatlar mavjud emas.

V.3-§.

334. 1).$a = 2^{64}$ sonini $m = 360$ ga bo'lishdan chiqqan qoldiqni topish uchun$2^{64} \equiv r(mod 360)$ taqqoslamadan r ni manfiy bo'lmagan eng kichik chegirma sifatida aniqlash kerak bo'ladi. $2^{64} = (2^{15})^4 \cdot 2^4 =$

$32768^4 \cdot 16 \equiv (360 \cdot 91 + 8)^4 \cdot 16 \equiv 8^4 \cdot 16 \equiv 4096 \cdot 16 \equiv (360 \cdot 11 + 136) \cdot 16 \equiv 136 \cdot 16 \equiv (360 \cdot 6 + 16) \equiv 16(mod 360)$. Shuning uchun ham izlanayotgan qoldiq 16 ga teng. **Javob:** 16.

2).$a = 1532^5 - 1$ sonini $m = 9$ ga bo'lishdan chiqqan qoldiqni topish uchun $1532^5 - 1 \equiv r(mod 9)$ taqqoslamadan r ni manfiy bo'lmagan eng kichik chegirma sifatida aniqlash kerak bo'ladi. $1532^5 - 1 \equiv (9 \cdot 170 + 2)^5 - 1 \equiv 2^5 - 1 \equiv 31 \equiv 4(mod 9)$. Shuning uchun ham izlanayotgan qoldiq 4 ga teng.**Javob:** 4.

3).$a = (12371^{56} + 34)^{28}$sonini $m = 111$ ga bo'lishdan chiqqan qoldiqni topish uchun$(12371^{56} + 34)^{28} \equiv r(mod 111)$ taqqoslamadan r ni manfiy bo'lmagan eng kichik chegirma sifatida aniqlash kerak bo'ladi. $(12371^{56} + 34)^{28} \equiv (50^{56} + 34)^{28} \equiv ((50^4)^{14} + 34)^{28} \equiv (34^{14} + 34)^{28} \equiv ((34^2)^7 + 34)^{28} \equiv (46^7 + 34)^{28} \equiv ((46^2)^3 \cdot 46 + 34)^{28} \equiv (7^3 \cdot 46 + 34)^{28} \equiv (16 + 34)^{28} \equiv 50^{28} \equiv (50^4)^7 \equiv 34^7 \equiv (34^2)^3 \cdot 34 \equiv 46^3 \cdot 34 \equiv 7 \cdot 46 \cdot 34 \equiv 70(mod 111)$. Shuning uchun ham izlanayotgan qoldiq 70 ga teng. **Javob:** 70.

4). $a = 8!$ sonini $m = 11$ ga bo'lishdan chiqqan qoldiqni topish uchun $8! \equiv r(mod 11)$ taqqoslamadan r ni manfiy bo'lmagan eng kichik chegirma sifatida aniqlash kerak bo'ladi. $8! \equiv 4! \cdot 5 \cdot 6 \cdot 7 \cdot 8 \equiv 2 \cdot 5 \cdot 6 \cdot 1 \equiv 5(mod 11)$. Shuning uchun ham izlanayotgan qoldiq 5 ga teng. **Javob:** 5.

335. Agar $a^x \equiv 2(mod 13)$va $a^{x+1} \equiv 6(mod 13)$ bo'lsa, a sonini $m = 13$ ga bo'lishdan chiqqan qoldiqni topish uchun ikkinchi taqqoslamani birinchi taqqoslamaga hadlab bo'lamiz. U holda $a \equiv 3(mod 13)$ hosil bo'ladi. Shuning uchun ham izlanayotgan qoldiq 3 ga teng. **Javob:** 3.

336. 1). $(13, 174) = 1$ bo'lgani uchun Eyler teoremasiga asosan $174^{\varphi(13)} \equiv 1(mod 13) \rightarrow (13 \cdot 13 + 5)^{12} \equiv 1(mod 13) \rightarrow 5^{12} \equiv 1(mod 13)$ bajarilishi kerak. Bundan $174^{249} \equiv (5^{12})^{20} \cdot 5^9 \equiv (5^3)^3 \equiv (-5)^3 \equiv 5(mod 13)$. Shuning uchun ham izlanayotgan qoldiq 5 ga teng. **Javob:** 5.

2).$1863^5 - 5 \equiv r(mod 10)$ taqqoslamadan r ni manfiy bo'lmagan eng kichik chegirma sifatida aniqlash kerak bo'ladi. Bu yerda $1863^5 - 5 \equiv 3^5 - 5(mod 10)$ va $(3, 10) = 1$ bo'lgani uchun Eyler teoremasiga asosan $3^{\varphi(10)} \equiv 1(mod 10) \rightarrow 3^4 \equiv 1(mod 10)$ bajarilishi kerak. Bundan $3^5 - 5 \equiv 3^4 \cdot 3 - 5 \equiv 3 - 5 \equiv 8(mod 10)$.Shuning uchun ham izlanayotgan qoldiq 8 ga teng. **Javob:** 8.

3). $2^{37\cdot73-1} \equiv r(mod37\cdot73)$ taqqoslamadan r ni manfiy bo'lmagan eng kichik chegirma sifatida aniqlash kerak bo'ladi. Eyler teoremasiga asosan $2^{\varphi(37)} \equiv 1(mod37) \to 2^{36} \equiv 1(mod37) \to 2^{72} \equiv 1(mod37) \to 2^{73} \equiv 2(mod37)$ (1) bajarilishi kerak. Ikkinchi tomondan $2^{\varphi(73)} \equiv 1(mod73) \to 2^{72} \equiv 1(mod73) \to 2^{73} \equiv 2(mod73)(2)$ bajariladi. (1) va (2) lardan $2^{73} \equiv 2(mod37\cdot73)$ (3) kelib chiqadi. Shuningdek $2^9 \equiv 1(mod73) \to 2^{36} \equiv 1(mod73) \to 2^{37} \equiv 2(mod73)$ va $2^{37} \equiv 2(mod37)$ bo'lgani uchun $2^{37} \equiv 2(mod37\cdot73)$ (4). (3)va (4) larga ko'ra $(2^{37})^{73} \equiv 2^{73} \equiv 2(mod37\cdot73)$. Bundan$2^{37\cdot73-1} \equiv 1(mod37\cdot73)$ ga ega bo'lamiz. Shuning uchun ham izlanayotgan qoldiq 1 ga teng. **Javob:** 1.

337. Berilgan sonning oxirgi ikkita raqamini topish uchun uni 100 bo'lishdan chiqqan qoldig'ini topish yetarli bo'ladi.

1).$203^{20} \equiv r(mod100)$ dan $r \geq 0$ ni aniqlaymiz. Bu yerda $203^{20} = (100\cdot2+3)^{20} \equiv (3^5)^4 \equiv 243^4 \equiv 43^4 \equiv (43^2)^2 \equiv 1849^2(mod100) \equiv 49^2 \equiv 2401(mod100)$. Bu yerdan berilgan sonning oxirgi ikki raqami 0 va 1 ekanligi kelib chiqadi. **Javob:** 0 va 1.

2).$243^{402} \equiv 43^{402}(mod100)$ dan $r \geq 0$ ni aniqlaymiz. Bu yerda $43^{402} = (43^2)^{201} \equiv 49^{201} \equiv (49^2)^{100}\cdot49 \equiv 49(mod100))$. Bundan berilgan sonning oxirgi ikki raqami 4 va 9 ekanligi kelib chiqadi. **Javob:** 4 va 9.

3). Bu yerda$1812\cdot1941\cdot1965 \equiv 12\cdot41\cdot65 \equiv 492\cdot65 \equiv -8\cdot65 \equiv 8\cdot35 \equiv 280\,(mod100)$ Bundan berilgan sonning oxirgi ikki raqami 8 va 0 ekanligi kelib chiqadi. **Javob:** 8 va 0.

4).$(116+17^{17})^{21} \equiv (16+17^{17})^{21}$ dan $r \geq 0$ ni aniqlaymiz. Bu yerda $17^{17} \equiv (17^2)^8\cdot17 \equiv 289^8\cdot17 \equiv (-11)^8\cdot17 \equiv 121^4\cdot17 \equiv 21^4\cdot17 \equiv (21^2)^2\cdot17 \equiv 41^2\cdot17 \equiv 1681\cdot17 \equiv (-19)\cdot17 \equiv -323 \equiv -23(mod100)$ bo'lgani uchun $(16-23)^{21} \equiv (-7)^{21} \equiv ((-7)^4)^5\cdot(-7) \equiv 2401^5\cdot(-7) \equiv -7 \equiv 93(mod100)$. Bundan berilgan sonning oxirgi ikki raqami 9 va 3 ekanligi kelib chiqadi. **Javob:** 9 va 3.

338.1).$2^{32}+1$ ning 641 ga bo'linishini isbotlash uchun $2^{32}+1 \equiv 0(mod641)$ taqqoslamaning bajarilishini ko'rsatamiz. Bu taqqoslamadan $2^{32} \equiv -1(mod641) \to 2^{32} \equiv 640(mod641) \to 2^{25} \equiv 5 \to 2^{25} = (2^{12})^2\cdot2 = 4096\cdot2 = (641\cdot6+250)^2\cdot2 \equiv 250^2\cdot2 \equiv 125000 \equiv$

$641\cdot 195+5\equiv 5\ (mod 641)$. Demak, berilgan $2^{32}+1$ soni 641 ga bo'linadi.

2). $A=222^{555}+555^{222}$ ning 7 ga bo'linishini isbotlash uchun $222^{555}+555^{222}\equiv 0(mod7)$ taqqoslamaning bajarilishini ko'rsatamiz. Bu taqqoslamadan $A=(7\cdot 31+5)^{555}+(7\cdot 79+2)^{222}\equiv 5^{555}+2^{222}\equiv(-2)^{555}+2^{222}\equiv -2^{555}+2^{222}\equiv 2^{222}(-2^{333}+1)$. Bu yerda $2^{222}\equiv(2^3)^{74}\equiv 1(mod7)$ va$2^{333}\equiv(2^3)^{111}\equiv 1(mod7)$ bo'lgani uchun $A\equiv 1\cdot(-1+1)\equiv 0\ (mod7)$. Demak, berilgan A soni 7 ga bo'linadi.

3). $A=220^{119^{69}}+69^{220^{119}}+119^{69^{220}}$ ning 102 ga bo'linishini isbotlash uchun $220^{119^{69}}+69^{220^{119}}+119^{69^{220}}\equiv 0(mod102)$ taqqoslamaning bajarilishini ko'rsatamiz. Bu yerda $102=2\cdot 3\cdot 17$ bo'lgani uchun $220^{119^{69}}+69^{220^{119}}+119^{69^{220}}\equiv 0(mod2), 220^{119^{69}}+69^{220^{119}}+119^{69^{220}}\equiv 0(mod3)$,

$220^{119^{69}}+69^{220^{119}}+119^{69^{220}}\equiv 0(mod17)$larning bajarilishini ko'rsatamiz. Bundan esa $A\equiv 0(mod102)$ kelib chiqadi. Bu yerda $220^{119^{69}}\equiv 0(mod2),\ 69^{220^{119}}\equiv 1(mod2)va119^{69^{220}}\equiv 1(mod2)$bo'lgani uchun $A\equiv(0+1+1)\equiv 0\ (mod2)$ bo'ladi. $220^{119^{69}}\equiv 1(mod3),\ 69^{220^{119}}\equiv 0(mod3)va(-1)^{69^{220}}\equiv -1(mod3)$bo'lgani uchun $A\equiv(1+0-1)\equiv 0\ (mod3)$ bo'ladi. $220^{119^{69}}\equiv -1(mod17),\ 69^{220^{119}}\equiv 1(mod17)va119^{69^{220}}\equiv 0(mod17)$bo'lgani uchun $A\equiv(-1+1+0)\equiv 0\ (mod17)$ bo'ladi. Demak, berilgan A soni 102 ga bo'linadi.

4). $A=6^{2n+1}+5^{n+2}$ ning 31 ga bo'linishini isbotlash uchun $6^{2n+1}+5^{n+2}\equiv 0(mod31)$ taqqoslamaning bajarilishini ko'rsatamiz. Bu yerda $6^{2n+1}=(6^2)^n\cdot 6\equiv 36^n\cdot 6\equiv 5^n\cdot(-25)\equiv -5^{n+2}$ bo'lgani uchun $A=6^{2n+1}+5^{n+2}\equiv -5^{n+2}+5^{n+2}\equiv 0(mod31)$ bo'ladi. Demak, berilgan A soni 31 ga bo'linadi.

339.$4^{\varphi(m)-1}\equiv r(modm)$ dan $m>1-$toq son bo'lganida $0\le r<m$ shartni qanoatlantiruvchi r ni aniqlaymiz. $4^{\varphi(m)-1}\equiv r(modm)\to 4^{\varphi(m)}\equiv 4r(modm)$ va bundan $(4,m)=1$ bo'lgani uchun Eyler teoremasiga asosan $4r\equiv 1(modm)$. Bu yerda $m>1-$toq son bo'lgani uchun uni 4 moduli bo'yicha

$m = 4q \pm 1$ ko'rinishlarida yozish mumkin. Agar $m = 4q + 1$ ko'rinishida bo'lsa, $4r \equiv 1(modm) \to 4r = 1 + 3m(modm) \equiv 12q + 4(modm) \to r \equiv 3q + 1(modm) \to 3 \cdot \frac{m-1}{4} + 1 \equiv \frac{3m+1}{4}(modm)$. Bu yerda $\frac{3m+1}{4} < m$ va $m > 1$ bo'lganda izlanayotgan qoldiqni beradi. Agar $m = 4q - 1$ ko'rinishida bo'lsa, $4r \equiv 1(modm) \to 4r = 1 + m(modm) \equiv 4q(modm) \to r \equiv q(modm) \to r \equiv \frac{m+1}{4}(modm)$.

Demak, bu holda $\frac{m+1}{4} < m, (m > 1)$ izlanayotgan qoldiq bo'ladi.

Javob: Agar $m = 4q + 1$ ko'rinishida bo'lsa, $\frac{3m+1}{4}$ ga va agar r $m = 4q - 1$ ko'rinishida bo'lsa, $\frac{m+1}{4}$ ga teng.

340.1).Indekslardan foydalanib berilgan$a = 10^{10}$ sonini $m = 67$ga bo'lishdan chiqqan qoldiqni topish talab qilnayapti. Buning uchun $10^{10} \equiv r(mod67)$ dan $0 \le r < 67$ shartni qanoatlantiruvchi r ni aniqlaymiz. Taqqoslamaning ikkala tomonini indekslaymiz. U holda $10ind10 \equiv indr(mod66)$ga ega bo'lamiz. Bu yerda 67 moduli bo'yicha indekslar jadvalidan $ind10 = 16$ ekanligini aniqlaymiz. U holda $10 \cdot 16 \equiv indr(mod66) \to indr \equiv 28(mod66)$. Anti indekslar jadvalidan foydalanib bu yerdan $r \equiv 23(mod67)$ ekanligini topamiz. Demak, izlanayotgan qoldiq 23 ga teng ekan. **Javob: 23.**

2). Indekslardan foydalanib berilgan$a = 178^{52}$ sonini $m = 11$ga bo'lishdan chiqqan qoldiqni topish talab qilnayapti. Buning uchun $178^{52} \equiv r(mod11)$ dan $0 \le r < 11$ shartni qanoatlantiruvchi r ni aniqlaymiz. Taqqoslamani $178^{52} \equiv r(mod11) \to (11 \cdot 16 + 2)^{52} \equiv r(mod11) \to 2^{52} \equiv r(mod11)$ ko'rinishda yozib olamiz va uning ikkala tomonini indekslaymiz. U holda $52ind2 \equiv indr(mod10)$ga ega bo'lamiz. Bu yerda 67 moduli bo'yicha indekslar jadvalidan $ind2 = 1$ ekanligini aniqlaymiz. U holda $52 \equiv indr(mod10) \to indr \equiv 2(mod10)$. Anti indekslar jadvalidan foydalanib, bu yerdan $r \equiv 4(mod11)$ ekanligini topamiz. Demak, izlanayotgan qoldiq 4 ga teng ekan. **Javob: 4.**

3). Indekslardan foydalanib berilgan$a = 2017^{2018}$ sonini $m = 11$ga bo'lishdan chiqqan qoldiqni topish talab qilnayapti. Buning uchun $2017^{2018} \equiv r(mod11)$ dan $0 \le r < 11$ shartni qanoatlantiruvchi r ni aniqlaymiz. Taqqoslamani $2017^{2018} \equiv r(mod11) \to (11 \cdot 183 +$

$4)^{2018} \equiv r(mod 11) \to 4^{2018} \equiv r(mod 11)$ ko'rinishda yozib olamiz va uning ikkala tomonini indekslaymiz. U holda $2018 ind 4 \equiv ind r(mod 10) \to (10 \cdot 201 + 8) ind 4 \equiv ind r(mod 10) \to 8 ind 4 \equiv ind r(mod 10)$ga ega bo'lamiz. Bu yerda 11 moduli bo'yicha indekslar jadvalidan $ind 4 = 2$ ekanligini aniqlaymiz. U holda $16 \equiv ind r(mod 10) \to ind r \equiv 6(mod 10)$. Anti indekslar jadvalidan foydalanib, bu yerdan $r \equiv 9(mod 11)$ ekanligini topamiz. Demak, izlanayotgan qoldiq 9 ga teng ekan. **Javob:** 9.

341. Paskalning umumiy bo'linish belgisini ifodalovchi nazaruy qismdagi (1)-formuladan foydalanamiz.Unga ko'ra $N = a_0 + a_1 \cdot 10 + a_2 \cdot 10^2 + \dots + a_n \cdot 10^n \equiv a_0 + a_1 \cdot r_1 + a_2 \cdot r_2 + \dots + a_n \cdot r_n \ (mod m)$(1) bajariladi. Bu yerda $10^k \equiv r_k(mod m)$, $k = 1, 2, \dots, n$.

1). 6 ga bo'linish belgisini keltirib chiqarish uchun yuqoridagi formulada $m = 6$ deb olamiz. U holda $10 \equiv 4(mod 6)$, $10^2 \equiv 4(mod 6), 10^3 \equiv 4(mod m), \dots$ bo'lgani uchun $10^k \equiv 4(mod 6), k = 1,2,3, \dots, n$ bo'ladi. Shuning uchun (1) dan $N \equiv a_0 + 4(a_1 + a_2 + \dots + a_n)\ (mod 6)$ ni hosil qilamiz. Bu yerdan quyidagi xulosaga kelamiz. Berilgan N sonining 6 ga bo'linishi uchun $a_0 + 4(a_1 + a_2 + \dots + a_n)$ ifodaning 6 ga bo'linishi zarur va yetarlidir. Misol uchun 26676 sonining 6 ga bo'linish yoki bo'linmasligini tekshiraylik. Bu yerda $6 + 4(7 + 6 + 6 + 2) = 6 + 84 = 90$ bo'lib, 90 soni 6 ga bo'linadi. Shuning uchun berilgan son ham 6 ga bo'linadi. Endi 22593 sonining 6 ga bo'linish yoki bo'linmasligini tekshiraylik. Bu yerda $3 + 4(9 + 5 + 2 + 2) = 3 + 72 = 75$ bo'lib, 75 soni 6 ga bo'linmaydi. Shuning uchun berilgan son ham 6 ga bo'linmaydi.

2). 8 ga bo'linish belgisini keltirib chiqarish uchun yuqoridagi formulada $m = 8$ deb olamiz. U holda $10 \equiv 2(mod 8)$, $10^2 \equiv 4(mod 8)$va $l \geq 3$ bo'lsa, $10^l \equiv 0(mod 8)$ bo'lgani uchun (1) dan $N \equiv (a_0 + 2a_1 + 4a_2)\ (mod 8)$ ni hosil qilamiz. Bu yerdan quyidagi xulosaga kelamiz. Berilgan N sonining 8 ga bo'linishi uchun $a_0 + 2a_1 + 4a_2$ ifodaning 8 ga bo'linishi zarur va yetarlidir. Misol uchun 38624 sonining 8ga bo'linish yoki bo'linmasligini tekshiraylik. Bu yerda $4 + 2 \cdot 2 + 4 \cdot 6 = 32$ bo'lib, 32 soni 8 ga bo'linadi. Shuning uchun berilgan son ham 8 ga bo'linadi. Endi 24674 sonining 8 ga bo'linish yoki bo'linmasligini tekshiraylik. Bu yerda $4 + 2 \cdot 7 + 4 \cdot 6 = 42$ bo'lib, 42 soni 8 ga bo'linmaydi. Shuning uchun berilgan son ham 8 ga bo'linmaydi.

3). 12 ga bo‘linish belgisini keltirib chiqarish uchun yuqoridagi formulada $m = 8$ deb olamiz. U holda $10 \equiv 10(mod 12)$, $10^2 \equiv 4(mod 12)$ va $l \geq 2$ bo'lsa, $10^l \equiv 4(mod 12)$ bo‘lgani uchun (1) dan $N \equiv 4(a_n + a_{n-1} + \cdots + a_2) + \overline{a_1 a_0}\ (mod 12)$ ni hosil qilamiz. Bu yerdan quyidagi xulosaga kelamiz. Berilgan N sonining 12 ga bo‘linishi uchun $4(a_n + a_{n-1} + \cdots + a_2) + \overline{a_1 a_0}$ ifodaning 12 ga bo‘linishi zarur va yetarlidir. Misol uchun 264816 sonining 12 ga bo‘linish yoki bo‘linmasligini tekshiraylik. Bu yerda $4(2 + 6 + 4 + 8) + 16 = 96$ bo‘lib, 96 soni 12 ga bo‘linadi. Shuning uchun berilgan son ham 12 ga bo‘linadi. Endi 24674 sonining 8 ga bo‘linish yoki bo‘linmasligini tekshiraylik. Bu yerda $4 + 2 \cdot 7 + 4 \cdot 6 = 42$ bo‘lib, 42 soni 8 ga bo‘linmaydi. Shuning uchun berilgan son ham 8 ga bo‘linmaydi.

4). a).15 ga bo‘linish belgisini keltirib chiqarish uchun yuqoridagi formulada $m = 15$ deb olamiz. U holda $10 \equiv 10(mod 15)$, $10^2 \equiv 10(mod 15)$va $l \geq 2$ bo'lsa, $10^l \equiv 10(mod 15)$ bo‘lgani uchun (1) dan $N \equiv 10(a_n + a_{n-1} + \cdots + a_2 + a_1) + a_0\ (mod 15)$ ni hosil qilamiz.

b).18 ga bo‘linish belgisini keltirib chiqarish uchun yuqoridagi formulada $m = 18$ deb olamiz. U holda $10 \equiv 10(mod 18)$, $10^2 \equiv 10(mod 18)$va $l \geq 2$ bo'lsa, $10^l \equiv 10(mod 15)$ bo‘lgani uchun (1) dan $N \equiv 10(a_n + a_{n-1} + \cdots + a_2 + a_1) + a_0\ (mod 18)$ ni hosil qilamiz.

c). 45 ga bo‘linish belgisini keltirib chiqarish uchun yuqoridagi formulada $m = 45$ deb olamiz. U holda $10 \equiv 10(mod 45)$, $10^2 \equiv 10(mod 45)$va $l \geq 2$ bo'lsa, $10^l \equiv 10(mod 45)$ bo‘lgani uchun (1) dan $N \equiv 10(a_n + a_{n-1} + \cdots + a_2 + a_1) + a_0\ (mod 45)$ ni hosil qilamiz.

Bu yerdan quyidagi xulosaga kelamiz. Berilgan N sonining 15, 18 va 45 ga bo‘linish belgisi bir xil ekan, ya'ni berilgan N sonining 15, 18 va 45 ga bo‘linishi uchun $10(a_n + a_{n-1} + \cdots + a_2 + a_1) + a_0$ ifodaning mos ravishda shu sonlarga bo‘linishi zarur va yetarlidir.

342. 792 ga bo‘linadigan $13xy45z$ ko‘rinishidagi barcha sonlarni topish uchun $13xy45z \equiv 0\ (mod\ 792)$ shartni qanoatlantiruvchi barcha x, y, z raqamlarni aniqlashimiz kerak. Bu yerda $792 = 8 \cdot 9 \cdot 11$ bo‘lgani uchun yuqoridagi taqqoslama ushbu taqqoslamalar sistemasi

$$\begin{cases} 13xy45z \equiv 0\ (mod\ 8) \\ 13xy45z \equiv 0\ (mod\ 9) \\ 13xy45z \equiv 0\ (mod\ 11) \end{cases}$$

ga teng kuchli. Bu sistemaning 1-taqqoslamasidan 8 ga bo'linish belgisiga ko'ra

$\overline{45z} \equiv 0 \ (mod\, 8) \rightarrow 4 \cdot 10^2 + 5 \cdot 10 + z \equiv 0 \ (mod\, 8) \rightarrow 450 + z \equiv 0 (mod 8) \rightarrow z \equiv 6 (mod 8)$ ga ega bo'lamiz. Bu yerdan z raqam bo'lgani uchun $z = 6$ ekanligi kelib chiqadi. Shuningdek, yuqoridagi sistemaning 2 va 3-taqqoslamalaridan 9 ga va 11 ga bo'linish belgilariga asosan

$$\begin{cases} 1+3+x+y+4+5+6 \equiv 0 \ (mod\, 9) \\ 1-3+x-y+4-5+6 \equiv 0 \ (mod\, 11) \end{cases} \rightarrow \begin{cases} x+y \equiv 8 \ (mod\, 9) \\ x-y \equiv 8 \ (mod\, 11) \end{cases} \rightarrow$$

$x = 8, y = 0$ ekanligi kelib chiqadi. Demak, izlanayotgan son yagona va u 1380456 ga teng. **Javob**: 1380456.

343. Agar $\frac{a}{b}$ – qisqarmas kasr berilgan bo'lib, (10,b)=1 bo'lsin va m soni b moduli bo'yicha 10 soni tegishli bo'lgan daraja ko'rsatkichi, $10^m \equiv 1 (mod b)$ taqqoslama o'rinli bo'lgan eng kichik ko'rsatkich bo'lsin. U holda berilgan kasrni cheksiz o'nli kasrlarga aylantirganda davr uzunligi m ga teng bo'ladi. Davr uzunligi kasrning suratiga bog'liq emas.

1). Bunda $b = 21$ va $10^m \equiv 1(mod 21)$ dan m ni aniqlaymiz: $10 \equiv 10(mod 21)$; $10^2 \equiv -5(mod 21)$; $10^3 \equiv -8(mod 21)$; $10^4 \equiv 4(mod 21)$; $10^5 \equiv -2(mod 21)$; $10^6 \equiv 1(mod 21)$. Demak, $m = 6$. **Javob:** 6.

2). Bunda $b = 91$ va $10^m \equiv 1(mod 91)$ dan m ni aniqlaymiz: $10 \equiv 10(mod 91)$; $10^2 \equiv 9(mod 91)$; $10^3 \equiv -1(mod 91)$; $10^4 \equiv -10(mod 91)$; $10^5 \equiv -9(mod 91)$; $10^6 \equiv 1(mod 91)$. Demak, $m = 6$. **Javob**: 6.

3). Bunda $b = 43$ va $10^m \equiv 1(mod 43)$ dan m ni aniqlaymiz. Buning uchun indekslardan foydalanish qulay: $m\, ind 10 \equiv 0(mod 42)$. Bu yerda $ind_{43} 10 = 10$ bo'lgani uchun $10m \equiv 0(mod 42) \rightarrow 5m \equiv 0(mod 21) \rightarrow m \equiv 0(mod 21)$. Demak, $m = 6$. **Javob:** 21.

5). Bunda $b = 97$ va $10^m \equiv 1(mod 97)$ dan m ni aniqlaymiz. Buning uchun indekslardan foydalanish qulay: $m\, ind 10 \equiv 0(mod 96)$. Bu yerda $ind_{97} 10 = 35$ bo'lgani uchun $35m \equiv 0(mod 96) \rightarrow m \equiv 0(mod 96)$. Demak, $m = 96$.

Javob: 96.

344. 1). $\frac{10}{17 \cdot 23}$ oddiy kasrlarni o'nli kasrlarga aylantirganda hosil bo'ladigan davr uzunligini aniqlaymiz. Bunda $b = 17 \cdot 23$ va $10^m \equiv$

$1(mod 17\cdot 23)$ dan m ni aniqlaymiz. Bu taqqoslama ushbu taqqoslamalar sistemasi $\begin{cases}10^m \equiv 1(mod 17)\\ 10^m \equiv 1(mod 23)\end{cases}$ ga teng kuchli. Bundan $\begin{cases} m\, ind_{17}10 \equiv 0(mod 16)\\ m\, ind_{23}10 \equiv 0(mod 22)\end{cases}$.

Bu yerda $ind_{17}10 = 3$ va $ind_{23}10 = 7$ bo'lgani uchun $\begin{cases}3m \equiv 0(mod 16)\\ 7m \equiv 0(mod 22)\end{cases} \rightarrow \begin{cases}m \equiv 0(mod 16)\\ m \equiv 0(mod 22)\end{cases} \rightarrow m \equiv 0(mod[16;22]) \rightarrow$ $m \equiv 0(mod 176)$.

Demak, $m = 176$. **Javob:** 176.

2). $\frac{1}{53\cdot 59}$ oddiy kasrlarni o'nli kasrlarga aylantirganda hosil bo'ladigan davr uzunligini aniqlaymiz. Bunda $b = 53\cdot 59$ va $10^m \equiv 1(mod\, 53\cdot 59)$ dan m ni aniqlaymiz. Bu taqqoslama ushbu taqqoslamalar sistemasi $\begin{cases}10^m \equiv 1(mod 53)\\ 10^m \equiv 1(mod 59)\end{cases}$ ga teng kuchli. Bundan $\begin{cases} m\, ind_{53}10 \equiv 0(mod 52)\\ m\, ind_{59}10 \equiv 0(mod 58)\end{cases}$.

Bu yerda $ind_{53}10 = 48$ va $ind_{59}10 = 7$ bo'lgani uchun $\begin{cases}48m \equiv 0(mod 52)\\ 7m \equiv 0(mod 58)\end{cases} \rightarrow \begin{cases}12m \equiv 0(mod 13)\\ m \equiv 0(mod 58)\end{cases} \rightarrow \begin{cases}m \equiv 0(mod 13)\\ m \equiv 0(mod 58)\end{cases} \rightarrow$ $m \equiv 0(mod[13;58]) \rightarrow m \equiv 0(mod 734)$. Demak, $m = 734$. **Javob:** 734.

3). $\frac{1}{7\cdot 23\cdot 31}$ oddiy kasrlarni o'nli kasrlarga aylantirganda hosil bo'ladigan davr uzunligini aniqlaymiz. Bunda $b = 7\cdot 23\cdot 31$ va $10^m \equiv 1(mod\, 7\cdot 23\cdot 31)$dan m ni aniqlaymiz. Bu taqqoslama ushbu taqqoslamalar sistemasi $\begin{cases}10^m \equiv 1(mod 7)\\ 10^m \equiv 1(mod 23)\\ 10^m \equiv 1(mod 31)\end{cases}$ ga teng kuchli. Bundan

$$\begin{cases} m\, ind_7 3 \equiv 0(mod 6)\\ m\, ind_{23}10 \equiv 0(mod 22).\\ m\, ind_{31}10 \equiv 0(mod 30)\end{cases}$$

Bu yerda $ind_7 3 = 1$, $ind_{23} 10 = 3$ va$ind_{31} 10 = 14$ bo'lgani uchun $\begin{cases} m \equiv 0(mod6) \\ 3m \equiv 0(mod22) \\ 14m \equiv 0(mod30) \end{cases} \rightarrow \begin{cases} m \equiv 0(mod6) \\ m \equiv 0(mod22) \\ 7m \equiv 0(mod15) \end{cases} \rightarrow$

$$\begin{cases} m \equiv 0(mod6) \\ m \equiv 0(mod22) \\ m \equiv 0(mod15) \end{cases} \rightarrow m \equiv 0(mod[6;22;15]) \rightarrow m \equiv 0(mod330).$$

Demak, $m = 330$. **Javob:** 330.

4). $\frac{1}{11\cdot13\cdot17}$ oddiy kasrlarni o'nli kasrlarga aylantirganda hosil bo'ladigan davr uzunligini aniqlaymiz. Bunda $b = 11\cdot13\cdot17$ va $10^m \equiv 1(mod\ 11\cdot13\cdot17)$ dan m ni aniqlaymiz. Bu taqqoslama ushbu taqqoslamalar sistemasi $\begin{cases} 10^m \equiv 1(mod11) \\ 10^m \equiv 1(mod13) \\ 10^m \equiv 1(mod17) \end{cases}$ ga teng kuchli. Bundan

$$\begin{cases} m\, ind_{11} 10 \equiv 0(mod10) \\ m\, ind_{13} 10 \equiv 0(mod12). \\ m\, ind_{17} 10 \equiv 0(mod16) \end{cases}$$

Bu yerda $ind_{11} 10 = 5$, $ind_{13} 10 = 10$ va $ind_{17} 10 = 3$ bo'lgani uchun $\begin{cases} 5m \equiv 0(mod10) \\ 10m \equiv 0(mod12) \\ 3m \equiv 0(mod16) \end{cases} \rightarrow \begin{cases} m \equiv 0(mod2) \\ 5m \equiv 0(mod6) \\ m \equiv 0(mod16) \end{cases} \rightarrow$

$$\begin{cases} m \equiv 0(mod2) \\ m \equiv 0(mod6) \\ m \equiv 0(mod16) \end{cases} \rightarrow m \equiv 0(mod[2;6;16]) \rightarrow m \equiv 0(mod48).$$

Demak, $m = 48$. **Javob:** 48.

5). $\frac{1}{13\cdot37}$ oddiy kasrlarni o'nli kasrlarga aylantirganda hosil bo'ladigan davr uzunligini aniqlaymiz. Bunda $b = 13\cdot37$ va $10^m \equiv 1(mod\ 13\cdot37)$ dan m ni aniqlaymiz. Bu taqqoslama ushbu taqqoslamalar sistemasi $\begin{cases} 10^m \equiv 1(mod\ 13) \\ 10^m \equiv 1(mod\ 37) \end{cases}$ ga teng kuchli. Bundan $\begin{cases} m\, ind_{13} 10 \equiv 0(mod12) \\ m\, ind_{37} 10 \equiv 0(mod36) \end{cases}$. Bu yerda $ind_{13} 10 = 10$, va$ind_{37} 10 = 24$ bo'lgani uchun

$$\begin{cases}10m \equiv 0(mod12) \\ 24m \equiv 0(mod36)\end{cases} \to \begin{cases}5m \equiv 0(mod6) \\ 2m \equiv 0(mod3)\end{cases} \to m \equiv 0(mod6).$$

Demak, $m = 6$. **Javob:** 6.

345. Agar $\frac{a}{b}$ – qisqarmas kasr berilgan bo'lib, $(10, b) = 1$ bo'lmasa, b ni $b = 2^{\alpha} \cdot 5^{\beta} \cdot b_1$ ko'rinishda yozib olamiz, bunda $(b_1, 10) = 1$ va m soni $b_1$moduli bo'yicha 10 soni tegishli bo'lgan daraja ko'rsatkichi, $10^m \equiv 1(modb_1)$ taqqoslama o'rinli bo'lgan eng kichik ko'rsatkich bo'lsin. U holda berilgan kasrni cheksiz o'nli kasrlarga aylantirganda davr uzunligi m ga teng bo'ladi. Davr uzunligi kasrning suratiga bog'liq emas.

1). $\frac{a}{b} = \frac{1}{14}$. Bunda $b = 14 = 2 \cdot 7$ va $b_1 = 7$ bo'lgani uchun $10^m \equiv 1(mod7)$ dan m ni aniqlaymiz: $10 \equiv 3(mod7)$; $10^2 \equiv 2(mod7)$; $10^3 \equiv 6(mod7)$; $10^4 \equiv 4(mod7)$; $10^5 \equiv 5(mod7)$; $10^6 \equiv 1(mod7)$. Demak, $m = 6$.

Javob: 6.

2). $\frac{a}{b} = \frac{7}{550}$.Bunda $b = 550 = 2 \cdot 5^2 \cdot 11$ va $b_1 = 11$ bo'lgani uchun $10^m \equiv 1(mod11)$ dan m ni aniqlaymiz: $10 \equiv -1(mod11)$; $10^2 \equiv 1(mod11)$;

Demak, $m = 2$. **Javob**: 2.

3). $\frac{1}{5 \cdot 23 \cdot 31}$ oddiy kasrlarni o'nli kasrlarga aylantirganda hosil bo'ladigan davr uzunligini aniqlaymiz. Bunda $b_1 = 23 \cdot 31$ va $10^m \equiv 1(mod\ 23 \cdot 31)$ dan m ni aniqlaymiz. Bu taqqoslama ushbu taqqoslamalar sistemasi $\begin{cases}10^m \equiv 1(mod\ 23) \\ 10^m \equiv 1(mod\ 31)\end{cases}$ ga teng kuchli. Bundan $\begin{cases}m\ ind_{23}10 \equiv 0(mod22) \\ m\ ind_{31}10 \equiv 0(mod30)\end{cases}$. Bu yerda $ind_{23}10 = 3$ va$ind_{31}10 = 14$ bo'lgani uchun $\begin{cases}3m \equiv 0(mod22) \\ 14m \equiv 0(mod30)\end{cases} \to \begin{cases}m \equiv 0(mod22) \\ 7m \equiv 0(mod15)\end{cases} \to$ $\begin{cases}m \equiv 0(mod22) \\ m \equiv 0(mod15)\end{cases} \to m \equiv 0(mod330)$. Demak, $m = 330$. **Javob:** 330.

4). $\frac{1}{4 \cdot 53 \cdot 73}$ oddiy kasrlarni o'nli kasrlarga aylantirganda hosil bo'ladigan davr uzunligini aniqlaymiz. Bunda $b_1 = 53 \cdot 73$ va $10^m \equiv 1(mod\ 53 \cdot 73)$ dan m ni aniqlaymiz. Bu taqqoslama ushbu taqqoslamalar sistemasi $\begin{cases} 10^m \equiv 1(mod\ 53) \\ 10^m \equiv 1(mod\ 73) \end{cases}$ ga teng kuchli. Bundan $\begin{cases} m\ ind_{53}10 \equiv 0(mod 52) \\ m\ ind_{73}10 \equiv 0(mod 72) \end{cases}$. Bu yerda $ind_{53}10 = 48$ va $ind_{73}10 = 9$ bo'lgani uchun

$$\begin{cases} 48m \equiv 0(mod 52) \\ 9m \equiv 0(mod 72) \end{cases} \to \begin{cases} 12m \equiv 0(mod 13) \\ m \equiv 0(mod 8) \end{cases} \to$$

$\begin{cases} m \equiv 0(mod 13) \\ m \equiv 0(mod 8) \end{cases} \to m \equiv 0(mod 104)$. Demak, $m = 104$. **Javob:** 104.

5). $\frac{a}{b} = \frac{1}{10\cdot 37}$.Bunda $b = 10 \cdot 37$ va $b_1 = 37$ bo'lgani uchun $10^m \equiv 1(mod 37)$ dan m ni aniqlaymiz: $m\ ind_{37}10 \equiv 0(mod 36)$. Bu yerda $ind_{37}10 = 24$ bo'lgani uchun $24m \equiv 0(mod 36) \to 2m \equiv 0(mod 3) \to m \equiv 0(mod 3)$.Demak, $m = 3$. **Javob:** 32.

346. Berilgan tengliklarning to'g'ri yoki noto'g'ri ekanligini 11 moduli bo'yicha taqqoslamaga o'tish yo'li bilan tekshiramiz.

1). $4237 \cdot 27925 = 118275855$ dan $4237 \cdot 27925 \equiv 118275855(mod 11)$.

Bu yerda $4237 = 11 \cdot 385 + 2$; $27925 = 11 \cdot 2538 + 7$; $118275855 = 11 \cdot 10743259 + 6$ ekanligini e'tiborga olsak, $2 \cdot 7 \equiv 6(mod 11) \to 3 \equiv 6(mod 11) \to 1 \equiv 2(mod 11)$. Oxirgi taqqoslama o'rinli emas. Shuning uchun ham berilgan tenglik noto'g'ri.

2). $42981 : 8264 = 5201$dan$42981 = 5201 \cdot 8264$. Bundan $5201 \cdot 8264 \equiv 42981(mod 11)$.Bu yerda $5201 = 11 \cdot 472 + 9$; $8264 = 11 \cdot 751 + 3$; $42981 = 11 \cdot 3907 + 4$ ekanligini e'tiborga olsak, $9 \cdot 3 \equiv 4(mod 11) \to 5 \equiv 4(mod 11)$. Oxirgi taqqoslama o'rinli emas. Shuning uchun ham berilgan tenglik noto'g'ri.

3). $1965^2 = 3761225$dan$1965^2 \equiv 3761225(mod 11)$. Bu yerda $1965 = 11 \cdot 178 + 7$; $3761225 = 11 \cdot 341929 + 6$ ekanligini e'tiborga olsak, $7^2 \equiv 6(mod 11) \to 5 \equiv 6(mod 11)$. Oxirgi taqqoslama o'rinli emas. Shuning uchun ham berilgan tenglik noto'g'ri.

347. 1).$25041 + 91382 = 116423$. Bu tenglikdan qulaylik uchun 9 moduli bo'yicha taqqoslamaga o'tamiz. U holda $25041 + 91382 \equiv 116423(mod 9)$. Bu yerdagi sonlarni 9 moduli bo'yicha eng kichik manfiy bo'lmagan chegirmalar bilan almashtirsak $3 + 5 \equiv 8(mod 9) \to 8 \equiv 8(mod 9)$ ayniy taqqoslama hosil bo'ladi. Demak, berilgan tenglik to'g'ri.

2). $42932 - 18265 = 24667$. Bu tenglikdan qulaylik uchun 9 moduli bo'yicha taqqoslamaga o'tamiz. U holda $42932 - 18265 \equiv 24667(mod9)$. Bu yerdagi sonlarni 9 moduli bo'yicha eng kichik manfiy bo'lmagan chegirmalar bilan almashtirsak $2 - 4 \equiv 7(mod9) \rightarrow 7 \equiv 7(mod9)$ ayniy taqqoslama hosil bo'ladi. Demak, berilgan tenglik to'g'ri.

3). $13547 - 9862 = 3685$. Bu tenglikdan qulaylik uchun 9 moduli bo'yicha taqqoslamaga o'tamiz. U holda $13547 - 9862 \equiv 3685(mod9)$. Bu yerdagi sonlarni 9 moduli bo'yicha eng kichik manfiy bo'lmagan chegirmalar bilan almashtirsak $2 - 7 \equiv 4(mod9) \rightarrow 4 \equiv 4(mod9)$ ayniy taqqoslama hosil bo'ladi. Demak, berilgan tenglik to'g'ri.

4). $235463 - 25376 = 210087$. Bu tenglikdan qulaylik uchun 9 moduli bo'yicha taqqoslamaga o'tamiz. U holda $235463 - 25376 \equiv 210087(mod9)$. Bu yerdagi sonlarni 9 moduli bo'yicha eng kichik manfiy bo'lmagan chegirmalar bilan almashtirsak $5 - 5 \equiv 0(mod9) \rightarrow 0 \equiv 0(mod9)$ ayniy taqqoslama hosil bo'ladi. Demak, berilgan tenglik to'g'ri.

VI.1 -§.

348. 1).Berilgan kasr $\frac{127}{52}$ ni uzluksiz kasrga yoyish uchun Evklid algoritmidan foydalanamiz. Unga ko'ra: $127 = 52 \cdot \boxed{2} + 23$; $52 = 23 \cdot \boxed{2} + 6$; $23 = 6 \cdot \boxed{3} + 5$; $6 = 5 \cdot \boxed{1} + 1$; $5 = 1 \cdot \boxed{5}$.
Bundan $\frac{127}{52} = (2,2,3,1,5)$.

Javob: $(2,2,3,1,5)$.

2). Berilgan kasr $\frac{24}{35}$ ni uzluksiz kasrga yoyish uchun Evklid algoritmidan foydalanamiz. Unga ko'ra: $24 = 35 \cdot \boxed{0} + 35$; $35 = 24 \cdot \boxed{1} + 11$; $24 = 11 \cdot \boxed{2} + 2$; $11 = 2 \cdot \boxed{5} + 1$; $2 = 1 \cdot \boxed{2}$. Bundan $\frac{127}{52} = (0,1,2,5,2)$.

Javob: $(0,1,2,5,2)$.

3). Berilgan kasr $1{,}23 = \frac{123}{100}$ ni uzluksiz kasrga yoyish uchun Evklid algoritmidan foydalanamiz. Unga ko'ra: $123 = 100 \cdot \boxed{1} + 23$; $100 = 23 \cdot \boxed{4} + 8$; $23 = 8 \cdot \boxed{2} + 7$; $8 = 7 \cdot \boxed{1} + 1$; $7 = 1 \cdot \boxed{7}$. Bundan $1{,}23 = (1,4,2,1,7)$.

Javob: $(1,4,2,1,7)$.

4). Berilgan kasr $\frac{29}{37}$ ni uzluksiz kasrga yoyish uchun Evklid algoritmidan foydalanamiz. Unga ko'ra: $29 = 37 \cdot \boxed{0} + 29$; $37 = 29 \cdot \boxed{1} + 8$; $29 = 8 \cdot \boxed{3} + 5$; $8 = 5 \cdot \boxed{1} + 3$; $5 = 3 \cdot \boxed{1} + 2$, $3 = 2 \cdot \boxed{1} + 1$, $2 = 1 \cdot \boxed{2}$. Bundan $\frac{29}{37} = (0, 1,3,1,1,1,2)$.
Javob:$(0, 1,3,1,1,1,2)$.

349. Berilgan chekli uzluksiz kasrlarga mos qisqarmas oddiy kasr $\frac{a}{b}$ ni topish uchun munosib kasrlar $\frac{P_k}{Q_k}$ dan foydalanamiz. Bunda $(P_k, Q_k) = 1$ va $\frac{P_n}{Q_n} = \frac{a}{b}$.

1). $(1,1,2,1,2,1,2) = \frac{P_n}{Q_n} = \frac{a}{b}$ ni aniqlashimiz kerak. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		1	1	2	1	2	1	2
P_i	$P_0 = 1$	1	2	5	7	19	26	71
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	3	4	11	15	41

Demak, $(1,1,2,1,2,1,2) = \frac{71}{41}$. **Javob:** $\frac{71}{41}$.

2). $(0,1,2,3,4,5) = \frac{P_n}{Q_n} = \frac{a}{b}$ ni aniqlashimiz kerak. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		0	1	2	3	4	5
P_i	$P_0 = 1$	0	1	2	7	30	157
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	3	10	43	225

Demak, $(0,1,2,3,4,5) = \frac{157}{225}$. **Javob:** $\frac{157}{225}$.

3). $(5,4,3,2,1) = (5,4,3,3) = \frac{P_n}{Q_n} = \frac{a}{b}$ ni aniqlashimiz kerak. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		5	4	3	3
P_i	$P_0 = 1$	5	21	68	225
Q_i	$Q_0 = 0$	$Q_1 = 1$	4	13	43

Demak, $(5,4,3,2,1)=\frac{225}{43}$. **Javob:** $\frac{225}{43}$.

4). $(a,a,a,a,a)=\frac{P_n}{Q_n}=\frac{a}{b}$ ni aniqlashimiz kerak. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		a	a	a	a	a
P_i	$P_0=1$	a	a^2+1	a^3+2a	a^4+3a^2+1	a^5+4a^3+3a
Q_i	$Q_0=0$	$Q_1=1$	a	a^2+1	a^3+2a	a^4+3a^2+1

Demak, $(a,a,a,a,a)=\frac{a^5+4a^3+3a}{a^4+3a^2+1}$. **Javob:** $\frac{a^5+4a^3+3a}{a^4+3a^2+1}$.

5). $(a,b,a,b,a)=\frac{P_n}{Q_n}=\frac{a}{b}$ ni aniqlashimiz kerak. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		a	b	a	b	a
P_i	$P_0=1$	a	$ab+1$	a^2b+2a	$a^2b^2+3ab+1$	$a^3b^2+4a^2b+3a$
Q_i	$Q_0=0$	$Q_1=1$	b	$ab+1$	ab^2+2b	$a^2b^2+3ab+1$

Demak, $(a,a,a,a,a)=\frac{a^3b^2+4a^2b+3a}{a^2b^2+3ab+1}$. **Javob:** $\frac{a^3b^2+4a^2b+3a}{a^2b^2+3ab+1}$.

6). $(2,1,1,3,1,2)=\frac{P_n}{Q_n}=\frac{a}{b}$ ni aniqlashimiz kerak. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		2	1	1	3	1	2
P_i	$P_0=1$	2	3	5	18	23	64
Q_i	$Q_0=0$	$Q_1=1$	1	2	7	9	25

Demak, $(2,1,1,3,1,2)=\frac{64}{25}$. **Javob:** $\frac{64}{25}$.

7). $(1,1,2,3,4)=\frac{P_n}{Q_n}=\frac{a}{b}$ ni aniqlashimiz kerak. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		1	1	2	3	4
P_i	$P_0=1$	1	2	5	17	73
Q_i	$Q_0=0$	$Q_1=1$	1	3	10	43

Demak, $(1,1,2,3,4) = \frac{73}{43}$. **Javob:** $\frac{73}{43}$.

8). $(2,5,3,2,1,4,2,3) = \frac{P_n}{Q_n} = \frac{a}{b}$ ni aniqlashimiz kerak. $\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		2	5	3	2	1	4	2	3
P_i	$P_0 = 1$	2	11	35	81	116	545	1206	4163
Q_i	$Q_0 = 0$	$Q_1 = 1$	5	16	37	53	249	551	1902

Demak, $(2,5,3,2,1,4,2,3) = \frac{4163}{1902}$. **Javob:** $\frac{4163}{1902}$.

350. Berilgan $\frac{a}{b}$ kasrni uzluksiz kasrlarga yoyishdan foydalanib qisqartirish uchun uni chekli uzluksiz kasrlarga yoyib munosib kasrlari $\frac{P_k}{Q_k}$ ni hisoblaymiz hamda bunda $(P_k, Q_k) = 1$ va $\frac{P_n}{Q_n} = \frac{a}{b}$ ekanliklaridan foydalanamiz.

1). $\frac{3587}{2743}$ ni uzluksiz kasrlarga yoyamiz. U holda $3587 = 2743 \cdot \boxed{1} + 844$;

$2743 = 844 \cdot \boxed{3} + 211$; $844 = 211 \cdot \boxed{4}$ dan $\frac{3587}{2743} = (1,3,4)$. Munosib kasrlari $\frac{P_k}{Q_k}$ ni hisoblaymiz:

q_i		1	3	4
P_i	$P_0 = 1$	1	4	17
Q_i	$Q_0 = 0$	$Q_1 = 1$	3	13

Demak, $\frac{3587}{2743} = (1,3,4) = \frac{17}{13}$. Tekshirish $\frac{3587}{2743} = \frac{17 \cdot 211}{13 \cdot 211} = \frac{17}{13}$. **Javob:** $\frac{17}{13}$.

2). $\frac{1043}{3427}$ ni uzluksiz kasrlarga yoyamiz. U holda $1043 = 3427 \cdot \boxed{0} + 1043$; $3427 = 1043 \cdot \boxed{3} + 298$; $1043 = 298 \cdot \boxed{3} + 149$; $298 = 149 \cdot \boxed{2}$ dan $\frac{1043}{3427} = (0,3,3,2)$. Munosib kasrlari $\frac{P_k}{Q_k}$ ni hisoblaymiz:

q_i		0	3	3	2
P_i	$P_0 = 1$	0	1	3	7

Q_i	$Q_0 = 0$	$Q_1 = 1$	3	10	23

Demak, $\frac{1043}{3427} = (0,1,3,3) = \frac{7}{23}$. Tekshirish $\frac{1043}{3427} = \frac{7\cdot149}{23\cdot149} = \frac{7}{23}$.

Javob: $\frac{7}{23}$.

3). $\frac{3653}{3107}$ ni uzluksiz kasrlarga yoyamiz. U holda $3653 = 3107\cdot\boxed{1} + 546$; $3107 = 546\cdot\boxed{5} + 377$; $546 = 377\cdot\boxed{1} + 169$; $377 = 169\cdot\boxed{2} + 39$; $169 = 39\cdot\boxed{4} + 13$; $39 = 13\cdot\boxed{3}$ dan $\frac{3653}{3107} = (1,5,1,2,4,3)$. Munosib kasrlari $\frac{P_k}{Q_k}$ ni hisoblaymiz:

q_i		1	5	1	2	4	3
P_i	$P_0 = 1$	1	6	7	20	87	281
Q_i	$Q_0 = 0$	$Q_1 = 1$	5	6	17	74	239

Demak, $\frac{3653}{3107} = (1,5,1,2,4,3) = \frac{281}{239}$. Tekshirish: $\frac{3653}{3107} = \frac{281\cdot13}{239\cdot13} = \frac{281}{239}$.

Javob: $\frac{281}{239}$.

4). $\frac{11281}{6583}$ ni uzluksiz kasrlarga yoyamiz. U holda $11281 = 6583\cdot\boxed{1} + 4698$; $6583 = 4698\cdot\boxed{1} + 1885$; $4698 = 1885\cdot\boxed{2} + 928$; $1885 = 928\cdot\boxed{2} + 29$; $928 = 29\cdot\boxed{32}$ dan $\frac{11281}{6583} = (1,1,2,2,32)$. Munosib kasrlari $\frac{P_k}{Q_k}$ ni hisoblaymiz:

q_i		1	1	2	2	32
P_i	$P_0 = 1$	1	2	5	12	389
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	3	7	227

Demak, $\frac{11281}{6583} = (1,1,2,2,32) = \frac{389}{227}$. Tekshirish: $\frac{11281}{6583} = \frac{389\cdot29}{227\cdot29} = \frac{389}{227}$. **Javob:** $\frac{389}{227}$.

5). $\frac{1491}{2247}$ ni uzluksiz kasrlarga yoyamiz. U holda $1491 = 2247\cdot\boxed{0} + 1491$; $2247 = 1491\cdot\boxed{1} + 756$; $1491 = 756\cdot\boxed{1} + 735$; $756 =$

$735 \cdot \boxed{1} + 21;\ 735 = 21 \cdot \boxed{35}$ dan $\frac{1491}{2247} = (0,1,1,1,35)$. Munosib kasrlari $\frac{P_k}{Q_k}$ ni hisoblaymiz:

q_i		0	1	1	1	35
P_i	$P_0 = 1$	0	1	1	2	71
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	2	3	107

Demak, $\frac{1491}{2247} = (0,1,1,1,35) = \frac{71}{107}$. Tekshirish: $\frac{1491}{2247} = \frac{71 \cdot 21}{107 \cdot 21} = \frac{71}{107}$.

Javob: $\frac{71}{107}$.

351. 1). $(x,2,3,4) = \frac{73}{30}$ tenglamalarni yechish uchun uning chap tomoni orqali ifodalanuvchi qisqarmas kasrni topib olamiz. Buning uchun $\frac{P_n}{Q_n}$ –munosib kasrni hisoblaymiz:

q_i		x	2	3	4
P_i	$P_0 = 1$	x	$2x+1$	$7x+3$	$30x+13$
Q_i	$Q_0 = 0$	$Q_1 = 1$	2	7	30

Bundan $\frac{30x+13}{30} = \frac{73}{30} \to 30x + 13 = 73 \to 30x = 60 \to x = 2$.

Javob: $x = 2$.

2). $7(xyz + x + z) = 10(yz + 1)$ tenglamalarni yechish uchun uni

$$\frac{xyz + x + z}{yz + 1} = \frac{10}{7}$$

ko'rinishida yozib olib uning chap va o'ng tomonlarini uzluksiz kasrlarga yoyamiz. $xyz + x + z = (yz + 1) \cdot \boxed{x} + z;\ yz + 1 = z \cdot \boxed{y} + 1;\ z = 1 \cdot \boxed{z}$ bundan $\frac{xyz+x+z}{yz+1} = (x, y, z)$. Shuningdek $10 = 7 \cdot \boxed{1} + 3;\ 7 = 3 \cdot \boxed{2} + 1;\ 3 = 1 \cdot \boxed{3}$ dan $\frac{10}{7} = (1,2,3)$. Hosil bo'lgan yoyilmalarni yuqoridagi tenglamaga olib borib qo'ysak $(x, y, z) = (1,2,3)$ bundan esa $x = 1, y = 2, z = 3$ kelib chiqadi.

Javob: $x = 1, y = 2, z = 3$.

352. Berilgan kasrlarni uzluksiz kasrga yoyib, uni $\frac{P_4}{Q_4}$ – munosib kasr bilan almashtirib xatoligini aniqlash hamda almashtirishni taqribiy tenglik yordamida xatoligini ko'rsatgan holda yozish uchun berilgan kasrlarni uzluksiz kasrga yoymiz va $\frac{P_4}{Q_4}$ – munosib kasrni aniqlaymiz. Bundagi xatolik $\frac{1}{Q_4 Q_5}$ dan oshmaydi.

1). $\frac{29}{37}$ – kasrni uzluksiz kasrlarga yoyamiz. U holda $29 = 37 \cdot \boxed{0} + 29$; $37 = 29 \cdot \boxed{1} + 8$; $29 = 8 \cdot \boxed{3} + 5$; $8 = 5 \cdot \boxed{1} + 3$; $5 = 3 \cdot \boxed{1} + 2$; $3 = 2 \cdot \boxed{1} + 1$; $2 = 1 \cdot \boxed{2}$. Bundan $\frac{29}{37} = (0,1,3,1,1,1,2)$. Endi munosib kasrlarini aniqlaymiz.

q_i		0	1	3	1	1	1	2
P_i	$P_0 = 1$	0	1	3	4	7	11	29
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	4	5	9	14	37

Bu yerdan $\frac{P_4}{Q_4} = \frac{4}{5} = 0{,}8$. Bundagi xatolik $\frac{1}{Q_4 Q_5} = \frac{1}{5 \cdot 9} = \frac{1}{45} \approx 0{,}02$ ga teng. Bulardan foydalanib berilgan kasrni quyidagicha yozishimoz mumkin: $\frac{29}{37} \approx \frac{4}{5}(-0{,}02) = 0{,}78$. **Javob:** $\frac{29}{37} \approx \frac{4}{5}(-0{,}02)$.

2). $\frac{163}{159}$ – kasrni uzluksiz kasrlarga yoyamiz. U holda $163 = 159 \cdot \boxed{1} + 4$; $159 = 4 \cdot \boxed{39} + 3$; $4 = 3 \cdot \boxed{1} + 1$; $3 = 1 \cdot \boxed{3}$. Bundan $\frac{163}{159} = (1,39,1,3)$. Endi munosib kasrlarini aniqlaymiz.

q_i		1	39	1	3
P_i	$P_0 = 1$	1	40	41	163
Q_i	$Q_0 = 0$	$Q_1 = 1$	39	40	79

Bu yerdan $\frac{P_4}{Q_4} = \frac{163}{79}$. Bundan ko‘rinadiki, $\frac{P_4}{Q_4}$ – munosib kasr berilgan kasrning o‘ziga teng. Shuning uchun ham bu yerda xatolik nolga teng bo‘ladi.

Javob: $\frac{163}{159} = \frac{163}{159}(\pm 0)$.

3). $\frac{648}{385}$ – kasrni uzluksiz kasrlarga yoyamiz. U holda $648 = 385 \cdot \boxed{1} + 263$; $385 = 263 \cdot \boxed{1} + 122$; $263 = 122 \cdot \boxed{2} + 19$; $122 = 19 \cdot \boxed{6} + 8$; $19 = 8 \cdot \boxed{2} + 3$; $8 = 3 \cdot \boxed{2} + 2$; $3 = 2 \cdot \boxed{1} + 1$; $2 = 1 \cdot \boxed{2}$. Bundan $\frac{648}{385} = (1,1,2,6,2,2,1,2)$. Endi munosib kasrlarini aniqlaymiz.

q_i		1	1	2	6	2	2	1	2
P_i	$P_0 = 1$	1	2	5	32	69	170	239	648
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	3	19	41	101	142	385

Bu yerdan $\frac{P_4}{Q_4} = \frac{32}{19} = 0{,}6842$. Bundagi xatolik $\frac{1}{Q_4Q_5} = \frac{1}{19\cdot41} = \frac{1}{779} \approx 0{,}0013$ ga teng. Bulardan foydalanib berilgan kasrni quyidagicha yozishimoz mumkin:

$\frac{648}{385} \approx \frac{32}{19}(-0{,}0013) = 1{,}6831$. **Javob:** $\frac{648}{385} \approx \frac{32}{19}(-0{,}0013)$.

4). $\frac{1882}{1651}$ – kasrni uzluksiz kasrlarga yoyamiz. U holda $1882 = 1651\cdot\boxed{1} + 231$; $1651 = 231\cdot\boxed{7} + 34$; $231 = 34\cdot\boxed{6} + 27$; $34 = 27\cdot\boxed{1} + 7$; $27 = 7\cdot\boxed{3} + 6$; $7 = 6\cdot\boxed{1} + 1$; $6 = 1\cdot\boxed{6}$. Bundan $\frac{1882}{1651} = (1, 7, 6, 1, 3, 1, 6)$. Endi munosib kasrlarini aniqlaymiz.

q_i		1	7	6	1	3	1	6
P_i	$P_0 = 1$	1	8	49	57	220	277	1882
Q_i	$Q_0 = 0$	$Q_1 = 1$	7	43	50	193	207	1651

Bu yerdan $\frac{P_4}{Q_4} = \frac{57}{50} = 1{,}14$. Bundagi xatolik $\frac{1}{Q_4Q_5} = \frac{1}{50\cdot193} = \frac{1}{9650} \approx 0{,}000103$ ga teng. Bulardan foydalanib berilgan kasrni quyidagicha yozishimiz mumkin:

$\frac{1882}{1651} \approx \frac{57}{50}(-0{,}000103) = 1{,}139897$. **Javob:** $\frac{1882}{1651} \approx \frac{57}{50}(-0{,}000103)$.

Eslatma: Xatolikni $\Delta\alpha < \frac{1}{Q_4Q_5}\left(= \frac{1}{3\cdot5} = \frac{1}{15} < 0{,}067 < 0{,}1\right)$ ko'rinishda baholash ham mumkin.

4). $\alpha = \frac{2-\sqrt{3}}{5}$ sonini $\frac{P_4}{Q_4}$ munosib kasr bilan almashtirib uning natijasida hosil bo'ladigan xatolikni baholashimiz talab etilayapti. Buning uchun avvalo $\frac{2-\sqrt{3}}{5}$ ni uzluksiz kasrlarga yoyib munosiob kasrlarini topamiz: ($\sqrt{3} = 1{,}73050807$)

$\alpha = \frac{2-\sqrt{3}}{5} = 0 + \frac{1}{\frac{5}{2-\sqrt{3}}} = 0 + \frac{1}{\alpha_1}$, bunda $\alpha_1 = \frac{5}{2-\sqrt{3}} = 5(2+\sqrt{3}) = 10 + 5\sqrt{3} = 18 + (5\sqrt{3} - 8) = 18 + \frac{75-64}{5\sqrt{3}+8} = 18 + \frac{11}{5\sqrt{3}+8} = 18 + \frac{1}{\frac{8+5\sqrt{3}}{11}} = 18 + \frac{1}{\alpha_2}$;

$$\alpha_2 = \frac{8+5\sqrt{3}}{11} = 1 + \left(\frac{8+5\sqrt{3}}{11} - 1\right) = 1 + \frac{5\sqrt{3}-3}{11} = 1 + \frac{1}{\alpha_3};$$

$$\alpha_3 = \frac{11}{5\sqrt{3}-3} = \frac{11(5\sqrt{3}+3)}{66} = 1 + \left(\frac{5\sqrt{3}+3}{6} - 1\right) = 1 + \frac{5\sqrt{3}-3}{6} = 1 + \frac{1}{\alpha_4};$$

$$\alpha_4 = \frac{6}{5\sqrt{3}-3} = \frac{6(5\sqrt{3}+3)}{66} = \frac{5\sqrt{3}+3}{11} = 1 + \left(\frac{5\sqrt{3}+3}{11} - 1\right)$$

$$= 1 + \frac{5\sqrt{3}-8}{11} = 1 + \frac{1}{\frac{11}{5\sqrt{3}-8}} = 1 + \frac{1}{\alpha_5}; \alpha_5 = \frac{11}{5\sqrt{3}-8}$$

$$= 5\sqrt{3} + 8, \dots .$$

Demak, $\alpha = \frac{2-\sqrt{3}}{5} = (0,18,1,1,1,16,\dots)$ ekan. Endi $\frac{P_4}{Q_4}$ munosib kasrni aniqlaymiz. Uni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		0	18	1	1	1	16	...
P_i	$P_0 = 1$	0	1	1	$\boxed{2}$	3	50	...
Q_i	$Q_0 = 0$	$Q_1 = 1$	18	19	$\boxed{37}$	56	933	...

Demak, $\frac{P_4}{Q_4} = \frac{2}{37}$ va xatolik $\Delta\alpha = |\alpha - a| = \left|\frac{2-\sqrt{3}}{4} - \frac{2}{37}\right| = |0,06737299 - 0,05405405| < 0,014 < 0,02$ va shuning uchun ham $\frac{2-\sqrt{3}}{4} \approx \frac{2}{37}(+0,014) = 0,06805405$. Bu yerda xatolikni "−" ishora bilan olamiz, chunki $\frac{P_4}{Q_4} > \alpha$.

Javob: $\frac{2}{3}$, $\Delta\alpha = 0,05$.

Eslatma: Xatolikni $\Delta\alpha < \frac{1}{Q_4 Q_5}\left(= \frac{1}{37\cdot 56} = \frac{1}{2072} < 0,0005 < 0,001\right)$ ko'rinishda baholash ham mumkin.

5). $\alpha=\frac{1+\sqrt{5}}{2}$ sonini $\frac{P_4}{Q_4}$ munosib kasr bilan almashtirib uning natijasida hosil bo'ladigan xatolikni baholashimiz talab etilayapti. Buning uchun avvalo $\frac{1+\sqrt{5}}{2}$ ni uzluksiz kasrlarga yoyib munosiob kasrlarini topamiz: ($\sqrt{5}=2{,}236067975$)

$\alpha=\frac{1+\sqrt{5}}{2}=1+\left(\frac{1+\sqrt{5}}{2}-1\right)=1+\frac{\sqrt{5}-1}{2}=1+\frac{1}{\alpha_1}$, bunda $\alpha_1=\frac{2}{\sqrt{5}-1}=\frac{2(\sqrt{5}+1)}{4}=\frac{\sqrt{5}+1}{2}=\alpha$. Demak, $\alpha=\frac{1+\sqrt{5}}{2}=((1))$ ekan. Endi $\frac{P_4}{Q_4}$ munosib kasrni aniqlaymiz.$\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		1	1	1	1	1	1	...
P_i	$P_0=1$	1	2	3	$\boxed{5}$	8	13	...
Q_i	$Q_0=0$	$Q_1=1$	1	2	$\boxed{3}$	5	8	...

Demak, $\frac{P_4}{Q_4}=\frac{5}{3}$ va xatolik $\Delta\alpha=|\alpha-a|=\left|\frac{1+\sqrt{5}}{2}-\frac{5}{3}\right|=|1{,}618033989-1{,}66666666|<0{,}04864<0{,}05$ va shuning uchun ham $\frac{1+\sqrt{5}}{2}\approx\frac{5}{3}(-0{,}04864)=1{,}61802666$. Bu yerda xatolikni "$-$" ishora bilan olamiz, chunki $\frac{P_4}{Q_4}>\alpha$.

Javob: $\frac{5}{3}$, $\Delta\alpha=0{,}05$.

Eslatma: Bu yerdagi $\frac{P_i}{Q_i}\left(\frac{1}{1},\frac{2}{1},\frac{3}{2},\frac{5}{3},\frac{8}{5},\frac{13}{8},\frac{21}{13},\frac{34}{21},\dots\right)$sonlariga Fibonachchi ketma-ketligi deyiladi.

6). $\alpha=\frac{-1+\sqrt{2}}{2}$ sonini $\frac{P_4}{Q_4}$ munosib kasr bilan almashtirib uning natijasida hosil bo'ladigan xatolikni baholashimiz talab etilayapti. Buning uchun avvalo $\frac{-1+\sqrt{2}}{2}$ ni uzluksiz kasrlarga yoyib munosiob kasrlarini topamiz: ($\sqrt{2}=2{,}236067975$)

$\alpha=\frac{-1+\sqrt{2}}{2}=0+\frac{-1+\sqrt{2}}{2}=1+\frac{1}{\frac{2}{-1+\sqrt{2}}}=1+\frac{1}{\alpha_1}$, bunda $\alpha_1=\frac{2}{\sqrt{2}-1}=2(\sqrt{2}+1)=4+(2\sqrt{2}-2)=4+\frac{(2\sqrt{2}-2)(2\sqrt{2}+2)}{(2\sqrt{2}+2)}=4+\frac{2}{\sqrt{2}+1}=4+\frac{1}{\alpha_2}$;

$$\alpha_2 = \frac{\sqrt{2}+1}{2} = 1 + \frac{\sqrt{2}-1}{2} = 1 + \frac{1}{\alpha_3}; \alpha_3 = \frac{2}{\sqrt{2}-1} = \alpha_1.$$

Demak, $\alpha = \frac{-1+\sqrt{2}}{2} = (0,(4,1))$ ekan. Endi $\frac{P_4}{Q_4}$ munosib kasrni aniqlaymiz.$\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		0	4	1	4	1	4	...
P_i	$P_0 = 1$	0	1	1	$\boxed{5}$	6	29	...
Q_i	$Q_0 = 0$	$Q_1 = 1$	4	5	$\boxed{24}$	29	140	...

Demak, $\frac{P_4}{Q_4} = \frac{5}{24}$ va xatolik $\Delta\alpha = |\alpha - a| = \left|\frac{-1+\sqrt{2}}{2} - \frac{5}{24}\right| =$ $|0{,}2071067812 - 0{,}2083333333| < 0{,}001227 < 0{,}002$ va shuning uchun ham$\frac{-1+\sqrt{2}}{2} \approx \frac{5}{24}(-0{,}001227) = 0{,}20710633333$. Bu yerda xatolikni "$-$" ishora bilan olamiz, chunki $\frac{P_4}{Q_4} > \alpha$.

Javob: $\frac{5}{24}$, $\Delta\alpha = 0{,}002$.

353. Buning uchun berilgan $\frac{1261}{881}$ kasrni uzluksiz kasrga yoyamiz. Berilgan aniqlikni ta'minlash uchun k ni $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0{,}0001}} = 100$ bajariladigan tanlash kifoya. Avvalo $\frac{1261}{881}$ kasrni uzluksiz kasrga yoyamiz: $1261 = 881\cdot\boxed{1} + 380$; $881 = 380\cdot\boxed{2} + 121$; $380 = 121\cdot\boxed{3} + 17$; $121 = 17\cdot\boxed{7} + 2$; $17 = 2\cdot\boxed{8} + 1$, $2 = 1\cdot\boxed{2} + 0$. Demak, $\frac{1261}{881} = (1,2,3,7,8,2)$ ekan. Endi munosib kasrni aniqlaymiz:$\frac{P_k}{Q_k}$ larni topish uchun quyidagi jadvalni tuzib olamiz:

q_i		1	2	3	7	8	2
P_i	$P_0 = 1$	1	3	10	73	594	1261
Q_i	$Q_0 = 0$	$Q_1 = 1$	2	7	51	$\boxed{415}$	881

Jadvaldan $Q_k > 100$ shartni qanoatlantiruvchi eng kichik k bu $k = 5$ va $Q_5 = 415$. Shuning uchun ham $\frac{1261}{881} \approx \frac{594}{415}(-0.0001)$ deb yoza olamiz. Lekin $\left|\frac{1261}{881}-\frac{P_k}{Q_k}\right|<0{,}0001$shartni qanoatlantiruvchi eng kichik maxrajli munosib kasr bilan almashtirish talab etilgani uchun $\frac{P_4}{Q_4}$ tekshirib ko'ramiz. Bu holda

$\left|\frac{1261}{881}-\frac{P_4}{Q_4}\right|=\left|\frac{1261}{881}-\frac{73}{51}\right|=|1{,}43132803632-1{,}43137254901|=0{,}00004451269<0{,}00005<0{,}0001$ bajariladi va shu uchun $\frac{1261}{881}\approx\frac{73}{51}(-0.0001)$ deb yozish mumkin. Lekin $\left|\frac{1261}{881}-\frac{P_3}{Q_3}\right|=\left|\frac{1261}{881}-\frac{10}{7}\right|=|1{,}43132803632-1{,}42857142857|=0{,}00275660775>0{,}0001$. Berilgan shartlarni qanoatlantiruvchi $\frac{1261}{881}$ kasrga eng yaxshi yaqinlashish sifatida $\frac{P_4}{Q_4}=\frac{73}{51}$ munosib kasrni olsak bo'ladi. **Javob:** $\frac{73}{51}$.

354. 1).$\sqrt{2}=1+(\sqrt{2}-1)=1+\frac{1}{\sqrt{2}+1}=1+\frac{1}{2+(\sqrt{2}-1)}=1+\frac{1}{2+\frac{1}{(\sqrt{2}+1)}}=1+\frac{1}{2+\frac{1}{2+}}$=(1,(2)) bo'lgani uchun $Q_k>\sqrt{\frac{1}{\varepsilon}}=\sqrt{\frac{1}{0{,}001}}>31$, ya'ni $31<Q_k$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz.Buning uchun munosib kasrni aniqlaymiz:

q_i			1	2	2	2	2	2	...
P_i	$P_0=1$		1	3	7	17	41	99	...
Q_i	$Q_0=0$	$Q_1=1$		2	5	12	29	70	...

Jadvaldan $Q_k>31$ shartni qanoatlantiruvchi eng kichik k bu $k=6$ va $Q_6=70$. Shuning uchun ham $\frac{P_4}{Q_4}=\frac{99}{70}$, ya'ni $\sqrt{2}\approx\frac{99}{70}(-0{,}001)$ deb yoza olamiz. Lekin $\left|\sqrt{2}-\frac{P_k}{Q_k}\right|<0{,}001$shartni qanoatlantiruvchi eng kichik maxrajli munosib kasr bilan almashtirish talab etilgani uchun $\frac{P_5}{Q_5}$ tekshirib ko'ramiz. Bu holda $\left|\sqrt{2}-\frac{P_5}{Q_5}\right|=\left|\sqrt{2}-\frac{41}{29}\right|=|1{,}4142135624-1{,}41379310344|=0{,}000421<0{,}001$ bajariladi va shu uchun $\sqrt{2}\approx\frac{41}{29}(+0{,}001)$ deb yozish mumkin. Lekin $\left|\sqrt{2}-\frac{P_4}{Q_4}\right|=$

$\left|\sqrt{2}-\frac{17}{12}\right| = |1{,}4142135624 - 1{,}4166666667| = 0{,}00245310426 > 0{,}001$. Berilgan shartlarni qanoatlantiruvchi $\sqrt{2}$ ga eng yaxshi yaqinlashish sifatida $\frac{P_5}{Q_5} = \frac{41}{29}$ munosib kasrni olsak bo'ladi. **Javob:** $\frac{41}{29}$.

2).$\sqrt{3} = 1 + \left(\sqrt{3}-1\right) = 1 + \frac{2}{\sqrt{3}+1} = 1 + \frac{1}{\frac{\sqrt{3}+1}{2}} = 1 + \frac{1}{1+\left(\sqrt{3}-1\right)} =$

$1 + \frac{1}{1+\frac{2}{\sqrt{3}+1}} = 1 + \frac{1}{1+\frac{1}{\frac{\sqrt{3}+1}{2}}} = (1,(1,2))$ bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} =$

$\sqrt{\frac{1}{0{,}001}} > 31$, ya'ni $31 < Q_k$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz.Buning uchun munosib kasrni aniqlaymiz:

q_i		1	1	2	1	2	1	2	...
P_i	$P_0 = 1$	1	2	5	7	19	26	71	...
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	3	4	11	15	41	...

Jadvaldan $Q_k > 31$ shartni qanoatlantiruvchi eng kichik k bu $k = 7$ va $Q_7 = 41$. Shuning uchun ham $\frac{P_7}{Q_7} = \frac{71}{41}$, ya'ni $\sqrt{3} \approx \frac{71}{41}(+0{,}001)$ deb yoza olamiz. Lekin $\left|\sqrt{3}-\frac{P_k}{Q_k}\right| < 0{,}001$shartni qanoatlantiruvchi eng kichik maxrajli munosib kasr bilan almashtirish talab etilgani uchun $\frac{P_6}{Q_6}$ tekshirib ko'ramiz. Bu holda

$\left|\sqrt{3}-\frac{P_6}{Q_6}\right| = \left|\sqrt{3}-\frac{26}{15}\right| = |1{,}73050807 - 1{,}73333333| =$

$0{,}031 > 0{,}001$ bajariladi va shu uchun berilgan shartlarni qanoatlantiruvchi $\sqrt{3}$ga eng yaxshi yaqinlashish sifatida $\frac{P_7}{Q_7} = \frac{71}{41}$ munosib kasrni olsak bo'ladi. **Javob:** $\frac{71}{41}$.

3).$\sqrt{7} = 2 + \left(\sqrt{7}-2\right) = 2 + \frac{3}{\sqrt{7}+2} = 2 + \frac{1}{\frac{\sqrt{7}+2}{3}} = 2 + \frac{1}{\alpha_1}$,

bu yerda $\alpha_1 = \frac{\sqrt{7}+2}{3} = 1 + \left(\frac{\sqrt{7}+2}{3} - 1\right) = 1 + \frac{\sqrt{7}-1}{3} = 1 + \frac{1}{\alpha_2}$;

$$\alpha_2 = \frac{3}{\sqrt{7}-1} = \frac{3(\sqrt{7}+1)}{6} = \frac{\sqrt{7}+1}{2} = 1 + \frac{\sqrt{7}-1}{2} = 1 + \frac{1}{\frac{2}{\sqrt{7}-1}}$$
$$= 1 + \frac{1}{\alpha_3};$$
$$\alpha_3 = \frac{2}{\sqrt{7}-1} = \frac{2(\sqrt{7}+1)}{6} = \frac{\sqrt{7}+1}{3} = 1 + \frac{\sqrt{7}-2}{3} = 1 + \frac{1}{\frac{3}{\sqrt{7}-2}}$$
$$= 1 + \frac{1}{\alpha_4};$$
$\alpha_4 = \frac{3}{\sqrt{7}-2} = \frac{3}{\sqrt{7}-2} = \sqrt{7} + 2 = 4 + (\sqrt{7}-2) = 4 + \frac{3}{\sqrt{7}+2} = 4 +$ $\frac{1}{\frac{\sqrt{7}+2}{3}} = 4 + \frac{1}{\alpha_4}$; $\alpha_4 = \frac{\sqrt{7}+2}{3} = \alpha_1$. Demak, $\sqrt{7} = (2, (1,1,1,4))$

bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0,001}} > 31$, ya'ni $31 < Q_k$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz.Buning uchun munosib kasrni aniqlaymiz:

q_i		2	1	1	1	4	1	1	1	4	...
P_i	$P_0 = 1$	2	3	5	8	37	45	82	127	590	...
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	2	3	14	17	31	48	223	...

Jadvaldan $Q_k > 31$ shartni qanoatlantiruvchi eng kichik k bu $k = 8$ va $Q_8 = 48$. Shuning uchun ham $\frac{P_8}{Q_8} = \frac{127}{48}$, ya'ni $\sqrt{7} \approx \frac{127}{48}(+0,001)$ deb yoza olamiz. Lekin $\left|\sqrt{7} - \frac{P_k}{Q_k}\right| < 0,001$shartni qanoatlantiruvchi eng kichik maxrajli munosib

kasr bilan almashtirish talab etilgani uchun $\frac{P_7}{Q_7}$ tekshirib ko'ramiz. Bu holda

$\left|\sqrt{7} - \frac{P_7}{Q_7}\right| = \left|\sqrt{7} - \frac{82}{31}\right| = |2,645751311 - 2,64583333333| =$ $0,00008 < 0,001$ bajariladi va shu uchun berilgan shartlarni qanoatlantiruvchi $\sqrt{7}$ga eng yaxshi yaqinlashish sifatida $\frac{P_7}{Q_7} = \frac{82}{31}$ munosib kasrni olsak bo'ladi. **Javob:** $\frac{82}{31}$.

4).$\sqrt{11} = 3 + (\sqrt{11} - 3) = 3 + \frac{2}{\sqrt{11}+3} = 3 + \frac{1}{\frac{\sqrt{11}+3}{2}} = 3 + \frac{1}{\alpha_1},$

bu yerda $\alpha_1 = \frac{\sqrt{11}+3}{2} = 3 + \left(\frac{\sqrt{11}+3}{2} - 3\right) = 3 + \frac{\sqrt{11}-3}{2} = 3 + \frac{1}{\alpha_2};$

$$\alpha_2 = \frac{2}{\sqrt{11} - 3} = \sqrt{11} + 3 = 6 + (\sqrt{11} - 3) = 6 + \frac{2}{\sqrt{11} + 3}$$

$$= 6 + \frac{1}{\frac{\sqrt{11}+3}{2}} = 6 + \frac{1}{\alpha_3};$$

$\alpha_3 = \frac{\sqrt{11}+3}{2} = \alpha_1$. Demak, $\sqrt{11} = (3,(3,6))$ bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0,001}} > 31$, ya'ni $31 < Q_k$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz. Buning uchun munosib kasrni aniqlaymiz:

q_i		3	3	6	3	6	...
P_i	$P_0 = 1$	3	10	63	199	1257	...
Q_i	$Q_0 = 0$	$Q_1 = 1$	3	19	60	379	...

Jadvaldan $Q_k > 31$ shartni qanoatlantiruvchi eng kichik k bu $k = 4$ va $Q_4 = 60$. Shuning uchun ham $\frac{P_4}{Q_4} = \frac{199}{60} = 3{,}31(6)$, ya'ni $\sqrt{11} \approx \frac{199}{60}(-0{,}001)$ deb yoza olamiz. Lekin $\left|\sqrt{11} - \frac{P_k}{Q_k}\right| < 0{,}001$shartni qanoatlantiruvchi eng kichik maxrajli munosib kasr bilan almashtirish talab etilgani uchun $\frac{P_3}{Q_3}$ tekshirib ko'ramiz. Bu holda $\left|\sqrt{11} - \frac{P_3}{Q_3}\right| = \left|\sqrt{11} - \frac{63}{19}\right| = |3{,}3166247903 \;\; - 3{,}31578947368\,| < 0{,}00084 < 0{,}001$ bajariladi va shu uchun berilgan shartlarni qanoatlantiruvchi $\sqrt{11}$ ga eng yaxshi yaqinlashish sifatida $\frac{P_3}{Q_3} = \frac{63}{19}$ munosib kasrni olsak bo'ladi.

Javob: $\frac{63}{19}$.

355. 1).$x^2 - 5x + 2 = 0$ tenglamaning ildizlarini topamiz. $x_{1,2} = \frac{5 \pm \sqrt{25-4 \cdot 1 \cdot 2}}{2} = \frac{5 \pm \sqrt{17}}{2}$; $x_1 = \frac{5+\sqrt{17}}{2}$, $x_2 = \frac{5-\sqrt{17}}{2}$. Avvalo birinchi ildiz

$x_1 = \frac{5+\sqrt{17}}{2}$ ni qaraymiz. $x_1 = \frac{5+\sqrt{17}}{2} = 4 + \frac{\sqrt{17}-3}{2} = 4 + \frac{1}{\alpha_1}$, bu yerda

$\alpha_1 = \frac{2}{\sqrt{17}-3} = \frac{2(\sqrt{17}+3)}{8} = \frac{\sqrt{17}+3}{4} = 1 + \frac{\sqrt{17}-1}{4} = 1 + \frac{1}{\frac{4}{\sqrt{17}-1}} = 1 + \frac{1}{\alpha_2};$

$$\alpha_2 = \frac{4}{\sqrt{17}-1} = \frac{\sqrt{17}+1}{4} = 1 + \frac{\sqrt{17}-3}{4} = 1 + \frac{1}{\frac{4}{\sqrt{17}-3}} = 1 + \frac{1}{\alpha_3};$$

$\alpha_3 = \frac{4}{\sqrt{17}-3} = \frac{\sqrt{17}+3}{2} = 3 + \frac{\sqrt{17}-3}{2} = 3 + \frac{1}{\frac{2}{\sqrt{17}-3}} = 3 + \frac{1}{\alpha_1}$. Demak, $x_1 = \frac{5+\sqrt{17}}{2} = (4,(1,1,3))$ bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0,0001}} = 100$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz. Buning uchun munosib kasrni aniqlaymiz:

q_i		4	1	1	3	1	1	3	1	1	3	...
P_i	$P_0 = 1$	4	5	9	32	41	73	260	333	593	2112	...
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	2	7	9	16	57	73	130	463	...

Jadvaldan $Q_k > 100$ shartni qanoatlantiruvchi eng kichik k bu $k = 8$ va $Q_8 = 130$. Shuning uchun ham $\frac{P_8}{Q_8} = \frac{593}{130} = 4,56153846153$, ya'ni $\frac{5+\sqrt{17}}{2} \approx \frac{593}{130}(-0,0001)$ deb yoza olamiz. Bunda xatolik $< \frac{1}{Q_8 Q_9} = \frac{1}{130 \cdot 463} = \frac{1}{60190} < 0,000017 < 0,0001$ bo'ladi.

Endi ikkinchi $x_2 = \frac{5-\sqrt{17}}{2}$ ildizni qaraymiz. $x_2 = \frac{5-\sqrt{17}}{2} = 0 + \frac{1}{\frac{2}{5-\sqrt{17}}} = 0 + \frac{1}{\alpha_1}$, bu yerda $\alpha_1 = \frac{2}{5-\sqrt{17}} = \frac{2(\sqrt{17}+5)}{8} = \frac{\sqrt{17}+5}{4} = 2 + \frac{\sqrt{17}-3}{4} = 2 + \frac{1}{\frac{4}{\sqrt{17}-3}} = 2 + \frac{1}{\alpha_2};$

$$\alpha_2 = \frac{4}{\sqrt{17}-3} = \frac{\sqrt{17}+3}{2} = 3 + \frac{\sqrt{17}-3}{2} = 3 + \frac{1}{\frac{2}{\sqrt{17}-3}} = 3 + \frac{1}{\alpha_3};$$

$$\alpha_3 = \frac{2}{\sqrt{17}-3} = \frac{\sqrt{17}+3}{4} = 1 + \frac{\sqrt{17}-1}{4} = 1 + \frac{1}{\frac{4}{\sqrt{17}-1}} = 1 + \frac{1}{\alpha_4};$$

$$\alpha_4 = \frac{4}{\sqrt{17}-1} = \frac{\sqrt{17}+1}{4} = 1 + \frac{\sqrt{17}-3}{4} = 1 + \frac{1}{\frac{4}{\sqrt{17}-3}} = 1 + \frac{1}{\alpha_2}.$$

Demak, $x_2 = \frac{5-\sqrt{17}}{2} = (0{,}2,(3{,}1{,}1))$bo‘lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0{,}0001}} = 100$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz. Buning uchun munosib kasrni aniqlaymiz:

q_i		0	2	3	1	1	3	1	1	3	1	1	...
P_i	$P_0 = 1$	0	1	3	4	7	25	32	57	203	260	463	...
Q_i	$Q_0 = 0$	$Q_1 = 1$	2	7	9	16	57	73	130	463	593	1056	...

Jadvaldan $Q_k > 100$ shartni qanoatlantiruvchi eng kichik k bu $k = 8$ va $Q_8 = 130$. Shuning uchun ham $\frac{P_8}{Q_8} = \frac{57}{130} = 0{,}43846153846$, ya’ni $\frac{5-\sqrt{17}}{2} \approx \frac{57}{130}(+0{,}0001)$ deb yoza olamiz. Bunda xatolik $< \frac{1}{Q_8 Q_9} = \frac{1}{130 \cdot 463} = \frac{1}{60190} < 0{,}000017 < 0{,}0001$ bo‘ladi.

Javob: $x_1 = \frac{5+\sqrt{17}}{2} \approx \frac{593}{130}(-0{,}0001); x_2 = \frac{5-\sqrt{17}}{2} \approx \frac{57}{130}(+0{,}0001)$.

$4x^2 + 20x + 23 = 0$ tenglamaning ildizlarini topamiz.

$$x_{1,2} = \frac{-10 \pm \sqrt{100-92}}{4} = \frac{-10 \pm 2\sqrt{2}}{4} = \frac{-5 \pm \sqrt{2}}{2};\ x_1 = \frac{-5+\sqrt{2}}{2},\ x_2 = \frac{-5-\sqrt{2}}{2}.$$

Avvalo birinchi ildiz $x_1 = \frac{-5+\sqrt{2}}{2}$ ni qaraymiz.

$$x_1 = \frac{-5+\sqrt{2}}{2} = -2 + \frac{\sqrt{2}-5}{2} + 2 = -2 + \frac{\sqrt{2}-1}{2} = -2 + \frac{1}{\alpha_1},$$

bu yerda

$$\alpha_1 = \frac{2}{\sqrt{2}-1} = 2(\sqrt{2}+1) = 4 + 2\sqrt{2} - 2 = 4 + \frac{2}{\sqrt{2}+1} = 4 + \frac{1}{\frac{\sqrt{2}+1}{2}} =$$

$$4 + \frac{1}{\alpha_2};\ \alpha_2 = \frac{\sqrt{2}+1}{2} = 1 + \frac{\sqrt{2}-1}{2} = 1 + \frac{1}{\frac{2}{\sqrt{2}-1}} = 1 + \frac{1}{\alpha_1}.$$

Demak, $x_1 = \frac{-5+\sqrt{2}}{2} = (-2,(4,1))$bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0,0001}} = 100$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz. Buning uchun munosib kasrni aniqlaymiz:

q_i		-2	4	1	4	1	4	1	...
P_i	$P_0 = 1$	-2	-7	-9	-43	-52	-251	-303	...
Q_i	$Q_0 = 0$	$Q_1 = 1$	4	5	24	29	140	169	...

Jadvaldan $Q_k > 100$ shartni qanoatlantiruvchi eng kichik k bu $k = 6$ va $Q_6 = 140$. Shuning uchun ham $\frac{P_6}{Q_6} = -\frac{251}{140} = -1,79285714285$, bunda $\frac{-5+\sqrt{2}}{2} \approx -1,792893238$, ya'ni $\frac{-5+\sqrt{2}}{2} \approx -\frac{251}{140}(-0,0001)$ deb yoza olamiz. Bunda xatolik $<\frac{1}{Q_8Q_9} = \frac{1}{140\cdot169} = \frac{1}{23660}$ <0,000043<0,0001 bo'ladi.

Endi ikkinchi $x_2 = \frac{-5-\sqrt{2}}{2}$ ildizni qaraymiz. $x_2 = \frac{-5-\sqrt{2}}{2} = -4 + \frac{3-\sqrt{2}}{2} = -4 + \frac{1}{\frac{2}{3-\sqrt{2}}} = -4 + \frac{1}{\alpha_1}$, bu yerda

$$\alpha_1 = \frac{2}{3-\sqrt{2}} = \frac{2(\sqrt{2}+3)}{7} = 1 + \frac{2\sqrt{2}-1}{7} = 1 + \frac{1}{\frac{7}{2\sqrt{2}-1}} = 1 + \frac{1}{\alpha_2};$$

$$\alpha_2 = \frac{7}{2\sqrt{2}-1} = 2\sqrt{2}+1 = 3 + (2\sqrt{2}-2) = 3 + \frac{1}{\frac{\sqrt{2}+1}{2}} = 3 + \frac{1}{\alpha_3};$$

$$\alpha_3 = \frac{\sqrt{2}+1}{2} = ((1,4)).$$

Demak, $x_2 = \frac{-5-\sqrt{2}}{2} = (-4,1,3,(1,4))$ bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0,0001}} = 100$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz. Buning uchun munosib kasrni aniqlaymiz:

q_i		-4	1	3	1	4	1	4	1	...
P_i	$P_0 = 1$	-4	-3	-13	-16	-77	-93	-449	-547	...
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	4	5	24	29	140	169	...

Jadvaldan $Q_k > 100$ shartni qanoatlantiruvchi eng kichik k bu $k = 7$ va $Q_7 = 140$. Shuning uchun ham $\frac{P_7}{Q_7} = -\frac{449}{140} = -3{,}20714285714$, $\frac{-5-\sqrt{2}}{2} = -3{,}207106812$, ya'ni $-\frac{449}{140}(-0{,}0001)$ deb yoza olamiz. Bunda xatolik $< \frac{1}{Q_8 Q_9} = \frac{1}{140 \cdot 169} = \frac{1}{23660} < 0{,}0001$ bo'ladi.

Javob: $x_1 = \frac{-5+\sqrt{2}}{2} \approx -\frac{251}{140}(-0{,}0001)$; $x_2 = \frac{-5-\sqrt{2}}{2} \approx -\frac{449}{140}(-0{,}0001)$.

3). $x^2 + 9x + 6 = 0$ tenglamaning ildizlarini topamiz.

$$x_{1,2} = \frac{-9 \pm \sqrt{81-24}}{2} = \frac{-9 \pm \sqrt{57}}{2}; \; x_1 = \frac{-9+\sqrt{57}}{2},$$

$$x_2 = \frac{-5-\sqrt{57}}{2}.$$

Avvalo birinchi ildiz $x_1 = \frac{-9+\sqrt{57}}{2}$ ni qaraymiz.

$$x_1 = \frac{-9+\sqrt{57}}{2} = -1 + \frac{\sqrt{57}-9}{2} + 1 = -1 + \frac{\sqrt{57}-7}{2} = -1 + \frac{1}{\alpha_1},$$

bu yerda

$$\alpha_1 = \frac{2}{\sqrt{57}-7} = \frac{\sqrt{57}+7}{4} = 3 + \frac{\sqrt{57}-5}{4} = 3 + \frac{1}{\frac{4}{\sqrt{57}-5}} = 3 + \frac{1}{\alpha_2};$$

$$\alpha_2 = \frac{4}{\sqrt{57}-5} = \frac{\sqrt{57}+5}{8} = 1 + \frac{\sqrt{57}-3}{8} = 1 + \frac{1}{\frac{8}{\sqrt{57}-3}} = 1 + \frac{1}{\alpha_3};$$

$$\alpha_3 = \frac{8}{\sqrt{57}-3} = \frac{\sqrt{57}+3}{6} = 1 + \left(\frac{\sqrt{57}+3}{6} - 1\right) = 1 + \frac{\sqrt{57}-3}{6}$$

$$= 1 + \frac{1}{\alpha_4};$$

$$\alpha_4=\frac{6}{\sqrt{57}-3}=\frac{\sqrt{57}+3}{8}=1+\left(\frac{\sqrt{57}+3}{8}-1\right)=1+\frac{\sqrt{57}-5}{8}$$
$$=1+\frac{1}{\alpha_5};$$
$$\alpha_5=\frac{8}{\sqrt{57}-5}=\frac{\sqrt{57}+5}{4}=3+\frac{\sqrt{57}-7}{4}=3+\frac{1}{\frac{4}{\sqrt{57}-7}}=3+\frac{1}{\alpha_6};$$
$$\alpha_6=\frac{4}{\sqrt{57}-7}=\frac{\sqrt{57}+7}{2}=7+\frac{\sqrt{57}-7}{2}=7+\frac{1}{\frac{2}{\sqrt{57}-7}}=7+\frac{1}{\alpha_7};$$
$$\alpha_7=\frac{2}{\sqrt{57}-7}=\frac{\sqrt{57}+7}{4}=3+\frac{1}{\frac{4}{\sqrt{57}-5}}=3+\frac{1}{\alpha_8}; \quad \alpha_8=\alpha_2$$

Demak, $x_1=\frac{-9+\sqrt{57}}{2}=\left(-1,3,(1,1,1,3,7,3)\right)$ bo'lgani uchun $Q_k>\sqrt{\frac{1}{\varepsilon}}=\sqrt{\frac{1}{0,0001}}=100$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz.Buning uchun munosib kasrni aniqlaymiz:

q_i		-1	3	1	1	1	3	7	3	...
P_i	$P_0=1$	-1	-2	-3	-5	-8	-29	-211	-662	...
Q_i	$Q_0=0$	$Q_1=1$	3	4	7	11	40	291	913	...

Jadvaldan $Q_k>100$ shartni qanoatlantiruvchi eng kichik k bu $k=7$ va $Q_7=291$. Shuning uchun ham $\frac{P_7}{Q_7}=-\frac{211}{291}=-0,72508591065$, bunda $\frac{-9+\sqrt{57}}{2}\approx-0,725082783$, ya'ni $x_1=\frac{-9+\sqrt{57}}{2}\approx-\frac{211}{291}(+0,0001)$ deb yoza olamiz. Bunda xatolik $<\frac{1}{Q_7Q_8}=\frac{1}{40\cdot291}=\frac{1}{11640}$ <0,000086<0,0001 bo'ladi.

Endi ikkinchi $x_2=\frac{-9-\sqrt{57}}{2}$ ildizni qaraymiz. $x_2=\frac{-9-\sqrt{57}}{2}=-9+\frac{9-\sqrt{57}}{2}=-9+\frac{1}{\frac{2}{9-\sqrt{57}}}=-9+\frac{1}{\alpha_1}$, bu yerda

$$\alpha_1=\frac{2}{9-\sqrt{57}}=\frac{\sqrt{57}+9}{12}=1+\frac{\sqrt{57}-3}{12}=1+\frac{1}{\frac{12}{\sqrt{57}-3}}=1+\frac{1}{\alpha_2};$$

$\alpha_2 = \frac{12}{\sqrt{57}-3} = \frac{\sqrt{57}+3}{4} = 2 + \frac{\sqrt{57}-5}{4} = 2 + \frac{1}{\frac{4}{\sqrt{57}-5}} = 2 + \frac{1}{\alpha_3}$;bunda $x_1$ni hisoblaganmizdagi singari $\alpha_3 = \frac{4}{\sqrt{57}-5} = ((1,1,1,3,7,3))$.

Demak, $x_2 = \frac{-9-\sqrt{57}}{2} = (-9,1,2,(1,1,1,3,7,3))$bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0,0001}} = 100$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz.Buning uchun munosib kasrni aniqlaymiz:

q_i		-9	1	2	1	1	1	3	7	3	...
P_i	$P_0 = 1$	-9	-8	-25	−33	-58	-91	-331	-2408	-7555	...
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	3	4	7	11	40	291	913	...

Jadvaldan $Q_k > 100$ shartni qanoatlantiruvchi eng kichik k bu $k = 8$ va $Q_8 = 291$. Shuning uchun ham $\frac{P_8}{Q_8} = -\frac{2408}{291} = -8{,}27491408934$, $\frac{-9-\sqrt{57}}{2} = -8{,}2749172175$, ya'ni $x_2 = \frac{-9-\sqrt{57}}{2} \approx -\frac{2408}{291}(-0{,}0001)$ deb yoza olamiz. Bunda xatolik

$$< \frac{1}{Q_8 Q_9} = \frac{1}{291 \cdot 913} = \frac{1}{265683} < 0{,}000004 < 0{,}0001 \text{ bo'ladi.}$$

Javob: $x_1 = \frac{-9+\sqrt{57}}{2} \approx -\frac{211}{291}(+0{,}0001)$; $x_2 = \frac{-9-\sqrt{57}}{2} \approx -\frac{2408}{291}(-0{,}0001)$.

4). $2x^2 - 3x - 6 = 0$ tenglamaning ildizlarini topamiz. $x_{1,2} = \frac{3 \pm \sqrt{9+48}}{4} = \frac{3 \pm \sqrt{57}}{4}$; $x_1 = \frac{3+\sqrt{57}}{4}$, $x_2 = \frac{3-\sqrt{57}}{4}$. Avvalo birinchi ildiz $x_1 = \frac{3+\sqrt{57}}{4}$ ni qaraymiz.

$x_1 = \frac{3+\sqrt{57}}{4} = 2 + \frac{\sqrt{57}-5}{4} = 2 + \frac{1}{\frac{4}{\sqrt{57}-5}} = 2 + \frac{1}{\alpha_1}$, bu yerda

$$\alpha_1 = \frac{4}{\sqrt{57}-5} = \frac{\sqrt{57}+5}{8} = 1 + \frac{\sqrt{57}-3}{8} = 1 + \frac{1}{\frac{8}{\sqrt{57}-3}} = 1 + \frac{1}{\alpha_2};$$

$$\alpha_2 = \frac{8}{\sqrt{57}-3} = \frac{\sqrt{57}+3}{6} = 1 + \frac{\sqrt{57}-3}{6} = 1 + \frac{1}{\frac{6}{\sqrt{57}-3}} = 1 + \frac{1}{\alpha_3};$$

$$\alpha_3=\frac{6}{\sqrt{57}-3}=\frac{\sqrt{57}+3}{8}=1+\frac{\sqrt{57}-5}{8}=1+\frac{1}{\alpha_4};$$

$$\alpha_4=\frac{8}{\sqrt{57}-5}=\frac{\sqrt{57}+5}{4}=3+\frac{\sqrt{57}-7}{4}=3+\frac{1}{\alpha_5};$$

$$\alpha_5=\frac{4}{\sqrt{57}-7}=\frac{\sqrt{57}+7}{2}=7+\frac{\sqrt{57}-7}{2}=7+\frac{1}{\frac{2}{\sqrt{57}-7}}=7+\frac{1}{\alpha_6};$$

$$\alpha_6=\frac{2}{\sqrt{57}-7}=\frac{\sqrt{57}+7}{4}=3+\frac{\sqrt{57}-5}{4}=3+\frac{1}{\frac{4}{\sqrt{57}-5}}=3+\frac{1}{\alpha_7};$$

$$\alpha_7=\frac{4}{\sqrt{57}-5}=\alpha_1$$

Demak, $x_1=\frac{3+\sqrt{57}}{4}=\left(2,(1,1,1,3,7,3)\right)$ bo'lgani uchun $Q_k>\sqrt{\frac{1}{\varepsilon}}=\sqrt{\frac{1}{0,0001}}=100$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz. Buning uchun munosib kasrni aniqlaymiz:

q_i		2	1	1	1	3	7	3	1	...
P_i	$P_0=1$	2	3	5	8	29	211	662	873	...
Q_i	$Q_0=0$	$Q_1=1$	1	2	3	11	80	251	331	...

Jadvaldan $Q_k>100$ shartni qanoatlantiruvchi eng kichik k bu $k=7$ va $Q_7=251$. Shuning uchun ham $\frac{P_7}{Q_7}=\frac{662}{251}=2,6374501992,$

bunda $x_1=\frac{3+\sqrt{57}}{4}\approx 2,63745860875$, ya'ni $x_1=\frac{3+\sqrt{57}}{4}\approx\frac{662}{251}(+0,0001)$ deb yoza olamiz. Bunda xatolik $<\frac{1}{Q_7Q_8}=\frac{1}{251\cdot 331}=\frac{1}{83081}<0,000013<0,0001$ bo'ladi.

Endi ikkinchi $x_2=\frac{3-\sqrt{57}}{4}$ ildizni qaraymiz. $x_2=\frac{3-\sqrt{57}}{4}=-2+\frac{11-\sqrt{57}}{4}=-2+\frac{1}{\frac{4}{11-\sqrt{57}}}=-2+\frac{1}{\alpha_1}$, bu yerda

$$\alpha_1=\frac{4}{11-\sqrt{57}}=\frac{\sqrt{57}+11}{16}=1+\frac{\sqrt{57}-5}{16}=1+\frac{1}{\frac{16}{\sqrt{57}-5}}=1+\frac{1}{\alpha_2};$$

$$\alpha_2 = \frac{16}{\sqrt{57}-5} = \frac{\sqrt{57}+5}{2} = 6 + \frac{\sqrt{57}-7}{2} = 6 + \frac{1}{\frac{2}{\sqrt{57}-7}} = 6 + \frac{1}{\alpha_3};$$

$$\alpha_3 = \frac{2}{\sqrt{57}-7} = \frac{\sqrt{57}+7}{4} = 3 + \frac{\sqrt{57}-5}{4} = 3 + \frac{1}{\frac{4}{\sqrt{57}-5}} = 3 + \frac{1}{\alpha_4};$$

$$\alpha_4 = \frac{4}{\sqrt{57}-5}$$

bunda x_1 ni hisoblaganmizdagi singari $\alpha_4 = ((1,1,1,3,7,3))$.

Demak, $x_2 = \frac{3-\sqrt{57}}{4} = (-2,1,6,3,(1,1,1,3,7,3))$ bo'lgani uchun $Q_k > \sqrt{\frac{1}{\varepsilon}} = \sqrt{\frac{1}{0,0001}} = 100$ shartni qanoatlantiruvchi Q_k ning eng kichik qiymatini aniqlaymiz.Buning uchun munosib kasrni aniqlaymiz:

q_i		-2	1	6	3	1	1	1	3	7	3	...
P_i	$P_0 = 1$	-2	-1	-8	-25	-33	-58	-91	-331	-2408	-7555	...
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	7	22	29	51	80	291	2117	6642	...

Jadvaldan $Q_k > 100$ shartni qanoatlantiruvchi eng kichik k bu $k = 8$ va $Q_8 = 291$. Shuning uchun ham $\frac{P_8}{Q_8} = -\frac{331}{291} = -1,13745704467$, $x_2 = \frac{3-\sqrt{57}}{4} = -1,13745860875$, ya'ni $x_2 = \frac{3-\sqrt{57}}{4} \approx -\frac{331}{291}(-0,0001)$ deb yoza olamiz. Bunda xatolik $< \frac{1}{Q_8 Q_9} = \frac{1}{291 \cdot 2117} = \frac{1}{616047} < 0,000002 < 0,00001$ bo'ladi.

Javob: $x_1 = \frac{3+\sqrt{57}}{4} \approx \frac{662}{251}(+0,0001)$; $x_2 = \frac{3-\sqrt{57}}{4} \approx -\frac{331}{291}(-0,0001)$.

356. $A = \alpha - \frac{P_n + P_{n+1}}{Q_n + Q_{n+1}}$ ayirmani qaraymiz. Bu yerda $\alpha = \frac{P_{n+1}q_{n+2} + P_n}{Q_{n+1}q_{n+2} + Q_n}$ bo'lgani uchun

$$A = \frac{P_{n+1}q_{n+2} + P_n}{Q_{n+1}q_{n+2} + Q_n} - \frac{P_n + P_{n+1}}{Q_n + Q_{n+1}}$$
$$= \frac{P_{n+1}Q_nq_{n+2} + P_nQ_n + P_{n+1}Q_{n+1}q_{n+2} + P_nQ_{n+1} - P_nQ_{n+1}q_{n+2} - P_nQ_n}{(Q_{n+1}q_{n+2} + Q_n)(Q_n + Q_{n+1})}$$
$$- \frac{P_{n+1}Q_{n+1}q_{n+2} + P_{n+1}Q_n}{(Q_{n+1}q_{n+2} + Q_n)(Q_n + Q_{n+1})} =$$
$$\frac{(P_{n+1}Q_nq_{n+2} - P_nQ_{n+1}q_{n+2}) + P_nQ_{n+1} - P_{n+1}Q_n}{(Q_{n+1}q_{n+2} + Q_n)(Q_n + Q_{n+1})} =$$
$$\frac{(P_{n+1}Q_n - P_nQ_{n+1})q_{n+2} - (P_{n+1}Q_n - P_nQ_{n+1})}{(Q_{n+1}q_{n+2} + Q_n)(Q_n + Q_{n+1})}$$
$$= \frac{(P_{n+1}Q_n - P_nQ_{n+1})(q_{n+2} - 1)}{(Q_{n+1}q_{n+2} + Q_n)(Q_n + Q_{n+1})}$$
$$= \frac{(-1)^n(q_{n+2} - 1)}{(Q_{n+1}q_{n+2} + Q_n)(Q_n + Q_{n+1})}$$

bo'lgani uchun ayirmaning ishorasi n ning juft toqligiga bog'liq bo'lib, agar $n = 2k$ – juft son bo'lsa, $\alpha > \frac{P_n+P_{n+1}}{Q_n+Q_{n+1}}$; agar $n = 2k + 1$ – toq son bo'lsa, $\alpha < \frac{P_n+P_{n+1}}{Q_n+Q_{n+1}}$ bajariladi. Tushunarliki, $\frac{P_n+P_{n+1}}{Q_n+Q_{n+1}}$ kasr $\frac{P_n}{Q_n}$ va α sonlari orasida yotadi. Shuning uchun ham

$$\left|\alpha - \frac{P_n}{Q_n}\right| > \left|\frac{P_n + P_{n+1}}{Q_n + Q_{n+1}} - \frac{P_n}{Q_n}\right| = \frac{1}{Q_n(Q_n + Q_{n+1})}$$

bajariladi.

Eslatma. Isbotlangan tengsizlik $\left|\alpha - \frac{P_n}{Q_n}\right|$ uchub quyi chegarani beradi va shuning uchun ham u bizga ma'lum bo'lgan $\left|\alpha - \frac{P_n}{Q_n}\right| < \frac{1}{Q_nQ_{n+1}}$ tengsizlikni to'ldiradi.

357. Bu yerda $\frac{P_n}{Q_n} = \frac{P_{n-1}q_n+P_{n-2}}{Q_{n-1}q_n+Q_{n-2}}$ bo'lgani uchun

$$\frac{P_{n-1}(q_n + m) + P_{n-2}}{Q_{n-1}(q_n + m) + Q_{n-2}} - \frac{P_{n-1}q_n + P_{n-2}}{Q_{n-1}q_n + Q_{n-2}}$$
$$= \frac{(P_{n-1}Q_{n-2} - P_{n-2}Q_{n-1})m}{(Q_{n-1}(q_n + m) + Q_{n-2})(Q_{n-1}q_n + Q_{n-2})} =$$

$\frac{(-1)^{n-2}\cdot m}{(Q_{n-1}(q_n+m)+Q_{n-2})\,(Q_{n-1}q_n+Q_{n-2})}$ bo'lgani uchun juft tartibli munosib kasrlar ortadi, toq tartiblilari esa kamayadi.

358. Bu yerda

$$\left|\alpha-\frac{P_n}{Q_n}\right|+\left|\alpha-\frac{P_{n-1}}{Q_{n-1}}\right|=\left|\frac{P_n}{Q_n}-\frac{P_{n-1}}{Q_{n-1}}\right|=\frac{1}{Q_{n-1}Q_n}<\frac{1}{2Q_n^2}+\frac{1}{2Q_{n-1}^2}$$

munosabat o'rinli bo'lgani uchun $\left|\alpha-\frac{P_{n-1}}{Q_{n-1}}\right|$ ifoda aynan $\frac{1}{2Q_{n-1}^2}$ dan kichik bo'lishi mumkin. Chunki 372- masalaga ko'ra $\left|\alpha-\frac{P_{n-1}}{Q_{n-1}}\right|>\frac{1}{Q_{n-1}(Q_{n-1}+Q_n)}$ bo'lgani uchun albatta $\left|\alpha-\frac{P_{n-1}}{Q_{n-1}}\right|>\frac{1}{\frac{1}{2Q_n^2}}$ bajariladi.

VI.2-§.

359. 1).$(\overline{2,3})$ uzluksiz kasr yordamida berilgan kvadrat irratsionallikni topish uchun berilgan ifodani $x=(2,3,x)$ ko'rinishda yozib olib uning munosib kasrlarini topamiz:

q_i		2	3	x
P_i	$P_0=1$	2	7	$7x+2$
Q_i	$Q_0=0$	$Q_1=1$	3	$3x+1$

Bundan $\frac{7x+2}{3x+1}=x\rightarrow 3x^2-6x-2=0$ kvadrat tenglamaga kelamiz. Uning ildizlarini aniqlaymiz. U holda

$$x_{1,2}=\frac{3\pm\sqrt{9+6}}{3}=\frac{3\pm\sqrt{15}}{3}=1\pm\frac{\sqrt{15}}{3}=1\pm\sqrt{\frac{5}{3}}=1\pm\sqrt{1,(6)}$$

hosil bo'ladi. Berilgan ifoda musbat bo'lgani uchun izlanayotgan kvadrat irratsionallik $1+\sqrt{1,(6)}$ dan iborat bo'ladi. **Javob:** $1+\sqrt{\frac{5}{3}}$.

2).$(\overline{1,1,2,2})$ uzluksiz kasr yordamida berilgan kvadrat irratsionallikni topish uchun berilgan ifodani $x=(1,1,2,2,x)$ ko'rinishda yozib olib uning munosib kasrlarini topamiz:

q_i		1	1	2	2	x
P_i	$P_0 = 1$	1	2	5	12	$12x + 5$
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	3	7	$7x + 3$

Bundan $\frac{12x+5}{7x+3} = x \to 7x^2 - 9x - 5 = 0$ kvadrat tenglamaga kelamiz. Uning ildizlarini aniqlaymiz. U holda $x_{1,2} = \frac{9 \pm \sqrt{81+28\cdot5}}{14} = \frac{9 \pm \sqrt{201}}{14}$ hosil bo'ladi. Berilgan ifoda musbat bo'lgani uchun izlanayotgan kvadrat irratsionallik $\frac{9+\sqrt{201}}{14}$ dan iborat bo'ladi. **Javob:** $\frac{9+\sqrt{201}}{14}$.

3). $(\overline{5,4,3})$ uzluksiz kasr yordamida berilgan kvadrat irratsionallikni topish uchun berilgan ifodani $x = (5,4,3, x)$ ko'rinishda yozib olib uning munosib kasrlarini topamiz:

q_i		5	4	3	x
P_i	$P_0 = 1$	5	21	68	$68x + 21$
Q_i	$Q_0 = 0$	$Q_1 = 1$	4	13	$13x + 4$

Bundan $\frac{68x+21}{13x+4} = x \to 13x^2 - 64x - 21 = 0$ kvadrat tenglamaga kelamiz. Uning ildizlarini aniqlaymiz. U holda

$$x_{1,2} = \frac{32 \pm \sqrt{1024 + 13 \cdot 21}}{13} = \frac{32 \pm \sqrt{1297}}{13}$$

hosil bo'ladi. Berilgan ifoda musbat bo'lgani uchun izlanayotgan kvadrat irratsionallik $\frac{32+\sqrt{1297}}{13}$ dan iborat bo'ladi. **Javob:** $\frac{32+\sqrt{1297}}{13}$.

4). $\alpha = (1,2,3,\bar{4})$ uzluksiz kasr yordamida berilgan kvadrat irratsionallikni topish uchun berilgan ifodni $\alpha = (1,2,3,\omega)$ ko'rinishda yozib olamiz. Bunda $\omega = (\bar{4}) = 4 + \frac{1}{\omega}$. Avvalo ω ni aniqlaymiz. $\omega = 4 + \frac{1}{\omega}$ dan $\omega^2 - 4\omega - 1 = 0$. Bu tenglamaning yechimi $\omega_{1,2} = 2 \pm \sqrt{5}$ dan iborat bo'lib, $\omega > 0$ bo'lgani uchun $\omega = 2 + \sqrt{5}$. Endi $\alpha = (1,2,3,\omega)$ dan foydalanib α ni topamiz. Buning uchun α ning munosib kasrlarini aniqlaymiz.

Bundan

$$\frac{10\omega+3}{7\omega+2}=\alpha \to \alpha=\frac{23+10\sqrt{5}}{16+7\sqrt{5}}=\frac{(23+10\sqrt{5})(16-7\sqrt{5})}{(16+7\sqrt{5})(16-7\sqrt{5})}$$
$$=\frac{18-\sqrt{5}}{11}$$

q_i		1	2	3	ω
P_i	$P_0=1$	1	3	10	$10\omega+3$
Q_i	$Q_0=0$	$Q_1=1$	2	7	$7\omega+2$

hosil bo'ladi. Shunday qilib izlanayotgan kvadrat irratsionallik $\frac{18-\sqrt{5}}{11}$ dan iborat bo'ladi. **Javob:** $\frac{18-\sqrt{5}}{11}$.

5).$\alpha=(0,1,1,1,1,\overline{2,2,2})$ uzluksiz kasr yordamida berilgan kvadrat irratsionallikni topish uchun berilgan ifodni $\alpha=(0,1,1,1,1,\omega)$ ko'rinishda yozib olamiz. Bunda $\omega=(\overline{2,2,2})$. Avvalo ω ni aniqlaymiz. $\omega=(2,2,2,\omega)$

q_i		2	2	2	ω
P_i	$P_0=1$	2	5	12	$12\omega+5$
Q_i	$Q_0=0$	$Q_1=1$	2	5	$5\omega+2$

dan $\frac{12\omega+5}{5\omega+2}=\omega \to 5\omega^2-10\omega-5=0 \to \omega^2-2\omega-1=0$ kvadrat tenglamaga kelamiz. Uning ildizlarini aniqlaymiz. U holda $\omega_{1,2}=\frac{1\pm\sqrt{1+1}}{1}=1\pm\sqrt{2}$ hosil bo'ladi. Berilgan ifodada ω musbat bo'lgani uchun $\omega=1+\sqrt{2}$. Endi $\alpha=(0,1,1,1,1,\omega)$ dan foydalanib α ni topamiz. Buning uchun α ning munosib kasrlarini aniqlaymiz.

q_i		0	1	1	1	1	ω
P_i	$P_0=1$	0	1	1	2	3	$3\omega+2$
Q_i	$Q_0=0$	$Q_1=1$	1	2	3	5	$5\omega+3$

Bundan

$$\frac{3\omega+2}{5\omega+3}=\alpha \to \alpha=\frac{5+3\sqrt{2}}{8+5\sqrt{2}}=\frac{(5+3\sqrt{2})(8-5\sqrt{2})}{(8+5\sqrt{2})(8-5\sqrt{2})}=\frac{10-\sqrt{2}}{14}$$

hosil bo'ladi. Shunday qilib izlanayotgan kvadrat irratsionallik $\frac{10-\sqrt{2}}{14}$ dan iborat bo'ladi. **Javob:** $\frac{10-\sqrt{2}}{14}$.

6). $\alpha=(a,\overline{a,2a,})=(a,\omega)=a+\frac{1}{\omega}$ uzluksiz kasr yordamida berilgan kvadrat irratsionallikni topish uchun berilgan ifodani $\alpha=(a,\overline{a,2a,})=(a,\omega)=a+\frac{1}{\omega}$ ko'rinishda yozib olamiz. Bunda $\omega=(\overline{a,2a})=(a,2a,\omega)$. Avvalo ω ni aniqlaymiz.

q_i		a	$2a$	ω
P_i	$P_0=1$	a	$2a^2+1$	$(2a^2+1)\omega+a$
Q_i	$Q_0=0$	$Q_1=1$	$2a$	$2a\omega+1$

dan $\frac{(2a^2+1)\omega+a}{2a\omega+1}=\omega \to 2\omega^2-2a\omega-1=0, (a\neq 0)$ kvadrat tenglamaga kelamiz. Uning ildizlarini aniqlaymiz. U holda $\omega_{1,2}=\frac{a\pm\sqrt{a^2+2}}{2}$ hosil bo'ladi. Berilgan ifodada ω musbat bo'lgani uchun $\omega=\frac{a+\sqrt{a^2+2}}{2}$. Endi $\alpha=(a,\omega)$ dan foydalanib α ni topamiz. Buning uchun α ning munosib kasrlarini aniqlaymiz. $\alpha=a+\frac{1}{\omega}=a+\frac{2}{a+\sqrt{a^2+2}}=\frac{(a+\sqrt{a^2+2})\cdot\sqrt{a^2+2}}{a+\sqrt{a^2+2}}=\sqrt{a^2+2}$ hosil bo'ladi. Shunday qilib izlanayotgan kvadrat irratsionallik $\sqrt{a^2+2}$ dan iborat bo'ladi. **Javob:** $\sqrt{a^2+2}$.

7). $\alpha=(\overline{2,2,1,1})=(2,2,1,1,\omega)$uzluksiz kasr yordamida berilgan kvadrat irratsionallikni topish uchun berilgan ifodani $\alpha=(2,2,1,1,\omega)$ ko'rinishda yozib olamiz. Bunda $\omega=(2,2,1,1,\omega)$. ω ni aniqlaymiz. Buning uchun esa munosib kasrlardan foydalanamiz.

q_i		2	2	1	1	ω
P_i	$P_0=1$	2	5	7	12	$12\omega+7$
Q_i	$Q_0=0$	$Q_1=1$	2	3	5	$5\omega+3$

dan $\frac{12\omega+7}{5\omega+3} = \omega \to 5\omega^2 - 9\omega - 7 = 0$ kvadrat tenglamaga kelamiz. Uning ildizlarini aniqlaymiz. U holda $\omega_{1,2} = \frac{-9\pm\sqrt{221}}{10}$ hosil bo'ladi. Berilgan ifodada ω musbat bo'lgani uchun $\omega = \frac{-9+\sqrt{221}}{10}$. Shunday qilib, izlanayotgan kvadrat irratsionallik $\frac{-9+\sqrt{221}}{10}$ dan iborat bo'ladi. **Javob:** $\frac{-9+\sqrt{221}}{10}$.

360. Bir xil chala bo'linmali cheksiz davriy uzluksiz kasrni $\alpha = (a, a, a, \ldots) = (a, \alpha) = a + \frac{1}{\alpha}$ ko'rinishida yozib olish mumkin. Bundan $\alpha^2 - a\alpha - 1 = 0$ kvadrat tenglamaga kelamiz. Uning ildizlarini aniqlaymiz.

U holda $\alpha = \frac{a+\sqrt{a^2+4}}{2}$ hosil bo'ladi. Shunday qilib, izlanayotgan kvadrat irratsionallik $\frac{a+\sqrt{a^2+4}}{2}$ dan iborat bo'ladi. Misol uchun: $a = 2$ bo'lsa, $\alpha = (2,2,\ldots) = (\bar{2}) = \frac{2+2\sqrt{2}}{2} = 1 + \sqrt{2}$; $a = 3$ bo'lsa, $\alpha = (3,3,\ldots) = (\bar{3}) = \frac{3+\sqrt{13}}{2}$ va hokazo. **Javob:** $\frac{a+\sqrt{a^2+4}}{2}$.

361.1). $\frac{P_k}{Q_k} = \frac{10}{3}$, $\alpha_{k+1} = \sqrt{2}$ bo'lsa, α ni topish kerak. $\frac{P_k}{Q_k} = \frac{10}{3}$ da $(P_k, Q_k) = 1$ bo'lgani uchun $P_k = 10, Q_k = 3$ ni hosil qilamiz. Ikkinchi tomondan $\frac{P_k}{Q_k} = \frac{10}{3} = 3 + \frac{1}{3}$ bo'lgani uchun $P_{k-1} = 3$, $Q_{k-1} = 1$ kelib chiqadi. Bu qiymatlarni $\alpha = \frac{P_k\alpha_{k+1}+P_{k-1}}{Q_k\alpha_{k+1}+Q_{k-1}}$ da foydalansak $\alpha = \frac{10\sqrt{2}+3}{3\sqrt{2}+1} = \frac{57-\sqrt{2}}{17}$ ekanligi kelib hiqadi.

Javob: $\alpha = \frac{57-\sqrt{2}}{17}$.

2). $\frac{P_k}{Q_k} = \frac{37}{13}$, $\alpha_{k+1} = \frac{1+\sqrt{3}}{2}$ bo'lsa, α ni topish kerak. $\frac{P_k}{Q_k} = \frac{37}{13}$ da $(P_k, Q_k) = 1$ bo'lgani uchun $P_k = 37, Q_k = 13$ ni hosil qilamiz. Ikkinchi tomondan

$$\frac{P_k}{Q_k} = \frac{37}{13} = 2 + \frac{11}{13} = 2 + \frac{1}{\frac{13}{11}} = 2 + \frac{1}{1+\frac{2}{11}} = 2 + \frac{1}{1+\frac{1}{\frac{11}{2}}} = 2 + \frac{1}{1+\frac{1}{5+\frac{1}{2}}} =$$

(2,1,5,2) bo'lgani uchun

q_i		2	1	5	2
P_i	$P_0 = 1$	2	3	17	37
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	6	13

dan $P_{k-1} = 17$, $Q_{k-1} = 6$ kelib chiqadi. Bu qiymatlarni $\alpha = \frac{P_k\alpha_{k+1}+P_{k-1}}{Q_k\alpha_{k+1}+Q_{k-1}}$ da foydalansak $\alpha = \frac{37\left(\frac{1+\sqrt{3}}{2}\right)+17}{13\left(\frac{1+\sqrt{3}}{2}\right)+6} = \frac{71+37\sqrt{3}}{25+13\sqrt{3}} = \frac{(71+37\sqrt{3})(25-13\sqrt{3})}{(25+13\sqrt{3})(25-13\sqrt{3})} = \frac{166+\sqrt{3}}{59}$

ekanligi kelib hiqadi. **Javob:** $\alpha = \frac{166+\sqrt{3}}{59}$.

362.1).$\alpha = \sqrt{x^2+1} = x + \left(\sqrt{x^2+1} - x\right) = x + \frac{1}{\sqrt{x^2+1}+x} = x + \frac{1}{\alpha_1}$, bunda

$\alpha_1 = \sqrt{x^2+1} + x = 2x + \left(\sqrt{x^2+1} - x\right) = 2x + \frac{1}{\sqrt{x^2+1}+x} = 2x + \frac{1}{\alpha_1}$. Demak, $\alpha = (x, \overline{2x})$. Misol uchun $x = 1$ da $\sqrt{2} = (1, \overline{2})$; $x = 2$ da $\sqrt{5} = (2, \overline{4})$; $x = 3$ da $\sqrt{10} = (3, \overline{6})$ va hokazo. Endi $\frac{P_3}{Q_3}$ aniqlaymiz.

q_i		x	$2x$	$2x$	...
P_i	$P_0 = 1$	x	$2x^2 + 1$	$4x^3 + 3x$	...
Q_i	$Q_0 = 0$	$Q_1 = 1$	$2x$	$4x^2 + 1$	...

Bundan $\frac{P_3}{Q_3} = \frac{4x^3+3x}{4x^2+1}$. **Javob:** $\alpha = (x, \overline{2x})$ va $\frac{P_3}{Q_3} = \frac{4x^3+3x}{4x^2+1}$.

2).$\alpha = \sqrt{a^4+2a} = a^2 + \left(\sqrt{a^4+2a} - a^2\right) = a^2 + \frac{2a}{\sqrt{a^4+2a}+a^2} = a\text{^}2 + \frac{1}{\frac{\sqrt{a^4+2a}+a^2}{2a}} = a^2 + \frac{1}{\alpha_1}$, bunda $\alpha_1 = \frac{\sqrt{a^4+2a}+a^2}{2a} = a + \left(\frac{\sqrt{a^4+2a}+a^2}{2a} - a\right) = a + \frac{\sqrt{a^4+2a}-a^2}{2a} = a + \frac{1}{\frac{2a}{\sqrt{a^4+2a}-a^2}} = a + \frac{1}{\alpha_2}$.

Bu yerda $\alpha_2 = \frac{2a}{\sqrt{a^4+2a}-a^2} = \sqrt{a^4+2a} + a^2 = 2a^2 + (\sqrt{a^4+2a} - a^2) = 2a^2 + \frac{2a}{\sqrt{a^4+2a}-a^2} = 2a^2 + \frac{1}{\alpha_1}$. Demak, $\alpha = (a^2, \overline{a, 2a^2})$. Endi $\frac{P_3}{Q_3}$ aniqlaymiz.

q_i		a^2	a	$2a^2$	...
P_i	$P_0 = 1$	a^2	a^3+1	$2a^5+3a^2$	...
Q_i	$Q_0 = 0$	$Q_1 = 1$	a	$2a^3+1$	...

Bundan $\frac{P_3}{Q_3} = \frac{2a^5+3a^2}{2a^3+1}$. **Javob:** $\alpha = (a^2, \overline{a, 2a^2})$ va $\frac{P_3}{Q_3} = \frac{2a^5+3a^2}{2a^3+1}$.

363. $\alpha = \sqrt{a^2+a+1}$ ni uzluksiz kasrga yoyamiz. U holda quyidagiga ega bo'lamiz:

$$\alpha = a + (\sqrt{a^2+a+1} - a) = a + \frac{a+1}{\sqrt{a^2+a+1}+a} = a + \frac{1}{\alpha_1},\ \text{bunda}$$

$$\alpha_1 = \frac{\sqrt{a^2+a+1}+a}{a+1} = \frac{(a+1)+(\sqrt{a^2+a+1}-1)}{a+1} = 1 + \frac{(\sqrt{a^2+a+1}-1)\cdot(\sqrt{a^2+a+1}+1)}{(a+1)\cdot(\sqrt{a^2+a+1}+1)} = 1 + \frac{a^2+a+1-1}{(a+1)\cdot(\sqrt{a^2+a+1}+1)} = 1 + \frac{a}{\sqrt{a^2+a+1}+1} =$$

$$1 + \frac{1}{\frac{\sqrt{a^2+a+1}+1}{a}} = 1 + \frac{1}{\alpha_2}\ \text{bo'lib}$$

$$\alpha_2 = \frac{\sqrt{a^2+a+1}+1}{a} = \frac{a+(\sqrt{a^2+a+1}+1-a)}{a} = 1 + \frac{\sqrt{a^2+a+1}+1-a}{a} = 1 + \frac{1}{\frac{a}{\sqrt{a^2+a+1}-(a-1)}} = 1 + \frac{1}{\alpha_3}$$

bo'ladi. Bulardan foydalanib $\frac{P_3}{Q_3}$ ni aniqlaymiz.

q_i		a	1	1	...
P_i	$P_0 = 1$	a	$a+1$	$2a+1$	...
Q_i	$Q_0 = 0$	$Q_1 = 1$	1	2	...

Bundan $\frac{P_3}{Q_3} = \frac{2a+1}{2}$ ekanligi kelib chiqadi.

364. Avvalo berilgan kvadrat uchhadning musbat ildizini aniqlaymiz.

$$bx^2 - abx - a = 0 \to x = \frac{ab+\sqrt{a^2b^2+4ab}}{2b} = a + \left(\frac{ab+\sqrt{a^2b^2+4ab}}{2b} - a\right) = a + \frac{\sqrt{a^2b^2+4ab}-ab}{2b} = a + \frac{1}{\frac{2b}{\sqrt{a^2b^2+4ab}-ab}} = a + \frac{1}{\alpha_1},$$ bunda $\alpha_1 = \frac{2b}{\sqrt{a^2b^2+4ab}-ab}$

$$\alpha_1 = \frac{2b}{\sqrt{a^2b^2+4ab}-ab} = \frac{\sqrt{a^2b^2+4ab}+ab}{2a}$$
$$= b + \frac{\sqrt{a^2b^2+4ab}-ab}{2a} = b + \frac{1}{\frac{2a}{\sqrt{a^2b^2+4ab}-ab}}$$
$$= b + \frac{1}{\alpha_2} \text{ bo‘lib} \alpha_2 = \frac{2a}{\sqrt{a^2b^2+4ab}-ab}$$
$$= \frac{\sqrt{a^2b^2+4ab}+ab}{2b} = x.$$

Demak, $x = \frac{ab+\sqrt{a^2b^2+4ab}}{2b} = (\overline{a,b})$, ya’ni berilgan tenglamaning musbat ildizi davr uzunligi 2 ga teng bo‘lgan sof davriy uzluksiz kasrga yoyilar ekan.

365. 380-misolda $x_1 = (\overline{a,b})$ ning $bx^2 - abx - a = 0$ tenglamaning musbat ildizi ekanligini ko‘rsatgan edik. Berilgan tenglamani $x^2 - ax - \frac{a}{b} = 0$ ko‘rinishda yozish mumkin. Bundan, Viyet teoremasiga asosan $x_1 + x_2 = a \to$

$$x_2 = a - x_1 = a - (\overline{a,b}) = a - \left(a + \frac{1}{b+\frac{1}{a+\frac{1}{b+\ddots}}}\right) = -\left(\frac{1}{b+\frac{1}{a+\frac{1}{b+\ddots}}}\right) =$$

$-\frac{1}{(\overline{b,a})}$ bo‘lishi kerak ekanligi kelib chiqadi. Shunday qilib $x_2 = -\frac{1}{(\overline{b,a})}$.

366. Bu holda $\alpha = (\overline{a_1, a_2, \dots, a_n})$ soni $x = \frac{P_{n-1}x+P_{n-2}}{Q_{n-1}x+Q_{n-2}}$ tenglamani qanoatlantiradi, ya’ni $f(x) = Q_{n-1}x^2 + (Q_{n-2} - P_{n-1})x - P_{n-2}$ ko‘phadning musbat ildizi bo‘lishi kerak. Bu ko‘phadning ikkinchi ildizi α ga qo‘shma $\bar{\alpha}$bo‘lib, $f(0) = -P_{n-2} < 0$ va $f(-1) = (Q_{n-1} - Q_{n-2}) + (P_{n-1} - P_{n-2}) > 0$ bo‘ladi, chunki n ning o‘sishi bilan cheksiz uzluksiz kasrning maxraji o‘sadi. Shuningdek cheksiz uzluksiz kasrning surati P_n monoton o'suvchi bo'ladi. Bu holda $\alpha > 1$ bo'lgani uchun $\bar{\alpha} \in (-1; 0)$ bo'lishi kerak.

367. Bu yerda $x = (a, \overline{b, c}) = a + \frac{1}{(\overline{b,c})}$ bo'lgani chun $x - a = \frac{1}{(\overline{b,c})} \to (\overline{b,c}) = \frac{1}{x-a}$ bo'ladi. Bunda $(\overline{b,c})$ soni (380-misol) soni $cx^2 - bcx - b = 0$ tenglamaning ildizi. U holda bu tenglamaning ikkinchi ildizi 381-misolga asosan $-\frac{1}{(\overline{c,b})} = \frac{1}{x-a}$ tenglikdan topish mumkin. Bundan $(\overline{c,b}) = -x + a \to x = a - (\overline{c,b})$ kelib chiqadi.

368. 381-misolga asosan $x_1 = (\overline{a,b})$ soni $bx^2 - abx - a = 0$ tenglamaning musbat ildizi ekanligini ko'rgan edik, uning ikkinchi ildizi $x_2 = -\frac{1}{(\overline{b,a})} = -\left(0, (\overline{b,a})\right)$ dan iborat bo'ladi. Berilgan tenglamani $x^2 - ax - \frac{a}{b} = 0$ ko'rinishda yozish mumkin. Bundan, Viyet teoremasiga asosan $x_1 \cdot x_2 = -\frac{a}{b} \to x_1 \cdot x_2 = (\overline{a,b}) \cdot \left(0, (\overline{b,a})\right) = \frac{a}{b}$.

Javob: $(\overline{a,b}) \cdot \left(0, (\overline{b,a})\right) = \frac{a}{b}$.

369. Bu yerda $\alpha = a + \frac{1}{b+\frac{1}{c}} = a + \frac{c}{bc+1} = \frac{abc+a+c}{bc+1}$ va

$\beta = c + \frac{1}{b+\frac{1}{a}} = c + \frac{a}{ab+1} = \frac{abc+a+c}{ab+1}$ bo'lgani uchun $\frac{\alpha}{\beta} = \frac{ab+1}{bc+1}$ ekanligi kelib chiqadi. $x = (\overline{a,b,c})$ va $y = (\overline{c,b,a})$ lar mos ravishda quyidagi tenglamalarni qanoatlantiradi:

$$x = a + \frac{1}{b + \frac{1}{c+\frac{1}{x}}} = a + \frac{1}{b + \frac{x}{cx+1}} = a + \frac{cx+1}{bcx+b+x}$$

$$= \frac{abcx + (a+c)x + ab + 1}{bcx + b + x}$$

$$\to \frac{(bc+1)x^2 + bx - [abcx + (a+c)x + ab + 1]}{bcx + b + x} = 0 \to$$

$$(bc+1)x^2 - (abc + a + c - b)x - (ab+1) = 0.$$

Shunga o'xshash $(ab+1)y^2 - (abc + a + c - b)y - (bc+1) = 0.$

Bu tenglamalarni yechib

$$x = \frac{(abc + a + c - b) + \sqrt{(abc + a + c - b)^2 + 4(bc+1)(ab+1)}}{2(bc+1)};$$

$$y = \frac{(abc + a + c - b) + \sqrt{(abc + a + c - b)^2 + 4(bc + 1)(ab + 1)}}{2(ab + 1)}$$

larga ega bo'lamiz. Bulardan

$$\frac{x}{y} = \frac{ab + 1}{bc + 1} = \frac{\alpha}{\beta}$$

kelib chiqadi.

370. Agar n natyral soni uchun $\sqrt{n} = (q_1, q_2, \dots)$ bo'lsa, u holda $\sqrt{n} + q_1 = (2q_1, q_2, \dots) > 1$ va $-1 < q_1 - \sqrt{n} < 0$ bajariladi. Shuning uchun ham $\sqrt{n} + q_1$ ifoda sof uzluksiz kasrga yoyiladi., ya'ni $\sqrt{n} + q_1 = (\overline{2q_1, q_2, \dots, q_n})$. Bundan $\sqrt{n} = (q_1, \overline{q_2, \dots, q_n, 2q_1})$. Bu esa isbotlanishi talab etilgan tasdiq. misol uchun $\sqrt{2} = (1, \overline{2})$; $\sqrt{8} = (1, \overline{2, 4})$.

FOYDALANILGAN ADABIYOTLAR

Asosiy adabiyotlar

1. Хожиев Ж.Х. Файнлейб А.С. Алгебра ва сонлар назарияси курси, Тошкент, «Ўзбекистон», 2001 й.

2. Аюпов Ш.А., Омиров В.А., Худойбердиев А.Х., Хайдаров А.Н.. Алгебра ва сонлар назарияси. Ўқув қўлланма. Тошкент, «ЎзМУ», 2020, 340 с.

3. Исроилов М. И., Солеев А. Сонлар назариясига кириш. Тошкент, «Фан», 2003, 190 с.

4. ManinYu.I., Panchishkin A.A. Introduction to modern number theory Germany, 2007, English.

5. Godfrey H. Hardy and Edward M. An Introduction to the Theory of Number. Four Edition. Wright. 2004. Publisher Oxford University Press.

Qo'shimcha adabiyotlar

1. Виноградов И.М. Основы теории чисел.–М.: Наука, 1981, 176 с.

2. Бухштаб А.А. Теория чисел. –М.: Просвещение,1966.-384с.

3. Гильберт Д. Избранные труды. Том 1. Теория инвариантов. Теория чисел. Алгебра. Геометрия. Основания математики. 1998.

4. Нестеренко Ю.В. Теория чисел. –М.: Издательский центр "Академия", 2008, 272с.

5. Hardy G.H., Wright E. M. An introduction to the Theory of Numbers. -6th.ed., Oxford University Press. -2008, 480p.

6. Кудреватов Г.А. Сборник задач по теории чисел. – М.: «Просвещение», 1970, 128с.

7. Грибанов В.У., Титов П.И. Сборник упражнений по теории чисел. –М.: «Просвещение», 1964, 143с.

Internet saytlar:

1. http://lib.mexmat.ru; http://www.mcce.ru.

2. http://lib.mexmat.ru.

3. http://techlibrary.ru.

GLOSARIY

Pifagor uchburchagi– tomonlari Pifagor teoremasi shartini qanoatlantiruvchi uchburchak.

Umumiy bo'luvchilar – berilgan sonlarning barchasi bo'linadigan sonlar.

Eng katta umumiy bo'luvchi (EKUB) – umumiy bo'luvchilarining eng kattasi.

Umumiy karralilar – berilgan sonlarning barchasiga bo'linadigan sonlar.

Eng kichik umumiy karrali (EKUK) – umumiy karralilarining eng kichigii.

Algoritm – chekli qadamdan keyin masalaning yechimiga olib keluvchi amallar ketma-ketligi.

Evklid algoritmi – dastavval Evklid tomonidan ikkita sonning EKUBini topish uchun qo'llanilgan algoritm.

Tub son – faqat o'ziga va birga bo'linadigan birdan katta natural sonlar.

Murakkab sonlar – tub son bo'lmagan birdan katta natural sonlar.

Arifmetik funksiya (sonli funksiya)– butun sonlar to'plamida aniqlangan va qiymatlari to'plami umuman olganda kompleks sonlardan iborat bo'lgan funksiya.

$\pi(x)$ *funksiyasi* – x ning musbat qiymatlarida aniqlangan, x dan katta bo'lmagan tub sonlarning sonini ifodalaydi.

$y=[x]$ *butun qism funksiyasi* – x ning barcha haqiqiy qiymatlarida aniqlangan, x dan katta bo'lmagan va unga eng yaqin turgan butun sonni ifodalaydi.

$y=\{x\}$ – *kasr qism funksiyasi* $\{x\}=x-[x]$ tenglik yordamida aniqlanuvchi funksiya.

$\tau(n)$ *funksiyasi* – n ning barcha natural qiymatlarida aniqlangan, n ning barcha natural bo'luvchilari sonini ifodalaydi.

$\sigma(n)$ *funksiyasi* – n ning barcha natural qiymatlarida aniqlangan, n ning barcha natural bo'luvchilari yig'indisini ifodalaydi.

Multiplikativ funksiya – ixtiyoriy a va b o'zaro tub natural sonlari uchun aynan nolga teng bo'lmagan va $f(ab)=f(a)f(b)$ tenglikni qanoatlantiruvchi f funksiya.

Eyler funksiyasi $\varphi(a)-a$ dan katta bo'lmagan va a bilan o'zaro tub bo'lgan sonlarning sonini ifodalaydi.

m moduli bo'yicha taqqoslanuvchi sonlar –agar ikkita butun *a* va *b* sonni *m* natural soniga bo'lganda hosil bo'lgan qoldiqlar o'zaro teng bo'lgan sonlar.

Berilgan modul bo'yicha chegirmalar sinfi – modulga bo'lganda bir xil qoldiq qoluvchi butun sonlar sinfi.

Berilgan modul bo'yicha chegirmalar to'la sistemasi – berilgan $m > 0$ modul bo'yicha *m* ta har xil sinf bo'ladi, shu sinflarning har biridan bittadan chegirma olib tuzilgan sistema.

Berilgan modul bo'yicha chegirmalar keltirilgan sistemasi – berilgan $m > 0$ modul bo'yicha *chegirmalarning to'la sistemasidan modul bilan o'zaro tublarini* olib tuzilgan sistema.

Kvadratik chegirma – $x^2 \equiv a(\bmod m)$ taqqoslama yechimga ega bo'lsa, *a* ga kvadrartik chegirma deyiladi.

n darajali chegirma – $x^n \equiv a(\bmod m)$ taqqoslama yechimga ega bo'lsa, *a* ga kvadrartik chegirma deyiladi.

Chekli zanjirli kasr – berilgan ratsional sonni Evklid algoritmiga yoyib uning chala bo'linmalarini ma'lum ko'rinishda joylashtirib tuzilgan ifoda.

Cheksiz zanjirli kasr – berilgan irratsional sonni Evklid algoritmiga o'xshash algoritm yordamida yoyib uning chala bo'linmalarini ma'lum ko'rinishda joylashtirib tuzilgan ifoda.

Ko'rsatkichga qarashli son – modul *m* bilan o'zaro tub bo'lgan *a* sonning bir bilan taqqoslanuvchi bo'lgan $a^\delta \equiv 1(\bmod m)$ manfiy bo'lmagan eng kichik darajasi δ bo'lsa, *a* soni *m* moduli bo'yicha δ ko'rsatkichga tegishli deyiladi.

Boshlang'ich ildiz – agar *a* soni *m* moduli bo'yicha $\varphi(m)$ ko'rsatkichga tegishli bo'lsa, *a* soni *m* moduli bo'yicha boshlang'ich ildiz deyiladi.

Algebraik son – biror ratsional koeffitsiyentli ko'phadning ildizi.

Transendent son – birorta ham ratsional koeffitsiyentli ko'phadning ildizi deb qarash mumkin bo'lmagan son.

MUNDARIJA

V BOB. BOSHLANG'ICH ILDIZLAR VA INDEKSLAR

VI BOB. UZLUKSIZ KASRLAR VA ULARNING TATBIQLARI

I.ALLAKOV

SONLAR NAZARIYASIDAN MISOL VA MASALALAR

Toshkent – «Инновацион ривожланиш нашриёт-матбаа уйи» – 2020

Muharrir:	M. Hayitova
Tex. muharrir:	A.Moydinov
Musavvir:	A.Shushunov
Musahhih:	Sh.Mirqosimova
Kompyuterda sahifalovchi:	M.Zoyirova

E-mail: nashr2019@inbox.ru Tel:+998999209035
Nashr.lits. AI№009, 20.07.2018.
Bosishga ruxsat etildi 02.11.2020.
Bichimi 60x84 1/16. «Timez Uz» garniturasi.
Ofset bosma usulida bosildi.
Shartli bosma tabog'i 22,5. Nashriyot bosma tabog'i 21,75.
Tiraji 200. Buyurtma № 141.

«Инновацион ривожланиш нашриёт-матбаа уйи»
bosmaxonasida chop etildi.
100066, Toshkent sh., Olmazor ko'chasi, 171-uy.