U.U.UMAROVA

DISKRET MATEMATIKA VA MATEMATIK MANTIQ FANIDAN MISOL VA MASALALAR TO'PLAMI

O`quv qo`llanma Oliy ta`lim muassasalarining 5130100-Matematika, 5130200-Amaliy matematika va informatika ta`lim yo`nalishlari talabalari uchun tayyorlangan

Umida, Umarova

Diskret matematika va matematik mantiq fanidan misol va masalalar to’plami: o’quv qo’llanma\ U. Umarova. 191 b.

ANNOTATSIYA

Mazkur o’quv qo’llanma o'n to’rt paragrafdan iborat bo'lib, ular to’rtta bobda guruhlangan: I. To’plamlar nazariyasi; II. Mulohazalar algebrasi; III. Bul funktsiyalari; IV. Mulohazalar hisobi. Har bir paragrafda misol va masalalarni yechish uchun ishlatiladigan tushunchalar tizimini ta'minlovchi nazariy xususiyatga ega qisqacha ma'lumot berilib, so’ng muammoli masala va topshiriqlar beriladi.

O`quv qo`llanma Oliy ta`lim muassasalarining 5130100-Matematika, 5130200-Amaliy matematika va informatika ta`lim yo`nalishlari fan dasturidagi mavzulari asosida tuzilgan. Misol va masalalar to'plami ta'lim muassasalarida diskret matematika va matematik mantiq asoslarini o'rganishni boshlagan har bir kishi uchun foydali bo'ladi.

АННОТАЦИЯ

Это Учебное пособие сборник задач предназначен для оказания практической поддержки теоретическим курсам по дискретной математике и математической логике. Сборник состоит из четырнадцати параграфов, которые сгруппированы в четыре главы: I. Множество; II. Алгебра высказываний; III. Булевые функции; IV. Формализованное исчисление высказываний. Каждый абзац содержит краткий обзор теоретической части, которые обеспечивают систему понятий, используемых для решения примеров и задач, за которыми следуют задачи для решения.

Учебное пособие предназначено для студентов бакалавриата направления 5130100-Математика, 5130200-Прикладная математика и информатика предусмотренная учебная программой.

Сборник задач и упражнений будет полезен всем, кто приступает к изучению основ дискретной математики и математической логики в учебных заведениях.

ANNOTATION

The collection consists of fourteen sections, which are grouped into four chapters: I. Sets; II. Propositional Algebra; III. Boolean functions IV. Propositional calculus. Each section contains a brief overview of the theoretical part, which provides a system of concepts used to solve examples and problems, followed by tasks to be solved.

The manual is intended for undergraduate students of the disciplines of 5130100-Mathematics, 5130200-Applied mathematics and informatics, and is designed according to the topics given in the curriculum. The collection of problems and exercises will be useful to everyone who begins to study the basics of discrete mathematics and mathematical logic in educational institutions.

TAQRIZCHILAR:

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SO‘Z BOSHI

Mamlakatimiz mustaqillikka erishgach, ta’lim tizimini tubdan islohot qilishga katta ahamiyat berildi. 1997-yil 29-avgust kuni O‘zbekiston Respublikasi Oliy Majlisining IX sessiyasida “Kadrlar tayyorlash milliy dasturi” qabul qilindi va unda ta’lim tizimini zamonaviy talablarga mos keltirish uchun bajarilishi lozim bo‘lgan vazifalar hamda ularni bosqichma–bosqich amalga oshirish belgilab berildi. Milliy dasturdagi eng asosiy vazifalaridan biri –yuksak ma’naviy va axloqiy talablarga javob beruvchi yuqori malakali mutaxassislar tayyorlashdan iboratdir. Bu vazifani amalga oshirishda o‘quv-tarbiya jarayoni uchun o‘quv adabiyotlarining yangi avlodini yaratish, uni yuqori sifatli o‘quv-uslubiy majmualar bilan ta’minlash muhim ahamiyatga ega ekanligi dasturda ta’kidlab o‘tilgan.

Yuqori malakali, raqobatbardosh, zamonaviy talablarga javob bera oladigan kadrlar tayyorlashda ularga chuqur matematik bilimlar berish va bu bilimlarni masalalar yechishga tatbiq eta olishga o‘rgatish katta ahamiyatga ega.

Misol va masalalar ishlamasdan matematikani o'rganishni tasavvur qilib bo'lmaydi. O'qitish jarayonida misol va masalalar turli funksiyalarni bajaradi - o'qitish, rivojlantirish, ta'lim berish. Biz misol va masalalar matematikani o'qitishning eng muhim vositasi deb bilamiz. Ular yordamida matematikani o'qitishda turli didaktik maqsadlar qo'yilishi mumkin: nazariy savollarni o'rganishga tayyorgarlik, olingan nazariy bilimlarni mustahkamlash, ko'nikmalarini shakllantirish, ilgari o'rganilgan materialni takrorlash, uni boshqa fanlarda va amaliyotda qo'llash, bilimlarning assimilyatsiyasini kuzatish.

Misol va masalalar yordamida o'z maqsadlariga erishishga o'rgatish uchun mashqlar va topshiriqlarning puxta o'ylangan tizimi kerak. Bunday tizimda vazifalar ketma-ketligi o'quvchilarning xususiyatlari va qobiliyatlari "oddiydan murakkabga" tamoyilini hisobga olgan holda to'g'ri tashkil etilishi kerak.

Misol va masalalar to'plami o'quv qo'llanmasida o'rganilayotgan diskret matematika va matematik mantiq fani bo'yicha nazariy mavzularni amaliy qo'llab-quvvatlashga mo'ljallangan. To'plam o'n to’rt paragrafdan iborat bo'lib, ular to’rtta bobda guruhlangan: I. To’plamlar nazariyasi; II. Mulohazalar algebrasi; III. Bul funksiyalari; IV. Mulohazalar hisobi. Har bir paragrafda misol va masalalarni yechish uchun ishlatiladigan tushunchalar tizimini ta'minlovchi nazariy xususiyatga ega qisqacha ma'lumot berilib, so’ng muammoli masala va topshiriqlar beriladi.

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1-BOB. TO'PLAMLAR NAZARIYASI

1.1-To'plamlar va ular ustida amallar.

$A=\{a,b,c,d,...\}$ - A to'plam $a,b,c,d...$ elementlardan tuzilgan.

$a\in A$ yoki $A\ni a$ - a narsa A to'plamning ***elementi***

$b\overline{\in}A$ yoki $A\overline{\ni}b$ - b narsa A to'plamning ***elementi emas***

$B\subseteq A$ yoki $A\supseteq B$ - B A ning ***qism to'plami***

$B\subset A$ yoki $A\supset B$ - B A ning ***xos qism*** to'plami.

$\varnothing$ -bo'sh to'plam har qanday A to'plamning qism to'plami bo'ladi va u ham A ning ***xosmas qismi***

$A\cup B = \{x \mid x \in A$ yoki $x \in B\}$ - A va B to'plamlarning ***yig'indisi yoki birlashmasi***

$\bigcup_{\alpha=1}^{n} A_\alpha = A_1 \cup A_2 \cup A_3 \cup ... \cup A_n$ - $A_1, A_2, A_3,..., A_n$ to'plamlarning ***yig'indisi yoki birlashmasi***

$A \cap B = A\ B = \{x \mid x \in A\ va\ x \in B\}$ - A, B to'plamlarning ***ko'paytmasi (kesishmasi yoki umumiy qismi)***

$\bigcap_{\alpha=1}^{n} A_\alpha = A_1 \cap A_2 \cap A_3 \cap ... \cap A_n$ - $A_1, A_2, A_3,..., A_n$ to'plamlarning ***ko'paytmasi (kesishmasi yoki umumiy qismi)***

$A-B$ yoki $A\setminus B$, $A - B = \{x \mid x \in A$ va $x \notin B\}$ - A va B ***to'plamlarning ayirmasi***

$\overline{B}$ yoki B' - B ni A to'plamigacha ***to'ldiruvchisi***

$\cup$ - universal to'plam.

$2^A = \{X \mid X \subseteq A\}$ —A to'plamning ***barcha qism to'plamlaridan tuzilgan to'plam***

$A \vartriangle B$ yoki $A \otimes B = (A - B) \cup (B - A)$ – A va B to'plamlarning ***simmetrik ayirmasi***.

$n(A)$ - A to'plamning ***elementlari soni***

Agar $A \subseteq B$ va $B \subseteq A$ bo'lsa, u vaqtda $A = B$.

Misol: $A = \{1,2,3,4,5\}$, $B = \{3,4,5,6,7\}$ $A \cup B = \{1,2,3,4,5,6,7\}$

Asosiy tengkuchliliklar.

1. $\overline{\overline{A}} = A$
2. $A \cap B = B \cap A$ - ko'paytmaga nisbatan kommutativlik qonuni.
3. $(A \cap B) \cap C = A \cap (B \cap C)$ - ko'paytmaga nisbatan assostiativlik qonuni.
4. $A \cup B = B \cup A$ - yig'indiga nisbatan kommutativlik qonuni.
5. $(A \cup B) \cup C = A \cup (B \cup C)$ -yig'indiga nisbatan assostiativlik qonuni.
6. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ - ko'paytmaga nisbatan distributivlik qonuni.
7. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ - yig'indiga nisbatan distributivlik qonuni.
8. $\overline{A \cap B} = \overline{A} \cup \overline{B}$. 9. $\overline{A \cup B} = \overline{A} \cap \overline{B}$. - de-Morgan qonuni
10. $A \cap A = A$. 11. $A \cup A = A$.
12. $A \cap U = A$. 13. $A \cup U = U$.
14. $\overline{A \cup A} = \overline{A}$. 15. $A \cup \varnothing = A$. 16. $A \cap \varnothing = \varnothing$.
17. $B \cup \overline{B} = U$. 18. $\overline{B} \cap B = \varnothing$ 19. $B \cup \overline{B} = A$.
20. $B - \overline{B} = B$, 21. $\overline{B} - B = \overline{B}$.
22. $A \cap (A \cup B) = A$ 23. $A \cup (A \cap B) = A$ yutish qonuni

Kesishma amali birlashma amaliga nisbatan mustahkamroq bog'laydi, shuning uchun qavslar qo'yilmagan taqdirda birinchi bajariladi.

Misol: $A \cup (A \cap B) = A \cup A \cap B$.

$(A \cap B) \cup C = A \cap B \cup C$

Muammoli masala va topshiriqlar:

1.1.1. Quyidagilardan qaysi to'g'ri:

1) $b \subset \{a, b\}$;
2) $b \in \{a,b\}$;
3) $\{b\} \subset \{a, b\}$;
4) $\{b\} \in \{a, b\}$;
5) $b \subset \{a,\{b\}\}$;
6) $b \in \{a,\{b\}\}$;
7) $\{b\} \subset \{a,\{b\}\}$;
8) $\{b\} \in \{a,\{b\}\}$;
9) $\varnothing \in \{\varnothing\}$;
10) $\varnothing \subseteq \{\varnothing\}$;
11) $\varnothing \in \varnothing$;
12) $\varnothing \subseteq \varnothing$?

1.1.2. Har bir to'plamning elementlar soni nechta:

1) $\{1, 2,3,\{1, 2,3\}\}$;
2) $\{1,\{1\},2,\{1,\{2,3\}\}, \varnothing\}$;
3) $\varnothing$;
4) $\{\varnothing\}$;
5) $\{\varnothing,\{\varnothing\}\}$;
6) $\{\{\varnothing,\{\varnothing\}\}\}$?

1.1.3. Agar $A \subseteq B$ va $a \in A$ bo'lsa, quyidagilardan qaysi to'g'ri:

1) $a \notin B$;
2) $a \in B$;
3) $A \in B$;
4) $a \in A \cup B$;
5) $a \in A \cap B$;
6) $a \in A - B$;
7) $a \in A \otimes B$;
8) $a \subseteq A$;
9) $\{a\} \subseteq A$;
10) $\{a\} \subseteq B$?

1.1.4. Agar $B \subseteq A \subseteq C$, $a \in A$ va $a \notin B$ bo'lsa, quyidagilardan qaysi to'g'ri:

1) $a \notin C$;
2) $a \in C$;
3) $a \in A \cap B$;
4) $a \in A \cup B$;
5) $a \in A - B$;
6) $a \in B - A$;
7) $a \in A \otimes B$;
8) $\{a\} \subseteq A \cap C$;
9) $a \in A \cup C$;
10) $\{a\} \subseteq A - C$;
11) $\{a\} \subseteq A \otimes C$;
12) $a \in (A \cap B) \cup C$;
13) $\{a\} \subseteq A \cap (B \cup C)$;
14) $\{a\} \subseteq B \cup (C - A)$;
15) $\{a\} \subseteq A \cap (B - C)$;
16) $\{a\} \subseteq B \otimes (A - C)$?

1.1.5. Universal to'plam $U=\{1,2,3,4,5,6,7,8\}$ va uning qism to'plamlari $A=\{x \mid 2<x\leq 6\}$, $B=\{x \mid x$ –juft $\}$, $C=\{x \mid x\geq 4\}$, $D=\{1,2,4\}$ bo'lsa, $A\cap B$, $A\cup B$, CD, $B\otimes C$, $\overline{A}$, $\overline{BD}$, $\overline{\overline{A}\cup\overline{B}\cup\overline{C}}$, $(A-B)\cup(C-D)$, $2^A\cap 2^B$, 2^D-2^B ni toping.

1.1.6. Universal to'plam $U=\{1,2,3,4,5,6,7,8\}$ va uning qism to'plamlari

$A=\{x \mid x$ – juft$\}$, $B=\{x \mid x$ –to'rtga karrali $\}$, $C=\{x \mid x$ – tub$\}$, $D=\{1,3,5\}$ bo'lsa, $A\cup B$, CD, $A\otimes B$, $A\cap(B\cup C\cup D)$, $C\otimes D$, $(A-B)\cup(C-D)$ $\overline{A}$, $\overline{BD}$, $\overline{\overline{A}\cup\overline{C}}$, $2^A\cap 2^B$ ni toping.

1.1.7. M_2, M_3, M_5 to'plamlar mos ravishda 2,3,5 ga bo'linuvchi sonlardan tuzilgan va universal N natural sonlar to'plamning qism to'plamlari bo'lsin. Shu to'plamlar yordamida quyidagi shartni bajaruvchi barcha sonlar to'plamini tuzing:

1) 6 ga bo'linuvchi;
2) 30 ga bo'linuvchi;
3) 30 bilan o'zaro tub;
4) 10 ga bo'linuvchi ammo 3 ga bo'linmaydigan;

To'plamlar nazariyasi simvollaridan foydalanib, quyidagilarni yozing:

1) 45 soni 15 ga bo'linadi;
2) 42 soni 6 ga bolinadi ammo 10 ga bo'linmaydi;
3) {8, 9, 10} to'plamdagi har bir son 2, 3, 5 sonlarning hech bo'lmaganda birtasiga bo'linadi ammo 6 ga bo'linmaydi.

1.1.8. $-$, $\otimes$ amallari uchun kommutativlik va assosiativlik xossalari bajarilishini tekshiring.

1.1.9. Ixtiyoriy A,B,C to'plamlar uchun quyidagi distributivlik xossalaridan qaysi bajarilishini aniqlang:

1) $A-(B\cup C)=(A-B)\cup(A-C)$;
2) $A-(B\cap C)=(A-B)\cap(A-C)$;
3) $A\otimes(B\cup C)=(A\otimes B)\cup(A\otimes C)$;

4) $A\otimes BC=(A\otimes B)(A\otimes C)$;

5) $A-(B\otimes C)=(A-B)\otimes(A-C)$;

6) $A\cup BC=(A\cup B)(A\cup C)$;

7) $A\cup(B-C)=(A\cup B)-(A\cup C)$;

8) $A(B-C)=AB-AC$;

9) $A\cup(B\otimes C)=(A\cup B)\otimes(A\cup C)$;

10) $A(B\otimes C)=AB\otimes AC$;

11) $A\otimes(B-C)=(A\otimes B)-(A\otimes C)$.

12) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$;

13) $A\cap(B\setminus C)=(A\cap B)\setminus(A\cap C)$;

14) $A\setminus(B\setminus C)=(A\setminus B)\cup(A\cap C)$;

15) $A\triangle(B\triangle C)=(A\triangle B)\triangle C$;

16) $A\cap(B\triangle C)=(A\cap B)\triangle(A\cap C)$;

17) $(A\cap B)\cup(C\cap D)=(A\cup C)\cap(B\cup C)\cap(A\cup D)\cap(B\cup D)$.

1.1.10. Isbotlang:

1) $A\cup AB=A$;

2) $A(A\cup B)=A$;

3) $A\cup\overline{A}\cap B=A\cup B$

4) $A(\overline{A}\cup B)=AB$

5) $A-(A-B)=AB$;

6) $A\text{-}AB=A\text{-}B$

13) $A(B-A)=\varnothing$;

14) $A\cup(B-A)=A\cup B$;

17) $A-(B\cup C)=(A-B)(A-C)$;

18) $A\otimes\overline{B}=\overline{A}\otimes B=AB\cup(\overline{A\cup B})$

7) $A\otimes B=A\cap\overline{B}\cup\overline{A}\cap B$

8) $A\otimes(A\otimes B)=B$;

9) $A-B=A\otimes AB$;

10) $A\cup B=(A\otimes B)\cup AB$;

11) $\overline{A\otimes B}=\overline{A}\otimes B=A\otimes B\otimes U$

12) $\overline{A\otimes B}=AB\cup(\overline{A}\overline{B})$

15) $A\cup\overline{A}B=A\otimes\overline{A}B=B\otimes A\overline{B}$

16) $AB\cup A\overline{B}=A$

19) $(A- B)-C = (A-C)-(B-C) = A-(B\cup C)$;

20) $A-(B-C) = (A-B)\cup AC = (A-B)\cup(\overline{A}-C)$;

21) $(A\cup B)-C = (A-C)\cup(B-C)$;

22) $A\cup B\cup C = (A- B)\cup(B-C)\cup(C - A)\cup ABC$;

23) $A- BC = (A- B)\cup(A-C) = ABC \otimes A$;

24) $A(B-C) = AB-C = ABC\otimes AB$;

25) $(AB \otimes A)\otimes(BC \otimes C) = (AB \otimes BC)\otimes(A\otimes C)$;

26) $(A\cup B)\otimes(B\cup C) = (A\otimes C) - B = (AB \otimes BC)\otimes(A\otimes C)$.

1.1.11. Ifodalang:

1) $\cup$ ni $\otimes$, $\cap$ amallar orqali;

2) $\cap$ ni $\otimes$, $\cup$ amallar orqali;

3) $\cup$ va $\cap$ ni $\otimes$,– amallar orqaliю

1.1.12. Tenglamani yeching:

1) $AX = B$;

2) $A\cup X = B$;

3) $A\otimes X = B$;

4) $A-X = B$;

5) $A\cup X = BX$;

6) $A\otimes X = BX$;

7) $A-X = X-B$;

8) $(A\cup X)\cup B = X \cup B$;

9) $AX = (X \cup B)- A$;

10) $\overline{X A} = (X - B)\cup A$;

11) $(A\cup X)\overline{B} = X - B$;

12) $(A-X)\cup B = B\otimes X$;

13) $(X-A)\cup B = \overline{AX}$;

14) $(X\otimes A)-B = BX$;

15) $AX\otimes B = B-\overline{X}$;

16) $A-X = BX-A$;

17) $A\cap \overline{X\ B} = (X - A)B$;

18) $\overline{A\cup X} = \overline{B\ X}$;

19) $X - A = B\cup(X - A)$;

20) $(A\cup X)-B = B-X A$;

21) $X A = B(X \cup A)$;

22) $(A\otimes X)X = X - B$;

23) $AX \cup \overline{A} = (X - B)B$;

24) $(A\cup X)B = \overline{A}\cup BX$;

25) $A\otimes(X\cup B) = BX$;

26) $BX = (A-X)X$;

27) $AX\otimes B = X-A$;

28) $(X \cup B)-A = A\cup X$;

29) $AX\cup B = A-X$;

30) $X \cup A = (B-X)\otimes A$.

1.1.13. Agar $A \subseteq B$ bo'lsa, $AB = \varnothing$ ekanligini isbotlang.

1.1.14. Agar $A = B$ bo'lsa, $A \otimes \overline{B} = \varnothing$ ekanligini isbotlang.

1.1.15. $U = \{0,1,2,3,4,5,6,7,8,9\}, A = \{1,2,3\}, B = \{0,1,2,5,8\}, C = \{0,2,5,8\},$

$D = \{3,6,8,9\}$ bo'lsa, quyidagi to'plamlarni toping:

1) $\overline{A \cup B} \cap (C \setminus D)$

2) $\overline{A / B} \cap (C \cup D)$

3) $\overline{A \cap B} \cap (C \setminus \overline{D})$

4) $(A \cup \overline{B \cup C}) \cap D$

5) $(A \otimes B) \cup \overline{C \cap D}$

6) $(A \cup B \cup C) \cap D$

7) $(B \cap C) \cup (A \cap D)$

8) $\overline{(\overline{A \cup B} \cup C) \cap D}$

1.1.16. $U = \{-1,-2,-3,-4,-5,5,4,3,2,1\}, A = \{1,2,3\}, B = \{-1,-2,-5,3\}, C = \{-3,-4,1,2,3\},$

$D = \{-2,1,4,5\}$ bo'lsa, quyidagi to'plamlarni toping:

1) $\overline{A \cup B} \cap (C \setminus D)$

2) $\overline{A / B} \cap (C \cup D)$

3) $\overline{A \cap B} \cap (C \setminus \overline{D})$

4) $(A \cup \overline{B \cup C}) \cap D$

5) $(A \otimes B) \cup \overline{C \cap D}$

6) $(A \cup B \cup C) \cap D$

7) $(B \cap C) \cup (A \cap D)$

8) $\overline{(\overline{A \cup B} \cup C) \cap D}$

1.1.17. Sistemani soddalashtiring:

1) $\begin{cases} A \subseteq BC \cup \overline{B}; \\ ABC \subseteq D; \\ AD \subseteq \overline{BC}. \end{cases}$

2) $\begin{cases} \overline{A} = B\overline{C}; \\ \overline{C} \subseteq D; \\ AD = \overline{B}CD; \\ B = CD. \end{cases}$

3) $\begin{cases} C \subseteq A \cup B; \\ A \cup D \subseteq B \cup C; \\ \overline{B} \subseteq D \subseteq \overline{C}; \\ BC \subseteq \overline{D}. \end{cases}$

1.1.18. Quyidagi sistemalar tengkuchlimi?

1) $\begin{cases} X \subseteq Z \subseteq \overline{W}, \\ Y \subseteq W, \\ X \cup Y \subseteq Z \cup W. \end{cases}$ va $\begin{cases} X = Z \\ Y = W. \end{cases}$

2) $\begin{cases} C\otimes D\subseteq A, \\ B\cup D\subseteq A\cup C, \\ A-D\subseteq C-B. \end{cases}$ va $\begin{cases} \overline{A}\subseteq CD, \\ B-C\subseteq \overline{A}, \\ A\subseteq C\cup D. \end{cases}$

3) $\begin{cases} A\subseteq C\otimes B, \\ C\subseteq B\otimes D, \\ AC\subseteq B-D. \end{cases}$ va $\begin{cases} B\subseteq \overline{CD}, \\ C-D\subseteq B, \\ AC\subseteq D, \\ A-B\subseteq BC. \end{cases}$

1.1.19. Berilgan tenglamalar sistemasidagi X to'plamni toping. Bu yerda A, B, C to'plamlar $B\subseteq A\subseteq C$ shartni qanoatlantiradi.

1) $\begin{cases} A-X=B \\ A\cup X=C. \end{cases}$ 2) $\begin{cases} AX=B \\ A\cup X=C. \end{cases}$

1.1.20. Tenglamalar sistemasini yeching.

1) $\begin{cases} (A\cup X)(B\cup X)=C\cup X \\ BX\cup C=\overline{AX}. \end{cases}$; 2) $\begin{cases} A-X=\overline{B} \\ A\cup X=\overline{C}. \end{cases}$

3) $\begin{cases} A\otimes B\otimes X=X\otimes C \\ AX\otimes B=AX\otimes C. \end{cases}$; 4) $\begin{cases} AX=C\cup X \\ A\cup X=BX. \end{cases}$

5) $\begin{cases} AX\cup B\overline{X}=C \\ BX\cup A\overline{X}=C. \end{cases}$; 6) $\begin{cases} AX=B \\ B\overline{X}=C \\ CX=A\cup B. \end{cases}$

1.1.21. Quyidagilardan qaysi to'g'ri va qaysi noto'g'ri? Javobingizni asoslang:

1) $\pi\in R$ 5) Ø∈ {Ø};

2) $cos\dfrac{\pi}{3}\in Q$ 6) $a\in\{\{a, b\}\}$;

3) 0,1010010001...∈ Q ; 7) $\{a, b\}\in\{\{a, b\}\}$;

4) Ø∈ Ø; 8) $\{a, b\}\in\{\{a, b\};\{a, c\}; a; b\}$.

1.1.22. Quyidagi to'plamlar tengmi:

1) {1, 3, 5} va {1, 3, 5, 1}; 4) $\{a, b, c\}$va $\{\{a\}, \{b\}, \{c\}\}$

2) {11, 13} va{{11, 13}}; 5) $\{\{a, b\}, c\}$ va $\{a, \{b, c\}\}$

3) $\{a, b, c\}$ va$\{a, b, a, c\}$; 6) $\{x \in R\ 22 \le x \le 3\}$ va Ø.

1.1.23. $\in$ yoki $\subseteq$ belgilardan qaysi qo'yilganda to'g'ri tasdiq hosil bo'ladi?

1) {1} va{1,{1, 2}};
2) {1, 2}va {1, 2, {1}, {2}};
3) {1, 2}va {1, 2, {1, 2}};
4) Ø va {1, 2, {1}, {Ø}};
5) Ø va {Ø};
6) Ø va {{Ø}};
7) {1, 2}va {1, 2}.

1.1.24. To'plamlarning asosiy xossalaridan foydalanib, quyidagilarni soddalashtiring:

1) $(A \cap \overline{B} \cap \overline{C}) \cup (\overline{A} \cap \overline{B} \cap \overline{C}) \cup B \cup C;$

2) $\overline{(A \cup B) \cap (\overline{A} \cup B) \cap (A \cup \overline{B})};$

3) $\overline{A} \cup (\overline{A \cup \overline{B} \cap \overline{C}}) \cup (B \cap (\overline{A \cup C}));$

4) $A \otimes A \otimes A$

5) $(A \cap B \cap C \cap \overline{D}) \cup (\overline{A} \cap C) \cup (\overline{B} \cap C) \cup (C \cap D)$

6) $(\overline{A} \cup B \cup \overline{C}) \cap (A \cap \overline{B} \cap C) \cap (\overline{A \cup C})$

1.1.25. Isboblang:

1) $(A \setminus B) \setminus C \subseteq (A \cup C) \setminus B$;

2) $A \triangle B = A \setminus B$, agar $B \subseteq A$;

3) $A \cap B \subseteq C$, agar$A \subseteq (\overline{B} \cup C)$;

4) $A = \overline{B}$, agar $A \cap B = \varnothing$ va $A \cup B = U$.

1.1.26. Sinfda 32 o'quvchi bor. Ulardan 18 nafari kimyo to'garagiga, 12 tasi biologiya to'garagiga, 8 talaba ushbu to'garaklarning hech biriga qatnashmaydi. Qancha talabalar kimyo va biologiya to'garaklarga qatnashadilar? Qancha talabalar faqat kimyo to'garagiga qatnashadilar?

1.1.27. Guruhda 30 talaba bor, ulardan 18 nafari suzishga, 17 nafari voleybolga qiziqadi. a) Ikkala sport turiga qiziqqan talabalar soni qancha bo'lishi mumkin?

b) Kamida bitta sport turiga qiziqqan o'quvchilar soni qancha bo'lishi mumkin?

1.1.28. Doskadagi tasodifiy yozilgan raqamlar orasida 65% -2, 70% - 3 ga, 75% ga - 5 ga bo'linadi, 30 ga bo'inadigan raqamlarning eng kichik foizi qancha?

1.1.29. PMIda o'qiyotgan barcha birinchi kurs talabalari uchta dasturlash tilini o'rganishadi. Bu yil 19 talaba Paskalni, 14 kishi Basicni, 17 kishi Delphini o'rganishga qaror qilishdi. Bundan tashqari, 4 talaba Paskal va Basic dasturlarida uchtasi Paskal va Delphi, uchtasi Delphi va Basic tillarini o'rganishadi. Ma'lumki, talabalarning hech biri birdaniga uchta kursga borishga rozi bo'lmagan. PMIda nechta talaba bor? Ularning qanchasi faqat Delphini o’rganadi?

1.1.30. Statistik mutaxassislar Golden Beach Sayohat agentligiga tashrif buyurgan 100 kishi bilan so'rov o'tkazdilar. Ma'lum bo'lishicha, so'nggi 5 yil ichida 50 kishi Turkiyada ta'tilga chiqqan, shulardan 20 kishi Gretsiyada, 18 kishi Misrda va besh kishi ushbu uchala mamlakatda uch yil bo'lgan. Respondentlardan 50 kishi Gretsiyaning diqqatga sazovor joylari bilan tanishishdi, shundan 26 kishi faqat ikki davlatga tashrif buyurishdi. Piramidalar mamlakatiga necha kishi tashrif buyurdi?

1.1.31. 40 talabadan o'tkazilgan imtihonlar natijalariga ko'ra 11 talaba matematikadan, 15 ta fizikadan, 13 ta kimyo fanidan, 4 ta matematika va fizikadan, 3 ta matematika va kimyodan, 3 ta fizika va kimyo fanlaridan a'lo baholarga ega bo'lishdi. uchala fandan – 1ta. Qancha talabalar kamida bitta a'lo bahoga ega bo'lishdi?

1.1.32. May oyida 12 yomg'irli, 8 shamolli, 4 sovuq, 5 yomg'irli va shamolli, 3 yomg'irli va sovuq, 2 shamolli va sovuq kunlar bo'lgan va bir kun yomg'irli, shamolli va sovuq bo'lgan. May oyida shamol va yomg'irsiz necha kun issiq edi?

1.1.33. Yozda Anapada dam olayotgan Kostromaning 100 aholisining har biri ekskursiyalarda, delfinariumda yoki akvaparkda edi. Ulardan 67 kishi akvaparkga tashrif buyurishdi, ekskursiyalar - 82, delfinarium - 67, ekskursiyalar va delfinarium - 53, ekskursiyalar va akvapark - 58, delfinarium va akvapark - 51. Uchala tadbirda nechta Kostromaliklar bo'lgan?

1.1.34. Guruh sardori jismoniy tarbiya bo'yicha quyidagi hisobotni taqdim etdi. Hammasi - 45 talaba. Futbol sektsiyasida - 25 kishi, basketbol sektsiyasida - 30, shaxmat sektsiyasida - 28, futbol va basketbolda - 16, futbol va shaxmatda- 18, basketbol va shaxmatda - 17. Uch sektsiyada bir vaqtning o'zida 15 kishi shug'ullanadi. Hisobot nima uchun qabul qilinmaganligini tushuntiring?

1.1.35. Odamlarning qo'rquvga qarshi kurashish klubida 100 kishi, o'rgimchaklardan 60, 54 kishi ilonlardan, 55 sichqonlardan, 38 o'rgimchak va ilonlardan, 34 ilonlar va sichqonlardan, 40 kishi o'rgimchak va sichqonlardan, 20 kishi yopiq joylardan qo'rqishadi. a) Qancha odamlar o'rgimchaklardan yoki sichqonlardan qo'rqishadi, lekin ilonlardan qo'rqmaydilar? b) Qancha odamlar faqat bittasidan qo'rqishadi? c) Qancha odamlar uchtasidan, ikkitasidan qo'rqishadi? d) Qancha odamlar ilon yoki o'rgimchaklardan qo'rqmaydi? e) Qancha odamlar faqat ilonlardan qo'rqishadi?

1.1.36. Oltin baliqni qo'lga kiritish uchun omadli bo'lganlar orasida yangi kvartirani orzu qilganlar 18 kishi, qimmatbaho mashina - 14, yaxshi ish - 28, kvartira va avtomobil - 5, kvartira va ish - 10, mashina va ish - 8 ta, har uchala tilaklar 3 kishi. Oltin baliqni qancha odam ushladi? Ulardan qanchasi bitta tilakni amalga oshirdi?

1.1.37. Chang'i, xokkey va konkida uchish sektsiyalarida 38 talaba qatnashadi. Ma'lumki, chang'i sektsiyasida 21 talaba shug'ullanadi, shulardan 3 talaba konkida uchish bilan shug'ullangan, 6 talaba xokkey sektsiyasida va bitta talaba bir vaqtning o'zida barcha uch seksiyalar bilan shug'ullangan. Konkida uchish bo'yicha 13 talaba tahsil oldi, shundan 5 talaba bir vaqtning o'zida ikkita sektsiyada tahsil olishdi. Xokkey bo'limida nechta talaba qatnashdi?

1.1.38. O'qituvchi guruhdagi 40 talabadan qaysi biri A, B va S kitoblarini o'qishini aniqlashga qaror qildi: so'rov natijalari quyidagicha: A kitobi 25 talaba tomonidan o'qilgan, B kitobni - 22, shuningdek S kitobni ham - 22. 33ta talabalar tomonidan A yoki B kitob o'qilgan. A yoki C - 32, B yoki C - 31; har uchala kitobni 10 talaba o'qidi. Qancha talabalar faqat bitta kitob o'qiydi? Qancha talabalar ushbu uchta kitobdan birini o'qimadilar?

1.2. Binar munosabatlar.

A^g to'plamning ixtiyoriy *R* qism to'plami *A* to'plamda **binar munosabat** deyiladi. Agar *(x, y)* ∈ *R* bo'lsa, u holda *x* element *u* element bilan *R* binar munosabatda deyiladi va *xR y* kabi yoziladi.

Matematikadagi muhim binar munosabatlar uchun ayrim belgilar kiritilgan.

Misollar: I) *R* haqiqiy sonlar to'plamida *x* va y sonlarning tenglik munosabati. Uning belgisi *x* = y. Bu munosabat R^2 tekisliqsagi y = *x* to'g'ri chiziq nuqtalari bilan beriladi.

2) *R* haqiqiy sonlar to'plamida *x* va *y* sonlarning tengmaslik munosabati. Uning belgisi *x* ≠ y. Bu munosabat R^2 tekislikda y = *x* to'g'ri chiziqqa kirmagan barcha nuqtalardan iborat bo'lgan to'plam bilan beriladi.

3) *R* da y sonning *x* sondan katta ekanligi munosabati: belgisi *y* > *x* yoki *x* < *y*. Bu munosabat R^2 da y = *x t*o'g'ri chiziqdan yuqorida yotuvchi nuqtalar to'plami bilan beriladi;

4) *A* = *V* — to'plamlarning tenglik munosabati;

5) *A* ≠ *V* — to'ilamlarning tengmaslik munosabati;

6) *A* ⊆ *V* yoki *V* ⊇ *A* — qism to'plam munosabati;

7) *A*⊂ *V* yoki *V* ⊇ *A* — xos qism to'plam munosabati;

8) α || β— to'g'ri chiziqlarning parallellik munosabati;

9) a ⊥ β — to'g'ri chiziqlarning perpendikulyarlik munosabati;

10) *a* => β — bir tenglamalar tizimi ikkinchisiningnatijasi ekanligi;

11) *a* <=> β — ikkita tenglamalar tizimining teng kuchlilik munosabati.

Agar *A* to'plamda berilgan biror *R* munosabat shunday bo'lsaki, har qanday *a*∈ *A* uchun *aRa* o'rinli bo'lsa, u **refleksiv** munosabat deyiladi. Agar *aRb* munosabatdan *a*≠*b* munosabat kelib chiqsa, (ya'ni *aRa* munosabat hech qanday *a*∈ *A* element uchun bajarilmasa), bunday munosabat **antirefleksiv** deyiladi.

Agar *aRb* munosabatning bajarilishidan b*Ra* munosabatning ham bajarilishi kelib chiqsa, bunday munosabat *A* da **simmetriklik munosabati** deyiladi.

Agar *aRb* va b*Rs* munosabatlarning bajarilishidan *aRs* bajarilishi kelib chiqsa, bunday munosabat **tranzitivlik** deyiladi va *ko'pincha* $a \sim b$ *belgi ishlatiladi.*

Ekvivalentlikka misollar:

1) haqiqiy sonlarning tenglik munosabati;
2) to'plamlarning tenglik munosabati;
3) tenglamalar tizimlarining teng kuchlilik munosabati;
4) funksiyalarning tenglik munosabati.
5) Muhim misol. *A* to'plamda *N* o'zgartirishlar guruhi berilgan bo'lsin.

Bu *N* o'zgartirishlar guruhi yordamida *A* da ekvivalentlik tushunchasini kiritamiz.

Agar *A* to'plamning *a* va *b* elementlari uchun shunday $h \in H$ bieksiya mavjud bo'lsaki, $h(a) = b$ bo'lsa, bu elementlar *N* — zkvivalent deyiladi va $a \sim b$ ko'rinishda yoziladi.

Agar ixtiyoriy $a \in A$ ni olib, $h \in N$ sifatida e_A ni olsak (e_A — birlik aks ettirish o'zgartirishlar guruhining ta'rifidagi d2) shartga ko'ra H ga tegishli, $e_A(a) = a$,ya'ni har qanday $a \in A$ uchun $a \sim a$ (refleksivlik).

Endi $a \sim b$ bo'lsin. U holda shunday $h \in N$ mavjudki, $h(a) = b$. O'zgartirishlar guruhining ta'rifidagi d3) shartga ko'ra. $h^{-1} \in N$. U holda $h(a) = b$ tenglikka h^{-1} tatbiq qilsak, $h^{-1}(h(a)) = h^{-1}(b)$. Bundan $a = h^{1}(b)$, ya'ni $b \sim a$ (simmetriklik).

Agar $a \sim b$ va $b \sim s$ bo'lsa, shunday $h_1 \in N$ va $h_2 \in N$ bieksiyalar mavjudki, $h_1(a) = b$, $h_2(b) = s$. Bulardan $h_2(b) = h_2 (h_1(a) = s$, ya'ni $(h_2h_1 (a) = s$. O'zgartirishlar guruhining d1) shartiga ko'ra $h_2h_1 \in N$. Bundan va $(h_2h_1)(d) = s$ tenglikdan $a \sim c$ munosabatni olamiz (tranzitivlik).

Demak, $a \sim b$ $(N$ — ekvivalentlik) haqiqatan ham ekvivalentlik munosabati ekan.

Misollar. 1. $\{<2,4>, <5,6>, <7,6>, <8,8>\}\}$ tartiblangan juftliklar to'plami binar munosabatga misol bo'la oladi.

2.Agar ρ ayniyat munosabatini bildirsa, u vaqtda $<x,y>\in\rho$ degani $x\equiv y$ ni bildiradi.

3.Agar ρ onalik munosabatini bildirsa, u vaqtda <Xurshida, Iroda>$\in\rho$ simvol Xurshida Irodaning onasi ekanligini bildiradi.

Misol: $\{<2,4>, <3,3>, <6,7>\}$ ρ munosabat berilgan bo'lsin. U vaqtda $D_\rho=\{2,3,6\}$, $R_\rho=\{4,3,7\}$.

Biror C to'plam $<x,y>$ tartiblangan juftliklar to'plami bo'lsin. Agarda x biror X to'plamning elementi va y boshqa Y to'plamning elementi bo'lsa, u vaqtda C to'plam X va Y to'plamlarning to'g'ri (dekart) ko'paytmasidan tuzilgan to'plam deyiladi va $C=X\times Y=\{<x,y>/x\in X$ va $y\in Y\}$ shaklida belgilanadi.

Har bir ρ munosabat ayrim olingan $X\times Y$ to'g'ri ko'paytmaning qism to'plami bo'ladi va $X\supseteq D_\rho$, $Y\supseteq R_\rho$. Agar $\rho\subseteq X\times Y$ bo'lsa, u vaqtda ρ X dan Y ga bo'lgan munosabat deb aytiladi. Agar $\rho\subseteq X\times Y$ va $Z\supseteq X\cup Y$ bo'lsa, u vaqtda ρ dan Z ga bo'lgan munosabat deb aytiladi. Z dan Z ga bo'lgan munosabatni Z ichidagi munosabat deb aytiladi.

X qandaydir to'plam bo'lsin. U vaqtda X ichidagi $X\times X$ munosabatni X ichidagi universal munosabat deb aytiladi.

$\{<x,x>/x\in X\}$ munosabat X ichidagi ayniyat munosabati deb aytiladi va i_X yoki i simvoli bilan belgilanadi. Har qanday X to'plamining x va y elementlari uchun x i_X y ifoda $x=y$ bilan teng kuchlidir.

A to'plam va ρ munosabat berilgan bo'lsin. U vaqtda $\rho[A]=\{y/A$ ning ayrim x lari uchun $x\,\rho\,y\}$. Bu to'plamga A to'plam elementlarining ρ - obrazlari to'plami deb aytiladi.

Misollar. $y=2x+1$ to'g'ri chiziqni $\{<x,y>\in R\times R/\ y=2x+1\}$ va $y<x$ munosabatini $\{<x,y>\in R\times R/\ y<x\}$ shakllarda yozish mumkin.

Muammoli masala va topshiriqlar:

1.2.1. $A=\{1, 2, 3\}$ va $B=\{a, b\}$ berilgan bo'lsa, $A\times B$, $A\times A$, $B\times A$, $B\times B$, $A\times\varnothing$, 2^A, 2^B, $B\times 2^B$ ni toping.

1.2.2. $A=\varnothing$, $B=\{\varnothing\}$ и $C=\{\varnothing,\{\varnothing\}\}$. $A\times B$, $A\times C$, $B\times C$,$B\times B$, $C\times C$, 2^A, 2^B, 2^C, $B\times 2^B$, $B\times 2^C$,$C\times 2^B$.

1.2.3. Ixtiyoriy A, B, C, D to'plamlar uchun quyidagi tenglilarning qaysi o'rinli:

1) $A\times B=B\times A$;
2) $(A\cup B)\times C=(A\times C)\cup(B\times C)$;
3) $(A\cap B)\times(C\cap D)=(A\times C)\cap(B\times D)$;
4) $(A\cup B)\times(C\cup D)=(A\times C)\cup(B\times D)$;
5) $(A-B)\times C=(A\times C)-(B\times C)$?

1.2.4. {1, 2, 3, 4, 5} to'plamlar uchun munosabatning qaysi xossalari o'rinli bo'lishini aniqlang:

R_1: $aR_1b \leftrightarrow |a-b|=1$;
R_2: $aR_2b \leftrightarrow 0<a-b<3$;
R_3: $aR_3b \leftrightarrow a+b$ – juft sonlar;
R_4: $aR_4b \leftrightarrow a\geq b^2$;
R_5: $aR_5b \leftrightarrow EKUB\,(a, b)=1$.

1.2.5. Quyidagi tasdiqlarning qaysi biri to'g'riligini aniqlang:

1) to'plamdagi barcha munosabatlar yo simmetrik, yoki antisimmetrikdir;

2) hech bir munosabat bir vaqtda ham simmetrik, ham antisimmetrik bo'la olmaydi;

3) ixtiyoriy R munosabat uchun $R \cup R^{-1}$ vaи $R \cap R^{-1}$ munosabat simmetrik bo'ladi;

4) ixtiyoriy R munosabat uchun $R-(R \cap R^{-1})$ munosabat antisimmetrik bo'ladi;

5) agar R munosabatda xossalardan birini qanoatlantirsa, u holda R^{-1} munosabatda ham shu xossani qanoatlantiradi;

6) ixtiyoriy R munosabat uchun $R \circ R^{-1}$ munosabat simmetrik bo'ladi;

7) ixtiyoriy R munosabat uchun $R \circ R^{-1}$ munosabat refleksiv bo'ladi;

8) Agar R_1 va R_2 ikkala munosabatda xossalardan birini qanoatlantirsa, u holda $R_1 \circ R_2$ munosabat ham shu xossani qanoatlantiradi.

1.2.6. Quyidagi tasdiqlarning qaysi biri to'g'riligini aniqlang:

1) Agar R_1 va R_2 ekvivalentlik munosabati bo'lsa, unda $R_1 \circ R_2$ ekvivalentlik munosabati bo'ladi;

2) Agar R_1 va R_2 ekvivalentlik munosabati bo'lsa, unda $R_1 \cap R_2$ ekvivalentlik munosabati bo'ladi;

3) Agar R_1 va R_2 ekvivalentlik munosabati bo'lsa, unda $R_1 \cup R_2$ ekvivalentlik munosabati bo'ladi;

1.2.7. $\{a, b\}$ ikki elementli to'plamda barcha munosabatlarni toping va ular orasida

1) barcha refleksiv;
2) barcha simmetrik;
3) barcha antisimmetrik;
4) barcha tranzitiv;
5) barcha ekvivalent;
6) barcha tartib munosabatlarni ko'sating.

1.2.8. $\{0,1,...,9\}$ to'plamda berilgan quyidagi munosabatlardan qaysi ekvivalentlik munosabati bo'lishini aniqlang;

$R_1: \ aR_1b \ \leftrightarrow \ a \equiv b (\text{mod } 3)$;

$R_2: \ aR_2b \ \leftrightarrow \ a^2 \equiv b^2 (\text{mod}10)$;

$R_3: \ aR_3b \ \leftrightarrow \ ab \equiv 0 \ (\text{mod } 2)$;

$R_4: \ aR_4b \ \leftrightarrow \ |2^a - 2^b| < 16$;

$R_5: \ aR_5b \ \leftrightarrow \ |2^a - 2^b| \leq 16$;

$R_6: \ aR_6b \ \leftrightarrow EKUB(a,b) = 1.$

1.2.9. Z - butun sonlar to'plamida $R : xR\,y \leftrightarrow x = y^2$ munosabat aniqlangan. $R \circ R^{-1}$, $R^{-1} \circ R$ munosabatlardan birortasi ekvivalentlik munosabati bo'ladimi?

1.2.10. Quyidagi munobatlardan qaysilari Z^2 da ekvivalentlik munosabati bo'ladi?

$R_1 : (x_1, y_1) R_1 (x_2, y_2) \leftrightarrow x_1 = x_2$;

$R_2 : (x_1, y_1) R_2 (x_2, y_2) \leftrightarrow x_1 = x_2$ yoki $y_1 = y_2$;

$R_3 : (x_1, y_1) R_3 (x_2, y_2) \leftrightarrow x_1 + y_1 = x_2 + y_2$;

$R_4 : (x_1, y_1) R_4 (x_2, y_2) \leftrightarrow x_1 + y_2 = y_1 + x_2$;

$R_5 : (x_1, y_1) R_5 (x_2, y_2) \leftrightarrow x_1 < x_2$ yoki $x_1 = x_2 , y_1 \leq y_2$?

1.2.11. n (n =1, 2, 3, 4) ta elementli to'plamda nechta turli ekvivalentlik munosabatlarini aniqlash mumkin?

1.2.12. U universal to'plaam berilgan. 2^U da quyidagi munosabatlardan qaysilari ekvivalentlik yoki tartib munosabati bo'ladi:

1) $AR_1B \ \leftrightarrow \ A \cap B = \varnothing$;

2) $AR_2B \leftrightarrow A-B=\varnothing$;

3) $AR_3B \leftrightarrow A-B=B-A$;

4) $AR_4B \leftrightarrow |A|=|B|$?

1.2.13. Uchta elementli to'plamda nechta turli tartib munosabatlarini aniqlash mumkin? Ular orasida chiziqlilari nechta?

1.2.14. $y=2x+1$ to'g'ri chiziqni $\{<x,y>\in R\times R/\ y=2x+1\}$ va $y<x$ munosabatini $\{<x,y>\in R\times R/\ y<x\}$ shakllarda yozish mumkinligini tushuntiring.

1.2.15. $\{<2,4>,\ <5,6>,\ <7,6>,\ <8,8>\}$ tartilangan juftliklar to'plami binar munosabat bo'la oladimi?

1.2.16. A-tekislikdagi barcha to‘g‘ri chiziqlar to‘plami bo‘lsin. Ixtiyoriy a,b tug‘ri chiziqlar uchun (atb)=(a\\b) bo‘lsa, A dagi: Parallelik munosabati: refleksiv bo‘ladimi?

1.2.17. A-tekislikdagi barcha to‘g‘ri chiziqlar to‘plami bo‘lsin. Ixtiyoriy a,b tug‘ri chiziqlar uchun (atb)=(a\\b) bo‘lsa, A dagi: Parallelik munosabati: simmetrik bo‘ladimi?

1.2.18. A-tekislikdagi barcha to‘g‘ri chiziqlar to‘plami bo‘lsin. Ixtiyoriy a,b tug‘ri chiziqlar uchun (atb)=(a\\b) bo‘lsa, A dagi: Parallelik munosabati: tranzitiv bo‘ladimi?

1.2.19. A-tekislikdagi barcha to‘g‘ri chiziqlar to‘plami bo‘lsin. Ixtiyoriy a,b tug‘ri chiziqlar uchun $a\tau b=a\perp b$ bo‘lsa, τ perpendikulyarlik munosabati: antirefleksiv bo‘ladimi?

1.2.20. A-tekislikdagi barcha to‘g‘ri chiziqlar to‘plami bo‘lsin. Ixtiyoriy a,b tug‘ri chiziqlar uchun $a\tau b=a\perp b$ bo‘lsa, τ perpendikulyarlik munosabati: simmetrik bo‘ladimi?

1.2.21. M_3 to‘plamda $\tau=\{<1,2>, <2;2>, <1;3>\}$ va $\sigma=\{<1;1>, <2;2>, <3;1>\}$ binar munosabatlar aniqlangan bo‘lsin, u xolda $\tau\cdot\sigma$ ni hisoblang?

1.2.22. N to'plamda aniqlangan quyidagi binar munosabatlar qanday xossaga ega ekanligini aniqlang, ularni aniqlanish va o'zgarish sohalarini toping: $\tau = \{<1;1>, <2;2>\} \subset N^2$.

1.2.23. N to'plamda aniqlangan quyidagi binar munosabatlar qanday xossaga ega ekanligini aniqlang, ularni aniqlanish va o'zgarish sohalarini toping: $\tau = \{<1;5>\} \subset N^2$.

1.2.24. N to'plamda aniqlangan quyidagi binar munosabatlar qanday xossaga ega ekanligini aniqlang, ularni aniqlanish va o'zgarish sohalarini toping: $\tau = \{<1;2>, <2;1>, <1;1>, <2;2>, <3;5>, <5;3>, <3;3>, <5;5>\}$

1.2.25. N to'plamda aniqlangan quyidagi binar munosabatlar qanday xossaga ega ekanligini aniqlang, ularni aniqlanish va o'zgarish sohalarini toping: $\tau = \{<1;3>, <3;1>, <4;5>, <5;4>\} \subset N^2$.

1.2.26. N to'plamda aniqlangan quyidagi binar munosabatlar qanday xossaga ega ekanligini aniqlang, ularni aniqlanish va o'zgarish sohalarini toping: $\forall (a,b \in N)$, $a\tau b \Leftrightarrow b<2a$.

1.2.27. N to'plamda aniqlangan quyidagi binar munosabatlar qanday xossaga ega ekanligini aniqlang, ularni aniqlanish va o'zgarish sohalarini toping: $\forall (a,b \in N)$, $a\tau b \Leftrightarrow a=b^2$.

1.2.28. N to'plamda aniqlangan quyidagi binar munosabatlar qanday xossaga ega ekanligini aniqlang, ularni aniqlanish va o'zgarish sohalarini toping: $\forall (a,b \in N)$, $a\tau b \Leftrightarrow a<b$.

1.2.29. N to'plamda aniqlangan quyidagi binar munosabatlar qanday xossaga ega ekanligini aniqlang, ularni aniqlanish va o'zgarish sohalarini toping: $\forall (a,b \in N)$, $a\tau b \Leftrightarrow a-b=12$.

1.2.30. N to'plamda aniqlangan quyidagi binar munosabatlar qanday xossaga ega ekanligini aniqlang, ularni aniqlanish va o'zgarish sohalarini toping: $\forall (a,b \in N)$, $a\tau b \Leftrightarrow |a-b|=12$.

1.2.31. N to‘plamda aniqlangan quyidagi binar munosabatlar qanday xossaga ega ekanligini aniqlang, ularni aniqlanish va o‘zgarish sohalarini toping: $\forall (a,b \in N)$, $a\tau b \Leftrightarrow (a-b):10$.

1.2.32. $M_n = \{\{1,2,\ldots,n\} \subset N\}$ to‘plamda bir vaqtda refleksiv va antirefleksif bo‘lmagan binar munosabatlar mavjudmi?

1.2.33. M_1, M_2, M_3 va $M_n = \{\{1,2,\ldots,n\} \subset N\}$ to‘plamlarning har birida nechtadan binar munosabat aniqlash mumkin?

1.2.34. M_{10} **(**$M_n = \{\{1,2,\ldots,n\} \subset N\}$**)**to‘plamda $\forall(a,b \in M10)$ $a\tau b \Leftrightarrow a-b=8$; aniqlangan binar munosabatlarni aniqlanish va o‘zgarish sohalarini toping. Ularning har biri qanday xossalarga ega ekanlngini toping.

1.2.35. M_{10} **(**$M_n = \{\{1,2,\ldots,n\} \subset N\}$**)**to‘plamda $\forall(a,b \in M10)$ $a\tau b \Leftrightarrow b=a^2$ aniqlangan binar munosabatlarni aniqlanish va o‘zgarish sohalarini toping. Ularning har biri qanday xossalarga ega ekanlngini toping.

1.2.36. M_{10} **(**$M_n = \{\{1,2,\ldots,n\} \subset N\}$**)**to‘plamda $\forall(a,b \in M10)$ $a\tau b \Leftrightarrow a-b-12$; aniqlangan binar munosabatlarni aniqlanish va o‘zgarish sohalarini toping. Ularning har biri qanday xossalarga ega ekanlngini toping.

1.2.37. M_{10} **(**$M_n = \{\{1,2,\ldots,n\} \subset N\}$**)**to‘plamda $\forall(a,b \in M10)$ $a\tau b \Leftrightarrow b>a^2$ aniqlangan binar munosabatlarni aniqlanish va o‘zgarish sohalarini toping. Ularning har biri qanday xossalarga ega ekanlngini toping.

1.2.38. M_3 to‘plamda aniqlangan $\tau=\{<1;1>,<2;1>,<2;3>,<3;3>\}$ munosabatga teskari τ^{-1} munosabatlarni toping.

1.2.39. M_3 to‘plamda aniqlangan $\sigma=\{<1;1>,<2;1>,<1;2>,<2;2>\}$ munosabatga teskari σ^{-1} munosabatlarni toping.

1.2.40. M_3 to‘plamda aniqlangan $\rho=\{<1;2>,<1;3>,<2;3>\}$ munosabatga teskari ρ^{-1} munosabatlarni toping.

1.2.41. M_3 to‘plamda aniqlangan $\tau=\{<1;1>,<2;3>,<1;2>\}$ va $\sigma=\{<1;1>,<1;3>,<3;3>\}$ binar munosabatlarni ko‘paytmasini toping.

1.2.42. M_4 to‘plamda aniqlangan $\tau=\{<1;1>,<2;3>\}$, $\sigma=\{<1;2>,<2;3>,<3;4>\}$ binar munosabatlarni ko‘paytmasini $\tau\sigma$ toping.

1.2.43. M_4 to‘plamda aniqlangan $\tau=\{<1;1>,<2;3>\}$, $\sigma=\{<1;2>,<2;3>,<3;4>\}$ binar munosabatlarni ko‘paytmasini $\sigma\tau$ toping.

1.2.44. M_4 to‘plamda aniqlangan $\tau=\{<1;1>,<2;3>\}$ binar munosabat uchun τ^2 toping.

1.2.45. M_4 to‘plamda aniqlangan $\sigma=\{<1;2>,<2;3>,<3;4>\}$ binar munosabat uchun τ^2 toping.

2-BOB. MULOHAZALAR ALGEBRASI.

2.1. Mulohazalar va ular ustida amallar. Formula, qism formulalar. Chinlik jadvali.

O’zgaruvchilar soni n ta bo’lsa, u vaqtda $C_n^0+C_n^1+C_n^2+...+C_n^n=2^n$ ta qiymatlar satriga ega bo’lamiz.

1. **Inkor amali.** $\overline{x}$ mulohaza “x emas” deb o’qiladi. x mulohazaning inkori deb atalgan $\overline{x}$ mulohaza shu bilan xarakterlanadiki, x mulohaza “**ch**” qiymatni qabul qilganda, $\overline{x}$ mulohaza “**yo**” qiymatni qabul qiladi va aksincha.
2. **Kon’yunkstiya (mantiqiy ko’paytma) amali.** x va y o’zgaruvchi mulohazalar ustida bajariladigan kon’yunkstiya (lotincha conjunctio - bog’layman so’zidan) $x\wedge y$ yoki $x\,\&\,y$ ko’rinishda belgilanadi. “Va” bog’lovchisiga mos keluvchi mantiqiy amalga kon’yunkstiya amali deb aytamiz.

   $x\wedge y$ ko’rinishdagi mulohaza «x va y» deb o’qiladi.
3. **Diz’yunkstiya (mantiqiy yig’indi) amali.** Rad etmaydigan ma’noda ishlatiladigan “yoki” mantiqiy amal diz’yunkstiya (lotincha disjunctio - farq qilaman so’zidan) deyiladi. Ikkita x va y mulohazaning diz’yunkstiyasi “$x\vee y$” kabi yoziladi va “x yoki y” deb o’qiladi.
4. **implikastiya** (lotincha implicatio - zich bog’layman so’zidan). Implikastiya amalini $\rightarrow$ ko’rinishida belgilaymiz.“$x\rightarrow y$” mulohaza “agar x, u holda y” deb o’qiladi.
5. **Ekvivalentlik (tengkuchlilik) amali.** Ko’p murakkab mulohazalar

elementar mulohazalardan “zarur va kifoya”, “faqat va faqat”, “shunda va faqat shundagina, qachonki”, “.......bajarilishi etarli va zarurdir” kabi bog’lovchilari yordamida tuziladi. “$\leftrightarrow$“ kabi belgilanadi. $x \leftrightarrow y$ murakkab mulohaza “**x** ekvivalent **y**” deb o’qiladi.

6. **Sheffer amali (shtrixi).** “ | “ kabi belgilanadi. Murakkab mulohaza “$x|y$” “x Sheffer shtrixi y ” deb o’qiladi.

7. **Pirs strelkasi “$\downarrow$”**kabi belgilanadi. Murakkab mulohaza “$x \downarrow y$” “x Pirs strelkasi y ” deb o’qiladi.

x	y	$x \wedge y$	$x \vee y$	$x \to y$	$x \leftrightarrow y$	$x \mid y$	$x \downarrow y$
ch	ch	ch	ch	ch	ch	yo	yo
ch	yo	yo	ch	yo	yo	ch	yo
yo	ch	yo	ch	ch	yo	ch	yo
yo	yo	yo	yo	ch	ch	ch	ch

Jadvalni barcha mantiqiy amalining ta’rifi sifatida qabul qilamiz va u **chinlik jadvali** deyiladi.

Propozisional o’zgaruvchilar bu, konkret mulohaza qo’yib bo’ladigan o’zgaruvchilardir: $x_1, x_2, x_3, ..., x_n$, A,B,C,…,X,Y,Z. Odatda formula tushunchasi quyidagicha kiritiladi:

1) har qanday $x_1, x_2, x_3, ..., x_n$ mulohazalarning istalgan biri formuladir;

2) agar A va B larning har biri formula bo’lsa, u holda $(A \wedge B)$, $(A \vee B)$, $(A \to B)$, $(A \leftrightarrow B)$ va $\overline{A}$ lar ham formulalardir.

3) 1 va 2-bandlarda ko’rsatilgan ifodalardan tashqari boshqa hech qanday ifoda ***formula*** bo’la olmaydi.

Mulohazalarni inkor, diz’yunkstiya, kon’yunkstiya, implikastiya va ekvivalenstiya mantiqiy amallar vositasi bilan ma’lum tartibda birlashtirib hosil etilgan murakkab mulohazaga **formula** deb aytamiz.

Masalan: $([x_1 \vee (x_2 \wedge x_3)] \to x)$; $([x_1 \wedge (x_2 \to x_3)] \vee (x_4 \leftrightarrow x_5))$;

$((x \leftrightarrow y) \wedge (x \vee y))$; $((x \to y) \wedge (x \to z) \to (z \to x))$ murakkab mulohazalar formulalar bo'ladi. Formula tashqaridan qavsga olinadi va qavslar mulohazalar ustida mantiqiy amallarning qay tartibda bajarilishini ko'rsatadi.

Formulaning qism formulalari deganda, berilgan formulaning formula bo'la oladigan barcha qismlariga aytiladi.

Misol: $A = ((\overline{x} \to y) \wedge \overline{\kappa})$ formulaning chinlik jadvali quyidagicha tuziladi:

x	y	κ	$\overline{x}$	$\overline{x} \to y$	$\overline{\kappa}$	A
0	0	0	1	0	1	0
0	0	1	1	0	0	0
0	1	0	1	1	1	1
0	1	1	1	1	0	0
1	0	0	0	1	1	1
1	0	1	0	1	0	0
1	1	0	0	1	1	1
1	1	1	0	1	0	0

Muammoli masala va topshiriqlar:

2.1.1. Quyidagilarning qaysi biri mulohaza bo'ladi:

1) Toshkent – O'zbekiston Respublikasining poytaxti;

2) $\sqrt{5} + 4\sqrt{3 - 30}$;

3) Oy Mars planetasining yo'ldoshi;

4) $a > 0$;

5) 2+2=5

6) Matematika fakul'teti talabalariga shon sharaflar bo'lsin!

7) Qizil atirgul.

8) x+3=7.

9) Soat nechada?

10) Muntazam uchburchak deyiladi, agar tomonlari teng bo'lsa.

11) Atirgul – eng chiroyli gul.

12) Ixtiyoriy a va b uchun, a + b = b + a tenglik o'rinli bo'ladi.

13) 28 soni 7 ga qoldiqsiz bo'linadi.

2.1.2. Quyidagi mulohazalarning chin yoki yolg'on ekanligini aniqlang:

1) $2 \in \{x \mid 2x^3 - 3x^2 + 1 = 0, \quad x \in R\}$; 2) $\{1\} \in N$;

2.1.3. Quyidagi implikastiyalarning qaysi birlari chin bo'ladi:

1) agar $2 \times 2 = 4$ bo'lsa, u holda $2 < 3$;

2) agar $2 \times 2 = 4$ bo'lsa, u holda $2 > 3$;

2.1.4. Quyidagi mulohazalarning inkorini tuzung:

1) 33 soni 7 ga qoldiqsiz bo'linadi.

2) $255 \prec 258$.

3) ΔABC - to'g'ri burchakli uchburchak..

4) Stol – oq.

5) Barcha tub sonlar, toq son bo'ladi.

6) ABCD to'rtburchak rombdir.

2.1.5. Quyidagi simvollar ketma-ketligi formula bo'la oladimi:

1) (PQ);

2) $((P \leftrightarrow Q) \wedge R) \rightarrow (P \vee Q)$;

3) $((\neg P \rightarrow Q) \rightarrow (R \wedge (Q \wedge S)))$;

4) $((\neg P \rightarrow Q) \neg (R \leftrightarrow P))$;

5) $(P \rightarrow ((Q \vee R) \rightarrow \neg P))$;

6) $\neg((\neg P \wedge \neg Q) \rightarrow (P \vee (R \wedge \neg S)))$;

7) $((P \vee \neg Q) \rightarrow (\neg \mathrm{P} \wedge \neg \mathrm{R} \wedge (\mathrm{Q} \leftrightarrow \mathrm{R})))$;

8) $P \rightarrow Q \rightarrow R$;

9) $\neg\neg P \rightarrow P$;

10) $((P \to Q) \to ((Q \wedge S)$;

11) $(((P \leftrightarrow Q)R) \to (P \vee Q))$;

12) $((P \wedge (\neg Q \to R)) \vee ((\neg P \leftrightarrow \mathrm{R}) \vee Q))$.

Yechim: 11) $(((P \leftrightarrow Q)R) \to (P \vee Q))$ ketma-ketlik formula bo'la olmaydi. Haqiqatdan ham ta'rifning 1) bandidan P,Q,R propozisional o'zgaruvchilar formula, ta'rifning 2) bandidan $(P \leftrightarrow Q)$, $(P \vee Q)$ formula ammo, $((P \leftrightarrow Q)R)$ formula emas, chunki $(P \leftrightarrow Q)$ va R formulalar biror mantiqiy amal vositasida bog'lanmagan.

2.1.6. Quyida berilgan simvollar ketma-ketligida turli xil usullar bilan qavslarni joylashtirib, hosil bo'lgan barcha formulalarni yozing:

1) $P \to Q \wedge \neg R \vee S$;

2) $P \to \neg Q \vee R \to \neg \mathrm{P} \to \neg \mathrm{R}$;

3) $\neg P \wedge Q \to R$;

4) $P \vee \neg Q \to \neg R \wedge Q$;

5) $\neg P \vee R \wedge Q \vee R$;

6) $\neg P \vee R \vee P \to Q$;

7) $P \leftrightarrow Q \wedge \neg R \to S$;

8) $P \vee Q \wedge \neg R \wedge P \vee R$;

9) $\neg P \wedge Q \vee R \to Q$;

10) $\neg P \wedge Q \to \neg P \vee R$;

11) $\neg P \leftrightarrow \neg Q \vee R \wedge Q$.

Yechim: 11) Soddalik uchun tashqi qavslarni tashlab yuboramiz va quyidagilarga ega bo'lamiz:

$(\neg P \leftrightarrow \neg Q) \vee (R \wedge Q)$;

$(\neg P \leftrightarrow (\neg Q \vee R)) \wedge Q$;

$\neg(P \leftrightarrow (\neg Q \vee R)) \wedge Q$;

$(\neg P \leftrightarrow \neg(Q \vee R)) \wedge Q$;

$\neg(P \leftrightarrow \neg(Q \vee R)) \wedge Q$;

$\neg((P \leftrightarrow \neg Q) \vee R) \wedge Q$;

$\neg P \leftrightarrow \neg(Q \vee (R \wedge Q))$;

$\neg P \leftrightarrow (\neg Q \vee (R \wedge Q))$;

$\neg P \leftrightarrow \neg((Q \vee R) \wedge Q)$;

$\neg P \leftrightarrow ((\neg Q \vee R) \wedge Q)$;

$\neg(P \leftrightarrow (\neg Q \vee R) \wedge Q)$;

$\neg(P \leftrightarrow ((\neg Q \vee R) \wedge Q))$;

$(\neg(P\leftrightarrow\neg Q)\vee R)\wedge Q$; $\neg((P\leftrightarrow(\neg Q\vee R))\wedge Q)$;

$\neg(P\leftrightarrow\neg(Q\vee(R\wedge Q)))$; $\neg P\leftrightarrow(\neg(Q\vee R)\wedge Q)$;

$\neg(P\leftrightarrow(\neg Q\vee(R\wedge Q)))$.

2.1.7. Quyidagi formulalarning barcha qism formulalarini yozing (tashqi qavslar tashlab yuborilgan):

1) $((P\vee Q)\vee\neg R)\wedge(\neg P\vee(\neg Q\vee R))$;

2) $(P\to Q)\to((P\to\neg Q)\to(P\wedge Q))$;

3) $(P\wedge(Q\vee\neg P))\wedge((\neg Q\to P)\vee Q)$;

4) $\neg(P\vee Q)\wedge((\neg P\leftrightarrow Q)\to(P\vee Q))$;

5) $((P\vee\neg Q)\leftrightarrow R)\vee(P\vee\neg(Q\wedge R))$;

6) $(P\leftrightarrow Q)\vee(\neg R\wedge(P\to(Q\vee R))\wedge(\neg P\vee(\neg Q\vee R)))$;

7) $((P\vee Q)\to(R\to\neg P))\to(\neg R\to\neg Q)$;

8) $(P\to(Q\to R))\to((P\to\neg R)\to(P\to\neg Q))$;

9) $P\vee(Q\to(R\leftrightarrow(P\wedge Q)))$;

10) $P\wedge((Q\wedge R)\vee S))\vee\neg S$;

11) $((P\leftrightarrow(Q\wedge\neg R))\vee(P\wedge Q))\to(P\vee(\neg Q\wedge R))$.

Yechim: 11) $((P\leftrightarrow(Q\wedge\neg R))\vee(P\wedge Q))\to(P\vee(\neg Q\wedge R))$ formulaning barcha qism formulalarini topish uchun quyidagich tartiblaymiz:

1) P, Q, R - propozisional o’zgaruvchilar;

2) $\neg R$, $(P\wedge Q)$, $\neg Q$ - birta mantiqiy bog’lovchi (amal) yordamida bog’langan formulalar;

3) $(Q\wedge\neg R)$, $(\neg Q\wedge R))$ - ikkita mantiqiy amal yordamida bog’langan formulalar;

4) $(P\leftrightarrow(Q\wedge\neg R))$, $(P\vee(\neg Q\wedge R))$ - uchta mantiqiy amal yordamida bog’langan formulalar;

To’rtta mantiqiy amal yordamida bog’langan formula yo’q;

5) $(P \leftrightarrow (Q \wedge \neg R)) \vee (P \wedge Q)$ - beshta mantiqiy amal yordamida bog’langan formulalar;

6) Nihoyat formulaning o’zi $((P \leftrightarrow (Q \wedge \neg R)) \vee (P \wedge Q)) \to (P \vee (\neg Q \wedge R))$.

Shunday qilib berilgan formulaning 12 ta qism formulalari mavjud ekan.

Eslatma: *Agar formulaning qism formulalarini topish talab qilinsa, yuqoridaqi usul yordamida bajariladi. Agar qism formulalari sonini topish talab qilinsa, berilgan formulaning propozisional o’zgaruvchilar soniga formuladagi barcha amallar soni qo’shiladi.*

Masalan: 11) 3 ta P, Q, R - propozisional o’zgaruvchilar va 9 ta amal $((P \overset{1}{\leftrightarrow} (Q \overset{2}{\wedge} \overset{3}{\neg} R)) \overset{4}{\vee} (P \overset{5}{\wedge} Q)) \overset{6}{\to} (P \overset{7}{\vee} (\overset{8}{\neg} Q \overset{9}{\wedge} R))$ Berilgan formulaning 12 ta qism formulalari mavjud.

2.1.8. Quyidagi formulalarning chinlik jadvallarini tuzing:

1) $(x\&y) \vee z$;
2) $x\&y \to (\bar{y} \vee x \to z)$;
3) $(x \to y) \to (x \vee y\&z)$;
4) $(x \vee z)\&(\bar{y} \to (u \to \bar{x}))$;
5) $(x\&y) \to x$;
6) $x \to (x \vee y)$;
7) $(x \to y) \to (\bar{y} \to \bar{x})$;
8) $(x \to y)\&(x \to \bar{y}) \to \bar{x}$;
9) $(x \leftrightarrow y)\&(x \vee y)$;
10) $(x \to y)\&(y \to z) \to (z \to x)$;
11) $(x \to y)\&(y \to z) \to (x \to z)$;
12) $(y \leftrightarrow z)\&(x \vee z)$;
13) $z\&y \to (y \vee z \to x)$;

14) $(A \vee \neg B) \wedge \neg C$;

15) $((\neg A \wedge B) \vee \neg C)$;

16) $((\neg A \wedge B) \vee (\neg C \wedge \neg B))$;

17) $((\neg A \wedge B) \to (B \vee C))$;

18) $(((A \to B) \to A) \to (A \to (B \wedge A)))$;

19) $((C \to A) \to (\neg(B \vee C) \to A))$;

20) $(\bar{x} \vee z) \wedge (y \to (u \to x))$;

21) $([x_1 \wedge (x_2 \to x_3)] \vee (x_4 \leftrightarrow x_5))$

22) $x_1 \to (x_2 \to (\ldots \to x_n)\ldots)$;

23) $x_1 \vee x_2 \vee \ldots x_n \to y_1 \wedge y_2 \wedge \ldots \wedge y_n$.

Eslatma: Berilgan formulaning chinlik jadvalini tuzishdan oldin satr va ustunlar sonini aniqlash kerek bo'ladi. Bilamizki, satrlar soni qism formula- 1ta va o'zgaruvchilar soni n ta berilganda 2^n ta satr qo'shiladi, jami; 2^n+1. Endi ustunlar soni qism formulalar soniga teng ya'ni o'zgaruvchilar soniga formulada qatnashgan mantiqiy amallar soni qo'shiladi.

2.1.9. Chinlik jadvalini tuzib formulalarni aynan chin, aynan yolg'on yoki bajariluvchi ekanligini aniqlang:

1) $(P \to Q) \to ((P \to Q) \to \neg P)$

2) $((P \to Q) \to P) \to Q$;

3) $(P \wedge (Q \vee \neg P)) \wedge ((\neg Q \to P) \vee Q)$;

4) $(P \leftrightarrow Q) \to (\neg P \vee Q)$;

5) $P \wedge (Q \wedge (\neg P \vee \neg Q))$;

6) $(((P \vee \neg Q) \wedge (Q \vee R)) \vee \neg R) \vee Q$;

7) $(P \wedge (Q \vee R)) \to ((R \to (P \to Q)) \leftrightarrow (Q \to (R \to P)))$;

8) $(((P \leftrightarrow Q) \leftrightarrow (P \leftrightarrow R)) \leftrightarrow (Q \leftrightarrow R)) \leftrightarrow P$;

9) $\neg((\neg R \to \neg(P \to \neg(Q \to R))) \to \neg(P \to \neg Q))$;

10) $(((P \to Q) \to Q) \to Q) \to Q$;

11) $((P \vee \neg Q) \to Q) \wedge (\neg P \vee Q)$.

Yechim: 11) $((P \vee \neg Q) \to Q) \wedge (\neg P \vee Q)$ formulani *F(P,Q)* bilan belgilab, chinlik jadvalini tuzamiz:

P	Q	$\neg Q$	$P \vee \neg Q$	$(P \vee \neg Q) \to Q$	$\neg P$	$\neg P \vee Q$	*F(P,Q)*
0	0	1	1	0	1	1	0
0	1	0	0	1	1	1	1
1	0	1	1	0	0	0	0
1	1	0	1	1	0	1	1

Chinlik jadvalidan ko'rinib turibdiki, *F(P,Q)* formula bajariluvchi ekan.

2.1.10. Chinlik jadvalini tuzmasdan, quyidagi formulalar chin bo'ladigan propozisional o'zgaruvchilar qabul qiluvchi qiymatlarni toping.

1) $\neg(\mathrm{P} \to \neg \mathrm{P})$;

2) $(\mathrm{P} \to Q) \to (Q \to \mathrm{P})$;

3) $(Q \to (\mathrm{P} \wedge \mathrm{R})) \wedge \neg((\mathrm{P} \vee \mathrm{R}) \to \mathrm{Q})$;

4) $\neg((\mathrm{P} \leftrightarrow \neg Q) \wedge R) \vee Q$;

5) $(((\mathrm{P} \to Q) \to (\mathrm{R} \to \mathrm{Q})) \to (\mathrm{R} \to \mathrm{P})) \to (\mathrm{P} \to \mathrm{Q})$;

6) $((\mathrm{Q} \to \neg P) \to P) \to (P \to (\neg P \to Q))$;

7) $(\mathrm{P} \to ((\mathrm{Q} \to \mathrm{R}) \to \mathrm{R})) \to ((\mathrm{P} \to (\mathrm{Q} \to \mathrm{R})) \to (\mathrm{P} \to \mathrm{R}))$;

8) $((\mathrm{P} \leftrightarrow \mathrm{Q}) \wedge (\mathrm{Q} \leftrightarrow \mathrm{R})) \to (\mathrm{R} \vee \mathrm{P})$;

9) $((\mathrm{P} \wedge \neg \mathrm{Q}) \vee (\neg \mathrm{P} \wedge \mathrm{Q})) \leftrightarrow (\mathrm{P} \leftrightarrow Q)$;

10) $((\mathrm{P} \wedge \mathrm{Q}) \to ((R \vee \mathrm{Q}) \to (\mathrm{Q} \wedge \neg \mathrm{Q}))$.

2.1.11. Chinlik jadvalini tuzmasdan, quyidagi formulalar yolg'on bo'ladigan propozisional o'zgaruvchilar qabul qiluvchi qiymatlarni toping.

1) $((X \to (Y \wedge Z)) \to (\neg Y \to \neg X)) \to \neg Y$;

2) $((X \vee Y) \vee Z) \to ((X \vee Y) \wedge (X \vee Z))$;

3) $((X \vee Y) \wedge ((Y \vee Z) \wedge (Z \vee X))) \to ((X \wedge Y) \wedge Z)$;

4) $(X \wedge Y) \vee (X \wedge Z) \vee (Y \wedge Z) \vee (U \wedge V) \wedge (\neg X \wedge \neg U)$

5) $((\neg P \to P) \to P) \to \neg((\neg Q \to \neg P) \to ((\neg Q \to P) \to Q))$;

6) $(((P \to Q) \to (R \to Q)) \to (P \to \neg Q))$;

7) $((Q \to \neg P) \to P) \to (P \to (P \to Q))$;

8) $(P \to ((Q \to R) \to R)) \to ((P \to (Q \to R)) \to (P \to \neg R))$;

9) $((P \to Q) \wedge (R \to \neg Q) \wedge (P \vee R)) \to R$;

10) $((P \wedge \neg Q) \wedge (Q \to R) \wedge (R \vee \neg S)) \to (S \wedge Q)$;

11) $(X \vee Y) \to ((\neg X \wedge Y) \vee (X \wedge \neg Y))$.

Yechim: 11) Implikasiya yolg'on qiymatni qabul qiladi, qachonki $X \vee Y$ chin bo'lib, $(\neg X \wedge Y) \vee (X \wedge \neg Y)$ yolig'on qiymatni qabul qilsa. Bundan 1 holda X yoki Y rost, 2 hol X va Y rost qiymatni qabul qilishi kerak. $(\neg X \wedge Y) \vee (X \wedge \neg Y)$ yolg'on qiymatni qabul qilishi uchun $(\neg X \wedge Y)$ va $(X \wedge \neg Y)$ bir vaqtda yolg'on bo'lishi kerak. Ko'rinib turibdiki, birinchi hol uchun $(\neg X \wedge Y)$ va $(X \wedge \neg Y)$ bir vaqtda yolg'on qiymat qabul qilmaydi. Demak, faqat va faqat X va Y rost qiymatni qabul qilgandagina $(X \vee Y) \to ((\neg X \wedge Y) \vee (X \wedge \neg Y))$ yolg'on qiymatni qabul qiladi. $F(1,1)=0$

2.1.12. A - "Bu raqam butun son", B - "bu raqam musbat son", C - "Bu tub son", D orqali - "Bu raqam 3 ga bo'linadi" degan mulohazani bildirsa, quyidagilarni o'qing:

1) $(A \vee B) \to \overline{C}$; 2) $(A \wedge C) \to D$;

3) $(A \wedge B) \to D$;

4) $(A \vee \overline{A}) \to (B \wedge C)$;

5) $(B \vee \overline{B}) \leftrightarrow (A \vee D)$;

6) $D \leftrightarrow (\overline{C} \wedge A)$;

7) $(A \wedge D) \to \overline{C}$;

8) $(A \vee B) \wedge (C \vee D)$;

9) $\overline{A} \vee \overline{D}$;

10) $(A \wedge B \wedge C) \vee D$;

2.1.13. Berilgan shartlardan foydalanib, oxirgi formulaning qiymatini toping:

1. $A \longrightarrow B = 1,\ A \leftrightarrow B = 0,\ B \longrightarrow A =$;
2. $A \longrightarrow B = 1,\ (\overline{A} \wedge B) \to (\overline{A} \vee B) =$;
3. $A \leftrightarrow B = 0,\ \overline{B} \longrightarrow A =$;
4. $A \wedge B = 0,\ A \to B = 1,\ B \longrightarrow \overline{A} =$;
5. $A \leftrightarrow B = 0,\ A \to B = 1,\ (\overline{A} \longrightarrow B) \leftrightarrow A =$;
6. $A \vee B = 1,\ A \to B = 1,\ \overline{B} \to A =$;
7. $A \wedge B = 0,\ A \leftrightarrow B = 0,\ A \to B = 1,\ A =$;
8. $A \wedge B = 0,\ A \leftrightarrow B = 0,\ A \to B = 1,\ B =$;
9. $A \wedge B = 0,\ A \vee B = 1,\ A \to B = 1,\ B \longrightarrow A =$;
10. $A \longrightarrow (B \leftrightarrow A) = 0,\ A \to B =$;
11. $(A \vee B) \to A = 1,\ A \longrightarrow B = 1,\ \overline{A} \leftrightarrow \overline{B} =$;
12. $A \leftrightarrow B = 1,\ (A \longrightarrow B) \wedge (\overline{A} \leftrightarrow \overline{B}) =$;

2.1.14. Quyidagi har bir formulaning qiymatini aniqlash uchun berilgan ma'lumot yetarlimi?

1. $A \wedge (B \to C),\ B \longrightarrow C = 0$;
2. $A \vee (B \to C),\ B = 0$;
3. $\overline{(A \vee B)} \leftrightarrow (\overline{B} \wedge \overline{A}),\ A = 1$;
4. $(A \to B) \to (\overline{A} \to \overline{B}),\ B = 1$;
5. $(A \wedge B) \to (A \vee C),\ A = 0$;
6. $(\overline{B \to A}) \leftrightarrow \overline{(A \vee C)},\ A = 0$;

7. $(A \leftrightarrow B) \vee (A \wedge C),\ A = 0.$

2.1.15. Quyidagi tengliklarni bir vaqtda qanoatlantiradigan A,B,C mulohazalar mavjudmi?

1. $A \wedge B = 1,\ A \wedge C = 0,\ A \wedge B \wedge \overline{C} = 1;$
2. $B \to A = 1,\ A \vee C = 0,\ A \leftrightarrow (B \wedge \overline{C}) = 0;$
3. $A \vee B = 0,\ \overline{B} \wedge C = 1,\ (A \vee \overline{C}) \leftrightarrow (\overline{B} \to \overline{C}) = 1;$
4. $A \wedge \overline{B} = 1,\ B \vee C = 1,\ \overline{(B \to A)} \vee C = 0;$
5. $\overline{A} \wedge B = 0,\ A \vee C = 0,\ (A \vee B) \wedge \overline{C} = 1;$
6. $A \vee B = 0,\ B \vee C = 1,\ (C \to A) \vee (C \to B) = 1;$
7. $A \to B = 0,\ A \to C = 1,\ (C \to A) \to (C \to B) = 1;$
8. $A \vee C = 1,\ A \vee B = 0,\ C \to (A \vee B) = 1;$
9. $B \vee C = 0,\ \overline{C} \to A = 0,\ A \to B = 0;$
10. $A \wedge C = 1,\ C \leftrightarrow \overline{B} = 0,\ A \to B = 1;$
11. $A \vee \overline{B} = 0,\ B \to (A \vee C) = 0,\ \overline{C} \to \overline{B} = 1.$

2.1.16. (Og'zaki) Quyidagilardan qaysi biri formula bo'ladi? Javobingizni asoslang:

1) $A \vee B$;
2) $A \vee (\wedge B)$
3) $(A \to B \vee C)$;
4) $(A \wedge B)$
5) $((A \vee B) \to C)$;
6) $(A \to (B \leftrightarrow C))$.

2.1.17. Quyidagi formulalarni amallar kuchi ta'rifidan foydalanib qavslarini qo'yib chiqing:

1) $A \to B \to B$;
2) $A \vee B \wedge C$;
3) $A \to B \wedge C \vee A \leftrightarrow B$;
4) $A \leftrightarrow B \to B \vee C \wedge A$;
5) $X \vee Y \to \neg X \to Z$;
7) $X \vee Y \vee Z \to X \vee Z$;
8) $X \to Y \to Z \to \neg X$;
9) $X \vee Y \to Z \to X$;
10) $X \to Y \to Y \wedge Z$;
11) $\neg X \wedge \neg Y \to X \wedge Y$;

6) $\neg X \to Y \vee \neg X \to Y$; 12) $\neg X \wedge \neg Y \vee Z \to Z \wedge \neg Y$.

2.1.18. Quyidagi formulalarning qiymatini saqlagan holda, iloji boricha ko'proq qavslarni olib tashlang:

1) $((A \to B) \vee (C \wedge D))$;

2) $((A \wedge B) \vee ((A \to B) \wedge D))$;

3) $((A \to B) \leftrightarrow (A \wedge D))$;

4) $(((A \vee B) \leftrightarrow (C \to D)) \vee (B \wedge C))$;

5) $(((\overline{A \vee B}) \wedge C) \to (A \vee C))$;

6) $(((B \leftrightarrow C) \to (A \vee B)) \wedge (A \vee D))$;

7) $((A \to (B \to (C \wedge D))) \vee A)$;

8) $(((A \leftrightarrow B) \wedge (C \wedge D)) \leftrightarrow B)$.

2.1.19. Quyidagi formulalarga teng kuchli keltirilgan formulalarni hosil qiling.

1) $((A \to B) \wedge (B \to A)) \to (A \vee B)$;

2) $((A \to B) \wedge (B \to \rceil A)) \to (C \to A)$;

3) $((A \leftrightarrow B) \wedge (\rceil A \leftrightarrow \rceil B)) \to ((A \vee B) \wedge (\rceil A \vee \rceil B))$;

4) $((A \leftrightarrow \rceil B) \to C) \to (A \to \rceil C)$;

5) $((A \to (B \leftrightarrow C)) \leftrightarrow ((A \to B) \leftrightarrow C)$.

2.2. Formulalarning teng kuchliligi. Asosiy teng kuchliliklar.

A va B formulalar berilgan bo'lsin. elementar mulohazalarning har bir qiymatlari satri uchun A va B formulalarning mos qiymatlari bir xil bo'lsa, A va B formulalarga **teng kuchli formulalar** deb aytiladi va bu $A = B$ tarzda belgilanadi.

J -***Aynan chin*** formulalar yoki ***tavtologiya.***

- ***Aynan yolg'on*** (doimo yolg'on) yoki ***bajarilmaydigan formula***lar.

Misol: $x \vee (y \wedge z) \equiv (x \vee y) \wedge (x \vee z)$ formulaning teng kuchliligini ko'rsating.

x	y	z	$y \wedge z$	$x \vee y$	$x \vee z$	$x \vee (y \wedge z)$	$(x \vee y) \wedge$ $\wedge (x \vee z)$	$x \vee (y \wedge z) \equiv$ $\equiv (x \vee y) \wedge (x \vee z)$
yo	yo	yo	yo	yo	yo	yo	yo	ch
yo	yo	ch	yo	yo	ch	yo	yo	ch
yo	ch	yo	yo	ch	yo	yo	yo	ch
yo	ch	ch	ch	ch	ch	ch	ch	ch
ch	yo	yo	yo	ch	ch	ch	ch	ch
ch	yo	ch	yo	ch	ch	ch	ch	ch
ch	ch	yo	yo	ch	ch	ch	ch	ch
ch	ch	ch	ch	ch	ch	ch	ch	ch

1. $x \wedge y = y \wedge x$ kon'yunksiyaning kommutativlik qonuni
2. $(x \wedge y) \wedge z = x \wedge (y \wedge z)$ kon'yunksiyaning assostiativlik qonuni
3. $x \vee y = y \vee x$ diz'yunksiyaning kommutativlik qonuni
4. $(x \vee y) \vee z = x \vee (y \vee z)$ diz'yunksiyaning assostiativlik qonuni
5. $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ kon'yunksiyaning diz'yunksiyaga nisbatan distributivlik qonuni
6. $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ diz'yunksiyaning kon'yunksiyaga nisbatan distributivlik qonuni
7. $x \to y \equiv \bar{x} \vee y$.
8. $\bar{\bar{x}} = x$
9. $\overline{x \vee y} \equiv \bar{x} \wedge \bar{y}$, de Morgan qonuni
10. $\overline{x \wedge y} \equiv \bar{x} \vee \bar{y}$. de Morgan qonuni
11. $x \vee y \equiv \overline{\overline{x \vee y}} \equiv \overline{\bar{x} \wedge \bar{y}}$
12. $x \wedge y \equiv \overline{\bar{x} \vee \bar{y}}$
13. $\overline{x \wedge y} \equiv x | y$ Sheffer amali

14. $x \leftrightarrow y = (x \to y) \wedge (y \to x)$

Misol sifatida $(x \to y)\ (y \to x) \to (\bar{x} \leftrightarrow \overline{y})$ ifodani shunday almashtiramizki, natijada faqat $\wedge$, $\vee$ va – belgilar qatnashsin. Buning uchun avvalo (7), (14) va (9) teng kuchliliklardan foydalanamiz:

$$(x \to y)\ (y \to x) \to (\bar{x} \leftrightarrow \overline{y})\ (x \to y) \wedge (y \to x) \to (\bar{x} \to \bar{y}) \wedge (\bar{y} \to \bar{x}) \equiv$$

$$\equiv (\bar{x} \vee y) \wedge (\bar{y} \vee x) \to (\bar{\bar{x}} \vee \bar{y}) \wedge (\bar{\bar{y}} \vee \bar{x}) \equiv \overline{(x \vee \bar{y}) \wedge (y \vee \bar{x})} \vee (\bar{x} \vee y) \wedge (\bar{y} \vee x)\,.$$

Kommutativlik va distributivlik qonunlaridan foydalanib, bu ifodani quyidagi ko'rinishda yozishimiz mumkin:

$$(x \to y) \wedge (y \to x) \to (\bar{x} \leftrightarrow \bar{y}) \equiv (\bar{x} \wedge y) \vee (\bar{y} \wedge x) \vee (\bar{x} \wedge \bar{y}) \vee (x \wedge y) \vee (x \wedge \bar{y}) \vee (\bar{x} \wedge y)\,.$$

15. $x \cdot \bar{x} \equiv$ yo qarama-qarshilik qonuni

16. $x \vee \bar{x} \equiv$ yo uchinchisi istisno qonuni

17. $x \cdot x \equiv x, \quad x \vee x \equiv x$ idempotentlik qonuni

18. $x \cdot (x \vee y) \equiv x, \quad x \vee x \cdot y \equiv x$ yutish qonunlari

19. $x \vee$ yo $\equiv x$, chch, $x \cdot$ ch, $x \cdot$ yoyo

Keltirilganteng kuchliliklar ixtiyoriy mantiqiy ifodalarni kerakli ko'rinishga keltirishga imkon beradi.

Muammoli masala va topshiriqlar:

2.2.1. Teng kuchliliklarni isbotlang:

1) $x \leftrightarrow y \equiv \bar{x} \leftrightarrow \bar{y}$;

2) $x \wedge y \vee \bar{x} \wedge y \vee \bar{x} \wedge \bar{y} \equiv x \to y$;

3) $x \to \bar{y} \equiv y \to \bar{x}$;

4) $x \to (y \to z) \equiv x \wedge y \to z$;

5) $x \equiv (x \wedge y \wedge z) \vee (x \wedge y \wedge \bar{z}) \vee (x \wedge \bar{y} \wedge z) \vee (x \wedge \bar{y} \wedge \bar{z})$;

6) $(x \vee y) \wedge (x \vee \bar{y}) \equiv x$;

7) $x \vee (\bar{x} \wedge y) \equiv x \vee y$;

8) $(x \vee y) \wedge (z \vee t) \equiv x \wedge z \vee y \wedge z \vee x \wedge t \vee y \wedge t$;

9) $x \wedge y \vee z \wedge t \equiv (x \vee z)(y \vee z)(x \vee t)(y \vee t)$;

10) $x_1 \wedge x_2 \wedge \ldots \wedge x_n \to y \equiv x_1 \to (x_2 \to (\ldots \to (x_n \to y)\ldots))$.

2.2.2. Agar $A \leftrightarrow B = 1$ $(A \leftrightarrow B = 0)$ bo’lsa, quyidagilarning qiymati nimaga teng;

1) $A \leftrightarrow \bar{B}$; 2) $\bar{A} \leftrightarrow B$; 3) $B \to A$; 4) $A \to \bar{B}$;

2.2.3. $A \wedge B = 1$, $A \wedge C = 0$, $(A \wedge B) \wedge \bar{C} = 0$ uchala shartni qanoatlantiruvchi *A,B,C* mulohazalar mavjudmi?

2.2.4. Agar quyidagilar berilgan bo’lsa, A mulohazaning qiymatini topish mumkinmi?

1) $A \wedge B = 1$

2) $A \wedge B = 0$

3) $A \wedge B = 0$, $B = 0$

4) $A \wedge B = 0$, $B = 1$.

2.2.5. Agar quyidagilar berilgan bo’lsa, $A \to (B \wedge C)$ mulohazalar algebrasi formulasining qiymatini aniqlang:

1) *A*=0 2) *A*=1, *B*=0 3) *B*=1, *C*=1.

2.2.6. $(A \wedge B) \to C$ implikasiyaning qiymatini aniqlash uchun berilganlardan qaysi ortiqcha?

1) *A=1, B=1, C=0;*

2) *A=0, B=0, C=0;*

3) *A=1, B=0, C=1;*

4) *A=0, B=0, C=1.*

2.2.7. Agar quyidagilar berilgan bo’lsa, $(A \to B) \vee C$ mulohazalar algebrasi formulasining qiymatini aniqlab bo’ladimi?

1) *A=0* 4) *A=1, C=0*

2) *C=0* 5) *A=0, B=1*

3) *A=0, B=0* 6) *B=0*

2.2.8. *A*,*B*,*C* ning qiymatini toping:

1) $\overline{A \wedge B} = 0;$

2) $\overline{A \to \overline{B}} = 1;$

3) $\overline{A \vee (A \leftrightarrow B)} \to C = 0;$

4) $(A \wedge B) \leftrightarrow (B \vee C) = 1;$

5) $A \vee (A \wedge B) = 0;$

6) $\begin{cases} A \to C = 0 \\ A \vee B = 1 \end{cases};$

7) $\begin{cases} \overline{A \wedge B} \leftrightarrow C = 1 \\ C \vee \overline{A} = 0 \end{cases};$

8) $\begin{cases} (A \wedge B) \vee C \to A = 1 \\ A \vee \overline{C} = 0 \end{cases};$

9) $\begin{cases} A \to \overline{B} = 0 \\ A \wedge B \leftrightarrow C = 1 \end{cases};$

10) $\begin{cases} A \leftrightarrow B = 0 \\ A \vee C = 0 \end{cases};$

11) $\begin{cases} (A \wedge B) \vee C \leftrightarrow A = 1 \\ C \vee \overline{B} = 0 \end{cases}$

2.2.9. Soddalashtiring:

1. $A \wedge (A \vee B) \wedge B;$

2. $A \vee \bar{A} \wedge B;$

3. $\bar{A} \vee A \wedge B;$

4. $(A \wedge B) \to (\bar{A} \vee B);$

5. $A \vee A \vee A \wedge A \wedge B \wedge C;$

6. $A \wedge B \vee \bar{A} \vee A;$

7. $(\bar{A} \vee B \leftrightarrow C) \wedge B \vee A$;

8. $(A \rightarrow B \leftrightarrow C) \wedge B \vee \bar{B}$;

9. $A \vee A \wedge B \wedge B \wedge (D \leftrightarrow C)$;

10. $A \leftrightarrow A \leftrightarrow A$;

11. $A \rightarrow A \rightarrow A \rightarrow A$;

12. $A \rightarrow (A \rightarrow A) \rightarrow A$;

13. $A \rightarrow (A \rightarrow (A \rightarrow A))$;

14. $(\overline{\bar{A} \rightarrow B} \rightarrow A \vee B) \wedge B$;

15. $\overline{\bar{A} \wedge \bar{B}} \vee (A \rightarrow B) \wedge A$;

16. $(A \leftrightarrow B) \wedge (A \vee B)$.

2.2.10. Quyidagi formulalar sodda shaklga olib keling:

1. $(A \vee C) \wedge (A \vee B \vee \bar{C})$;

2. $\overline{\bar{A} \rightarrow B} \rightarrow (A \vee B \rightarrow A)$;

3. $\overline{\bar{A} \wedge \bar{B}} \vee (A \rightarrow B) \wedge A$;

4. $(A \rightarrow B) \wedge (B \rightarrow A) \wedge (A \vee B)$;

5. $(A \wedge C) \vee (A \wedge \bar{C}) \vee (B \wedge C) \vee (\bar{A} \wedge B \wedge C)$;

6. $(A \leftrightarrow \bar{B}) \rightarrow C \rightarrow (A \leftrightarrow \bar{C})$;

7. $(A \rightarrow (B \leftrightarrow C)) \leftrightarrow (A \rightarrow B \leftrightarrow C)$.

2.2.11. Quyidagi formulalarni soddalashtirib, aynan yolg'on bo'lishini ko'rsating:

1) $(A \rightarrow B) \wedge (B \rightarrow A) \wedge ((A \wedge \overline{B}) \vee (\overline{A} \wedge B))$;

2) $(A \wedge \overline{B} \rightarrow \overline{A} \vee A \wedge B) \wedge (\overline{A} \vee A \wedge B \rightarrow \overline{B} \wedge A)$;

3) $(A \rightarrow B) \wedge (B \rightarrow C) \rightarrow \overline{(A \rightarrow C)}$;

4) $(A \rightarrow B) \wedge (A \rightarrow \overline{B}) \wedge A$;

5) $(A \wedge \overline{B} \vee A \wedge \overline{C}) \leftrightarrow ((A \rightarrow B) \wedge (A \rightarrow C))$.

2.2.12. Quyidagi shart berilgan bo'lsa, formulaning qiymatini toping:

1) $A \wedge C = 1$ bo'lsa, $A \wedge B \to C \leftrightarrow A \wedge C - ?$

2) $D = 1$ bo'lsa, $(A \leftrightarrow B) \wedge (A \to C) \to D \to \overline{D} - ?$

3) A =1, C =0 bo'lsa, $(A \vee B) \to A \wedge C - ?$

2.2.13. Quyidagi formulalar tavtologiya bo'lishini isbotlang:

1) $(\overline{P} \to (Q \wedge \overline{Q})) \to P;$

2) $((\overline{P} \to Q) \wedge (\overline{P} \to \overline{Q})) \to P;$

3) $((P \wedge \overline{Q}) \to (R \wedge \overline{R})) \to (P \to Q);$

4) $(P \wedge (P \to Q)) \to Q;$

5) $\left((P \to Q) \wedge \overline{Q}\right) \to \overline{P};$

6) $\left((P \to Q) \wedge (Q \to P)\right) \to (P \to R);$

7) $\left((P \to Q) \wedge (R \to S) \wedge \overline{(Q \vee S)}\right) \to \overline{(P \vee R)};$

8) $\left((P \to Q) \wedge (R \to S)\right) \to \left((P \wedge R) \to (Q \wedge S)\right);$

9) $\left((P \to Q) \vee R\right) \leftrightarrow \left(P \to (Q \vee R)\right);$

10) $P \to \left(Q \to \left((P \vee Q) \to (P \wedge Q)\right)\right).$

2.2.14. F va $F \to G$ formulalar tavtologiya bo'lsa, G ham formula ekanligini isbotlang. Ya'ni $\vDash F$ va $\vDash F \to G$ bo'lsa, $\vDash G$ bo'ladi.

2.2.15. Isbotlang:

1) Agar $\models F \wedge G$, $\models F \leftrightarrow H$ bo'lsa, $\vDash G \to H$ bo'ladi.

2) Agar $\vDash F \vee G$, $\vDash G \to H$ bo'lsa, $\vDash F \vee H$ bo'ladi.

3) Agar $\vDash \overline{F} \to G$, $\vDash \overline{G} \vee \overline{H}$ bo'lsa, $\vDash H \to F$ bo'ladi.

4) Agar $\vDash \overline{G} \wedge \overline{H}$, $\vDash F \vee G$ bo'lsa, $\vDash \overline{F} \to H$ bo'ladi.

5) Agar $\vDash F \vee G$, $\vDash F \leftrightarrow G$ bo'lsa, $\vDash G$ bo'ladi.

6) Agar $\vDash F$, $\vDash F \leftrightarrow G$, $\vDash F \leftrightarrow H$ bo'lsa, $\vDash G \wedge H$ bo'ladi.

7) Agar $\vDash \overline{F} \to G$, $\vDash \overline{G} \wedge \overline{H}$ bo'lsa, $F \vee H$ bo'ladi.

8) Agar $\vDash F \leftrightarrow G$, $\vDash G \leftrightarrow H$ bo'lsa, $\vDash F \leftrightarrow H$ bo'ladi.

9) Agar $\vDash F$, $\vDash G$, $\vDash H$ bo'lsa, $\vDash F \rightarrow (G \rightarrow H)$ bo'ladi.

10) Agar $\vDash F \wedge G$, $\vDash G \rightarrow \overline{H}$ bo'lsa, $\vDash F \wedge \overline{H}$ bo'ladi.

11) Agar $\vDash \overline{F} \vee G$, $\vDash \overline{G} \vee \overline{H}$ bo'lsa, $\vDash F \rightarrow \overline{H}$ bo'ladi.

12) Agar $\vDash G \rightarrow F$, $\vDash (\overline{F} \wedge H) \leftrightarrow G$ bo'lsa, $\vDash H$, $\vDash \overline{G} \wedge H$ bo'ladi.

2.2.16. Quyidagi tasdiqlar to'g'rimi?

1) $\vDash F \leftrightarrow G$ faqat va faqat shunda, qachon $\vDash (F \rightarrow G) \wedge (G \rightarrow F)$ bo'lsa.

2) $\vDash F \vee G$ faqat va faqat shunda, qachon $\vDash F$ yoki $\vDash G$ bo'lsa.

3) $\vDash F \leftrightarrow G$ faqat va faqat shunda, qachon $\vDash F \rightarrow G$ va $\vDash G \rightarrow F$ bo'lsa.

4) $\vDash F \vee G$ faqat va faqat shunda, qachon $\vDash F$ va $\vDash G$ bo'lsa.

5) $\vDash F \rightarrow G$ faqat va faqat shunda, qachon $\vDash F$ bo'lsa.

6) $\vDash F \rightarrow G$ faqat va faqat shunda, qachon $\vDash G$ bo'lsa.

7) $\vDash F \rightarrow G$ faqat va faqat shunda, qachon $\vDash \overline{F}$ yoki $\vDash G$ bo'lsa.

8) $\vDash F \wedge G$ faqat va faqat shunda, qachon $\vDash F$ va $\vDash G$ bo'lsa.

9) $\vDash F \leftrightarrow G$ faqat va faqat shunda, qachon $\vDash F$ va $\vDash G$ bo'lsa.

10) $\vDash F \vee G$ faqat va faqat shunda, qachon $\vDash F$ yoki $\vDash G$ bo'lsa.

11) $\vDash F \rightarrow G$ faqat va faqat shunda, qachon $\vDash F$ va $\vDash G$ bo'lsa.

12) $\vDash (\overline{F \vee G})$ faqat va faqat shunda, qachon $\vDash \overline{F}$ va $\vDash \overline{G}$ bo'lsa.

2.2.17. Quyidagi formulalar uchun chinlik jadvalini tuzib, ularning tavtologiya ekanligini ko'rsating:

1) $(P \rightarrow (Q \rightarrow P)$;

2) $(P \rightarrow Q) \rightarrow ((P \rightarrow (Q \rightarrow R)) \rightarrow (P \rightarrow R))$;

3) $(P \rightarrow (Q \rightarrow (P \wedge Q))$;

4) $(P \rightarrow (P \vee Q)$;

5) $(P \wedge Q) \rightarrow P$;

6) $(P \rightarrow (Q \wedge R)) \leftrightarrow ((P \rightarrow Q) \wedge (P \rightarrow R))$;

7) $(P\to Q)\wedge(P\to\neg Q))\to\neg \mathrm{P}$;

8) $(P\to Q)\to((P\to\neg Q)\to\neg P)$;

9) $(P\to R)\to((Q\to R)\to((P\vee Q)\to R))$;

10) $(P\to Q)\vee(Q\to P)$;

11) $(P\to Q)\vee((Q\to P)\to(P\leftrightarrow Q))$;

12) $(P\to(Q\to R))\leftrightarrow(Q\to(P\to R))$;

2.2.18. Asosiy tavtologiyalarni isbotlang:

1) $P\vee\neg P$;

2) $\neg\neg P\leftrightarrow P$;

3) $(P\to Q)\wedge(\neg Q\to\neg P)$;

4) $(P\leftrightarrow Q)\leftrightarrow(\neg P\leftrightarrow\neg Q)$;

5) $\neg P\to(P\to Q$;

6) $((P\to Q\wedge\neg Q)\to\neg P$;

7) $(P\to(Q\to R))\leftrightarrow(Q\to(P\to R))$;

8) $(P\to(Q\to R))\leftrightarrow((P\wedge Q)\to R)$;

9) $((P\to R)\wedge(Q\to R))\leftrightarrow((P\vee Q)\to R)$;

10) $((\neg P\to Q)\wedge(\neg P\to\neg Q))\to P$, $(\neg P\to(Q\wedge\neg Q))\to P$;

11) $\neg(P\wedge\neg P)$;

12) $P\to P$;

13) $((P\to Q)\wedge(Q\to R))\to(P\to R)$;

14) $P\leftrightarrow(Q\to P)$;

15) $(P\wedge(P\to Q))\to Q$.

Quyida keltiriladigan formulalar logik amallarning xossalarini ko'rsatib beradi. Ularning tavtologiya ekanligini ko'rsating:

2.2.19. Kon'yunksiya va diz'yunksiya xossalari:

1) $(P\wedge P)\leftrightarrow P$, $(P\vee P)\leftrightarrow P$;

2) $(P\wedge Q)\to P$, $P\to(P\vee Q)$;

3) $(P\wedge Q)\leftrightarrow(Q\wedge P)$, $(P\vee Q)\leftrightarrow(Q\vee P)$;

4) $(P\wedge(Q\wedge R))\leftrightarrow((P\wedge Q)\wedge R)$, $(P\vee(Q\vee R))\leftrightarrow((P\vee Q)\vee R)$;

5) $P\wedge(Q\vee R))\leftrightarrow((P\wedge Q)\vee(P\wedge R))$, $P\vee(Q\wedge R))\leftrightarrow((P\vee Q)\wedge(P\vee R))$;

6) $(P\wedge(P\vee Q))\leftrightarrow P$, $(P\vee(P\wedge Q))\leftrightarrow P$;

7) $\neg(P\wedge Q)\leftrightarrow(\neg P\vee\neg Q)$, $\neg(P\vee Q)\leftrightarrow(\neg P\wedge\neg Q)$.

2.2.20. Implikasiya va ekvivalentlik xossalari:

1) $(P\to(Q\to R))\to((P\to Q)\to(P\to R))$;

2) $P\to(Q\to(P\wedge Q))$;

3) $(P\to R)\to((Q\to R)\to((P\vee Q)\to R))$;

4) $(P\to Q)\to((P\to\neg Q)\to R))$;

5) $(\neg Q\wedge(P\to Q))\to\neg P$;

6) $(\neg P\wedge(P\vee Q))\to Q$;

7) $(P\to Q)\to((P\vee R)\to(Q\vee R))$;

8) $(P\to Q)\to(P\wedge R)\to(Q\wedge R))$;

9) $(P\to Q)\to((Q\to R)\to P\to R))$;

10) $(P\to Q)\vee(Q\to P)$;

11) $(\neg Q\to\neg P)\to((\neg Q\to P)\to Q))$;

12) $((P\to Q)\wedge(R\to Q))\leftrightarrow((P\vee R)\to Q)$;

13) $((P\to Q)\wedge(P\to R))\leftrightarrow(P\to(Q\wedge R))$;

14) $P\leftrightarrow P$;

15) $(P\leftrightarrow Q)\leftrightarrow(Q\leftrightarrow P)$;

16) $((P\leftrightarrow Q)\wedge(Q\leftrightarrow R))\to(P\leftrightarrow R)$.

2.2.21. Bir logik amallrni boshqa logik amallar orqali ifodalash:

1) $(P\to Q)\leftrightarrow(\neg P\vee Q)$

2) $(P\to Q)\leftrightarrow\neg(P\wedge\neg Q)$

3) $(P\wedge Q)\leftrightarrow\neg(\neg P\vee\neg Q)$

4) $(P\wedge Q)\leftrightarrow\neg(P\to\neg Q)$

5) $(P\vee Q)\leftrightarrow\neg(\neg P\wedge\neg Q)$

6) $(P\vee Q)\leftrightarrow(\neg P\to Q)$

7) $(P\leftrightarrow Q)\leftrightarrow((P\to Q)\wedge(Q\to P))$

2.3. Mulohazalar algebrasi formulasining normal shakllari.

Belgilash kiritamiz:

$$x^{\sigma} = \begin{cases} x, & agar \quad \sigma = ch, \\ \bar{x}, & agar \quad \sigma = yo. \end{cases}$$

σ^{σ} = ch ekanligi aniq.

$x_1^{\sigma_1} \wedge x_2^{\sigma_2} \wedge \cdots \wedge x_n^{\sigma_n}$ - ko'rinishdagi formulaga ***elementar kon'yunkstiya***

$x_1^{\sigma_1} \vee x_2^{\sigma_2} \vee \cdots \vee x_n^{\sigma_n}$ - ko'rinishdagi formulaga ***elementar diz'yunkstiya***

Elementar diz'yunkstiyalarning kon'yunkstiyasiga formulaning ***kon'yunktiv normal shakli*** (KNSh)

Elementar kon'yunkstiyalarning diz'yunkstiyasiga formulaning ***diz'yunktiv normal shakli*** (DNSh) deb aytiladi.

KNShga $(x \vee y) \wedge (\bar{x} \vee z) \wedge (x \vee \bar{y} \vee z)$ formula va DNShga $x\,y \vee \bar{x}z \vee x\bar{y}z$ formula misol bo'la oladi.

Misol. 1. $P = [(x \vee y) \wedge (\bar{x} \vee \bar{y})] \vee [x \wedge (\bar{x} \vee y)]$

$P = \{[(x \vee y) \wedge (\bar{x} \vee \bar{y})] \vee x\} \wedge \{[(x \vee y) \wedge (\bar{x} \vee \bar{y})] \vee (\bar{x} \vee y)\} =$

$= [(x \vee y) \vee x] \wedge [(\bar{x} \vee \bar{y}) \vee x] \wedge \ [(x \vee y) \vee (\bar{x} \vee y)] \wedge [(\bar{x} \vee \bar{y}) \vee (\bar{x} \vee y)] =$

$= (x \vee y) \wedge [J \vee \bar{y}] \wedge (J \vee y) \wedge (\bar{x} \vee J) = (x \vee y) \wedge \ J \wedge J \wedge J = x \vee y$;

$P = x \vee y$.

Shunday qilib, P formulaning KNSh bittagina diz'yunktiv $(x \vee y)$ haddan iborat ekan.

2. $P = \overline{x \vee y} \leftrightarrow x \wedge y$

$P = \bar{x} \wedge \bar{y} \leftrightarrow x \wedge y = \overline{x \vee y} \leftrightarrow (x \wedge y) = [\overline{\overline{x \vee y}} \vee (x \wedge y)] \wedge [(\overline{x \vee y}) \vee (\overline{x \wedge y})] =$

$= [(x \vee y) \vee (x \wedge y)] \wedge [(\bar{x} \wedge \bar{y}) \vee (\bar{x} \vee \bar{y})] = [(x \vee y) \vee (x \wedge y)] \wedge [(\bar{x} \wedge \bar{y}) \vee (\bar{x} \vee \bar{y})] =$

$$= [(x \vee y \vee x) \wedge (x \vee y \vee y)] \wedge [(\bar{x} \vee \bar{y} \vee \bar{x}) \wedge (\bar{x} \vee \bar{y} \wedge \bar{y})] =$$

$$= (x \vee y) \wedge (x \vee y) \wedge (\bar{x} \vee \bar{y}) \wedge (\bar{x} \vee \bar{y}) = (x \vee y) \wedge (\bar{x} \vee \bar{y}):$$

$P = (x \vee y) \wedge (\bar{x} \vee \bar{y}).$

Muammoli masala va topshiriqlar:

2.3.1. Quyidagi formulalarni kon'yunktiv normal shaklga keltiring.

1) $A \equiv (x \& \bar{y}) \vee z;$

2) $A \equiv x \& y \to (\bar{y} \vee x \to z);$

3) $A \equiv (x \to y) \to (x \vee y \& z);$

4) $A \equiv (x \vee z) \& (\bar{y} \to (u \to \bar{x}));$

5) $A \equiv (x \& y) \to x;$

6) $A \equiv (x \vee z) \to (x \vee \bar{y});$

7) $A \equiv (x \to y) \to (y \to \bar{x});$

8) $A \equiv (x \to y) \& (x \to \bar{y}) \to \bar{x};$

9) $A \equiv (x \leftrightarrow y) \& (x \vee y);$

10) $A \equiv (x \to y) \& (y \to z) \to (z \to x);$

11) $A \equiv (x \to y) \& (y \to z) \to (x \to z);$

12) $A \equiv (y \leftrightarrow z) \& (x \vee z);$

13) $A \equiv z \& y \to (y \vee z \to x).$

2.3.2. Quyidagi formulalarni keltirilgan formula shaklida yozing:

1. $(A \to B) \to (B \to A);$
2. $(A \to B) \to (\neg A \to \neg B);$

3. $(A \vee (B \wedge \neg C))$;

4. $((\neg A \wedge B) \rightarrow (B \vee C))$;

5. $(((A \rightarrow B) \rightarrow A) \rightarrow (A \rightarrow (B \wedge A)))$;

6. $((C \rightarrow A) \rightarrow (\neg(B \vee C) \rightarrow A))$;

7. $(\neg B \rightarrow \neg A) \rightarrow (A \rightarrow B)$;

8. $(A \rightarrow (\neg B \rightarrow \neg(A \rightarrow B)))$.

2.3.3. Quyidagi formulalarni DNSh va KNShlarini tuzing.

1. $(A \vee \neg B) \wedge \neg C$.

2. $((\neg A \wedge B) \vee \neg C)$.

3. $((\neg A \wedge B) \vee (\neg C \wedge \neg B))$.

4. $((\neg A \wedge B) \rightarrow (B \vee C))$.

5. $(((A \rightarrow B) \rightarrow A) \rightarrow (A \rightarrow (B \wedge A)))$.

6. $((C \rightarrow A) \rightarrow (\neg(B \vee C) \rightarrow A))$.

2.3.4. Quyidagi formulalarni diz'yunktiv normal shaklga keltiring:

1) $A \equiv (z \rightarrow x) \leftrightarrow (\overline{(y \vee z)} \rightarrow x)$;

2) $A \equiv (\overline{x \,\&\, (y \vee z)}) \leftrightarrow (x \& (y \vee z))$;

3) $A \equiv (x \& y \leftrightarrow y \& z) \rightarrow ((x \rightarrow y) \rightarrow (z \rightarrow y))$;

4) $A \equiv (\overline{(x \,\&\, y)} \rightarrow \overline{x}) \& \overline{((x \,\&\, y) \rightarrow \overline{y})}$;

5) $A \equiv (x \vee \overline{z}) \rightarrow y \leftrightarrow z$;

6) $A \equiv (\overline{x} \leftrightarrow z) | (\overline{y} \rightarrow \overline{x})$;

7) $A \equiv (\overline{x} \leftrightarrow \overline{y}) \rightarrow ((y \& z) \rightarrow (x \& z))$;

8) $A \equiv ((x \rightarrow y) \rightarrow \overline{x}) \rightarrow (x \rightarrow (y \& x))$;

9) $A \equiv \overline{((x \,\&\, y) \rightarrow \overline{x})} \leftrightarrow \overline{((x \,\&\, y) \rightarrow \overline{y})}$;

10) $A \equiv (\overline{(x \,\&\, y)} \leftrightarrow \bar{x}) | ((x\&y) \rightarrow \bar{y});$

11) $A \equiv (x \leftrightarrow \bar{z}) \rightarrow y\&z;$

12) $A \equiv (\bar{x} \rightarrow z) \rightarrow \overline{(\bar{y} \rightarrow \bar{x})};$

2.3.5. Asosiy tengkuchliliklardan foydalanib, shunday almashtirishlar kiritingki, quyidagi formulalarda faqat $(\wedge)$ konyunksiya, $(\vee)$ dizyunksiya va $(\neg)$ inkor amallari ishtirok etsin:

1) $((X \rightarrow Y) \wedge (Y \rightarrow X)) \rightarrow (X \vee Y);$

2) $((X \rightarrow Y) \wedge (Y \rightarrow \neg X)) \rightarrow (Z \rightarrow X);$

3) $((X \rightarrow Y) \wedge (\neg X \rightarrow \neg Y)) \rightarrow ((X \vee Y) \wedge (\neg X \vee \neg Y));$

4) $((X \leftrightarrow \neg Y) \rightarrow Z) \rightarrow (X \leftrightarrow \neg Z);$

5) $(X \rightarrow (Y \leftrightarrow Z)) \leftrightarrow ((X \rightarrow Y) \leftrightarrow Z);$

6) $(X \rightarrow Y) \rightarrow ((X \rightarrow Y) \rightarrow \neg X);$

7) $((X \wedge \neg Y) \rightarrow Y) \rightarrow (X \rightarrow \neg Y);$

8) $((X \rightarrow Y) \rightarrow Y) \rightarrow Y;$

9) $(X \rightarrow Y) \rightarrow ((X \rightarrow \neg Y) \rightarrow (X \wedge Y));$

10) $(X \rightarrow Z) \rightarrow ((X \vee Y) \rightarrow (\neg Z \vee Y);$

11) $X \rightarrow \neg(Y \leftrightarrow Z).$

Yechim: 11) Shartga ko'ra formuladagi implikasiya va ekvivalentlik amallarini $(\wedge)$ konyunksiya, $(\vee)$ dizyunksiya va $(\neg)$ inkor amallari ishtirok etgan tengkuchli formulalar bilan almashtiramiz:

$$X \rightarrow \neg(Y \leftrightarrow Z) = \neg X \vee \neg((Y \rightarrow Z) \wedge (Z \rightarrow Y)) =$$
$$\neg X \vee \neg((\overline{Y} \vee Z) \wedge (\overline{Z} \vee Y)).$$

2.3.6. Asosiy tengkuchliliklardan foydalanib, shunday almashtirishlar kiritingki, quyidagi formulalarda inkor amali faqat propozisional o'zgaruvchilarga tegishli bo'lsin:

1) $\neg((X \wedge (\neg Y \vee \neg Z)) \vee Z);$

2) $\neg((X \wedge Y) \vee \neg Z) \to \neg(X \wedge Z)$;

3) $\neg(U \to \neg(Z \wedge \neg(Y \wedge \neg X)))$;

4) $\neg(\neg(\neg(X \wedge Y) \to Y) \to (\neg X \wedge Z))$;

5) $\neg(\neg(X \vee (\neg Y \wedge Z) \vee \neg Z) \vee (Y \wedge Z))$;

6) $\neg((\neg X \wedge \neg Y) \to (X \vee (Z \wedge \neg T))$;

7) $\neg((X \leftrightarrow (\neg Y \vee Z)) \wedge Y)$;

8) $\neg((\neg X \leftrightarrow \neg Y) \vee Z) \wedge Y$;

9) $\neg((X \to Y) \to X) \to Y$;

10) $\neg((X \vee \neg Y) \to Y) \wedge (\neg X \vee Y))$;

11) $(X \to Y) \to \neg(X \leftrightarrow \neg Z)$.

Yechim: 11) Shartga ko'ra formuladagi inkor amalini amali faqat propozisional o'zgaruvchilarga tegishli bo'lishi uchun de-Morgan qonunidan va asosiy tengkuchliliklardan foydalanamiz: $(X \to Y) \to \neg(X \leftrightarrow \neg Z) =$

$$= (X \to Y) \to \neg((X \to \neg Z) \wedge (\neg Z \to X)) = (X \to Y) \to$$
$$\to \neg((\neg X \vee \neg Z) \wedge (\neg(\neg Z) \vee X)) = (X \to Y) \to$$
$$\to \neg((\neg X \vee \neg Z) \wedge (Z \vee X)) = (X \to Y) \to (X \wedge Z) \vee (\neg Z \wedge \neg X).$$

2.3.7. Asosiy tengkuchliliklardan foydalanib, shunday almashtirishlar kiritingki, quyidagi formulalarda faqat $(\wedge)$ konyunksiya va $(\neg)$ inkor amallari ishtirok etsin:

1) $(X \vee Y) \to (\neg X \to Z)$;

2) $(\neg X \to Y) \vee \neg(X \to Y)$;

3) $((X \vee Y \vee Z) \to X) \vee Z$;

4) $((X \to Y) \to Z) \to \neg X$;

5) $(X \vee (Y \to Z)) \to X$;

6) $(X \to Y) \to (Y \wedge Z)$;

7) $(\neg X \wedge \neg Y) \to (X \wedge Y)$;

8) $((\neg X \wedge \neg Y) \vee Z) \to (Z \wedge \neg Y)$;

9) $((X \to (Y \wedge Z)) \to (\neg Y \to \neg X)) \to \neg Y$;

10) $((X \to Y) \wedge (Y \to Z)) \to (X \to Z)$;

11) $(\neg X \leftrightarrow Y) \to Z$.

Yechim: 11) Quyidagi almashtirishlar bajaramiz: $(\neg X \leftrightarrow Y) \to Z =$

$$= \neg((\neg X \to Y)(Y \to \neg X)) \vee Z = \neg((\neg(\neg X) \vee Y) \wedge (\neg Y \vee \neg X)) \vee Z =$$
$$= \neg((X \vee Y) \wedge (\neg Y \vee \neg X)) \vee Z = \neg(\neg(\neg X \wedge \neg Y) \wedge \neg(Y \wedge X)) \vee Z =$$
$$= \neg((\neg(\neg X \wedge \neg Y) \wedge \neg(Y \wedge X)) \wedge \neg Z).$$

2.3.8. Asosiy tengkuchliliklardan foydalanib, shunday almashtirishlar kiritingki, yuqoridagi 2.3.7. misolning formulalarda faqat $(\vee)$ dizyunksiya va $(\neg)$ inkor amallari ishtirok etsin.

Yechim: 11) Quyidagi almashtirishlar bajaramiz: $(\neg X \leftrightarrow Y) \to Z =$

$$= \neg((\neg X \to Y)(Y \to \neg X)) \vee Z = \neg((\neg(\neg X) \vee Y) \wedge (\neg Y \vee \neg X)) \vee Z =$$
$$= \neg((X \vee Y) \wedge (\neg Y \vee \neg X)) \vee Z = \neg(X \vee Y) \vee \neg(\neg Y \vee \neg X)) \vee Z.$$

2.3.9. Asosiy tengkuchliliklardan foydalanib, 2.2.17. misolning formulalari tavtologiya ekanligini isbotlang.

Yechim: 11) Quyidagi almashtirishlar bajaramiz: $(P \to Q) \vee (Q \to P) =$

$$= (\overline{P} \vee Q) \vee (\overline{Q} \vee P) = (\overline{P} \vee P) \vee (Q \vee \overline{Q}) = J \vee J = J.$$

2.3.10. Asosiy tengkuchliliklardan foydalanib, quyidagi formulalari soddalashtiring:

1) $\neg(\neg P \vee Q) \to ((P \vee Q) \to P)$;

2) $\neg(\neg P \wedge \neg Q) \vee ((P \to Q) \wedge P)$;

3) $(P \to Q) \wedge (Q \to P) \wedge (P \vee Q)$;

4) $(P \to Q) \wedge (Q \to \neg P) \wedge (R \to P)$;

5) $(P \wedge R) \vee (P \wedge \neg R) \vee (Q \wedge R) \vee (\neg P \wedge Q \wedge R)$;

6) $(P \to Q) \to ((P \to \neg Q) \to \neg P)$;

7) $\neg((P \leftrightarrow \neg Q) \vee R) \wedge Q$;

8) $(P \leftrightarrow Q) \to (\neg P \to Q)$;

9) $(P \to \neg Q) \wedge ((P \to Q) \vee (R \to P))$;

10) $\neg((P \to Q) \wedge P) \wedge (\neg P \vee \neg Q)$;

11) $(P \leftrightarrow Q) \wedge (P \vee Q)$.

Yechim: 11) Quyidagi almashtirishlar bajaramiz: $(P \leftrightarrow Q) \wedge (P \vee Q) =$

$= (\neg P \vee Q) \wedge (P \vee \neg Q) \wedge (P \vee Q) = (\neg P \vee Q) \wedge (P \wedge (\neg Q \vee Q)) =$

$= (\neg P \vee Q) \wedge P = (\neg P \wedge P) \vee (Q \wedge P) = 0 \vee (Q \wedge P) = P \wedge Q.$

2.3.11. Asosiy tengkuchliliklardan foydalanib, quyidagi formulalarning aynan yolg'on formula ekanligini isbotlang:

1) $((P \to Q) \to P) \wedge \neg P$;

2) $(\neg((X \vee Y) \to \neg(X \to Y)) \vee \neg(Z \wedge Y)) \to \neg((Z \to \neg Z) \vee Z)$;

3) $\neg(((X \to Y) \wedge (Y \to Z)) \to (X \to Z))$;

4) $((X \to \neg Y) \to \neg(X \to Z)) \wedge \neg(Z \to Y)$;

5) $(Z \to \neg(X \wedge \neg Z)) \to (\neg(X \vee Z) \wedge X \wedge Y)$;

6) $((\neg P \to \neg Q) \to ((\neg P \to Q) \to P)) \to \neg((\neg P \to P) \to P)$;

7) $\neg Q \wedge P \wedge (P \to Q)$;

8) $(P \vee Q) \leftrightarrow (\neg P \wedge (Q \to \neg Q))$;

9) $(P \to (Q \to R)) \wedge (P \to Q) \wedge P \wedge \neg R$;

10) $((X \wedge \neg Y) \vee (X \wedge \neg Z)) \leftrightarrow ((X \to Y) \wedge (X \to Z))$;

11) $((P \to \neg Q) \to ((\neg R \to \neg S) \to (P \wedge Q))) \wedge \neg(R \to P)$.

Yechim: 11) Quyidagi almashtirishlar bajaramiz:

$((P \to \neg Q) \to ((\neg R \to \neg S) \to (P \wedge Q))) \wedge \neg(R \to P) =$

$= (\neg(\neg P \vee \neg Q) \vee (\neg(\neg\neg R \vee \neg S) \vee (P \wedge Q))) \wedge \neg(\neg R \vee P) =$

$= ((P \wedge Q) \vee (\neg R \wedge S) \vee (P \wedge Q)) \wedge (\neg\neg R \wedge \neg P) =$

$=((P\wedge Q)\vee(\neg R\wedge S))\wedge(R\wedge\neg P)=(P\wedge Q\wedge R\wedge\neg P)\vee$
$\vee(\neg R\wedge S\wedge R\wedge\neg P)=0\vee 0=0.$

2.3.12. Asosiy tengkuchliliklardan foydalanib, quyidagi tengliklarning qaysi bajarilishini aniqlang:

1) $P\to(Q\vee R)\cong(P\to Q)\vee(P\to R);$

2) $P\to(Q\wedge R)\cong(P\to Q)\wedge(P\to R);$

3) $P\to(Q\leftrightarrow R)\cong(P\to Q)\leftrightarrow(P\to R);$

4) $P\wedge(Q\leftrightarrow R)\cong(P\wedge Q)\leftrightarrow(P\wedge R);$

5) $P\vee(Q\leftrightarrow R)\cong(P\vee Q)\leftrightarrow(P\vee R);$

6) $P\wedge(Q\to R)\cong(P\wedge Q)\to(P\wedge R);$

7) $P\vee(Q\to R)\cong(P\vee Q)\to(P\vee P);$

8) $(P\to Q)\wedge R\cong(P\wedge R)\to(Q\wedge R);$

9) $(P\to Q)\vee R\cong(P\vee R)\to(Q\vee R);$

10) $P\to(P\leftrightarrow Q)\cong P\to Q;$

11) $P\to(P\wedge Q)\cong P\to Q;$

12) $P\to(P\vee Q)\cong P\to Q.$

Yechim: 11) Quyidagi almashtirishlar bajaramiz: $P\to(P\wedge Q)\cong$
$\cong\neg P\vee(P\wedge Q)=(\neg P\vee P)\wedge(\neg P\vee Q)=1\wedge(P\to Q)=P\to Q.$

12) $P\to(P\vee Q)\cong\neg P\vee(P\vee Q)\cong 1\vee Q=1.$ Tenglikning chap tomoni tavtologiya ammo, o'ng tomoni $P\to Q$ tavtologiya emasligi aniq.

2.3.13. Quyidagi formulalar nimaga teng:

1) $P\to 0;$	4) $1\to P;$	7) $0\to\neg P;$	10) $0\leftrightarrow\neg P;$
2) $P\to 1;$	5) $P\leftrightarrow 1;$	8) $\neg P\to 0;$	11) $1\to\neg P;$
3) $0\to P;$	6) $\neg P\to 1;$	9) $1\leftrightarrow\neg P;$	12) $P\leftrightarrow 0.$

Yechim: 11) implikasiya ta'rifiga ko'ra $1\to 0=0, 1\to 1=1,$ ya'ni $1\leftrightarrow\neg P$ formula $\neg P$ chin bo'lganda chin, yolg'on bo'lganda yolg'on qiymatni qabul qiladi. Demak, $1\to\neg P=\neg P$.

12) ekvivalentlik ta'rifiga ko'ra $1\leftrightarrow 0=0,\ \ 0\leftrightarrow 0=1,$ ya'ni formula $P\leftrightarrow 0$ - P mulohaza 1 bo'lganda 0, 0 bo'lganda 1 qiymatni qabul qilayapti. Demak, $P\leftrightarrow 0=\neg P.$

2.3.14. Ixtiyoriy $\neg$, $\vee$, $\wedge$ amallari yordamida berilgan formulaning inkorini topish uchun $\vee$ amali o'rniga $\wedge$, $\wedge$ amali o'rniga $\vee$ va barcha o'zgaruvchilarning inkor amali bilan berilgan bo'lsa olib tashlab, berilmagan bo'lsa kiritish usuli bilan topish mumkinligini isbotlang.

2.3.15. Asosiy tengkuchliliklardan foydalanib, shunday almashtirishlar kiritingki, quyidagi formulalarda faqat $(\wedge)$ konyunksiya, $(\vee)$ dizyunksiya va $(\bar{\ })$ inkor amallari ishtirok etsin so'ng, uning inkorini toping:

1) $A\equiv(x|y)\vee z\to(y\to z);$
2) $A\equiv x\&y\downarrow(y\to z)\to(\bar{y}\vee x\to z);$
3) $A\equiv(x\downarrow(y\to z)\to y)\to(x\vee y\&z);$
4) $A\equiv(x\vee z)\&(\bar{y}\to(u|\bar{x}));$
5) $A\equiv(x\to(y\downarrow z)\&y)\to x;$
6) $A\equiv x\to(x|y);$
7) $A\equiv((x\vee z)\&x\to y)\downarrow(\bar{y}\to\bar{x});$
8) $A\equiv(x\to(x|z)\&y)\&(x\to\bar{y})\downarrow\bar{x};$
9) $A\equiv(x\leftrightarrow y)\&(x\vee(x\mid z)\&y);$
10) $A\equiv(x\downarrow y)\&(y\to z)\to(z\to(x\vee z)\&x);$
11) $A\equiv(x\to y)\mid(x\vee z)\&(y\to z)\to(x\to z);$
12) $A\equiv(x\leftrightarrow y\to y\&z)\downarrow((x\to y)\to(z\to y));$

13) $A\equiv(y\leftrightarrow z) \mid ((x\vee z)\&x\vee z)$;

14) $A\equiv z\downarrow(x\vee z)\&y\to(y\vee z\to x)$.

2.3.16. Asosiy tengkuchliliklardan foydalanib, shunday almashtirishlar kiritingki, quyidagi formulalarda faqat ($\to$) implikasiya va ($\bar{\ }$) inkor amallari ishtirok etsin:

1) $A\equiv(x\&y)\to z$;

2) $A\equiv x\&y\to(\bar{y}\vee x\to z)$;

3) $A\equiv(x\&y)\to(x\vee y\&z)$;

4) $A\equiv(x\vee z)\&(y\to(u\&\bar{x}))$;

5) $A\equiv(x\&y)\to x$;

6) $A\equiv x\to(x\&y)$;

7) $A\equiv(x\&y)\to(\bar{y}\to\bar{x})$;

8) $A\equiv(x\to y)\leftrightarrow(x\to\bar{y})\&\bar{x}$;

9) $A\equiv(x\leftrightarrow y)\leftrightarrow(x\vee y)$;

10) $A\equiv(x\to y)\&(y\to z)\leftrightarrow(z\to x)$;

11) $A\equiv(x\to y)\vee(y\to z)\ v\ (x\to z)$;

12) $A\equiv(y\leftrightarrow z)\leftrightarrow(x\vee z)$;

13) $A\equiv z\to y\to(y\vee z\to x)$.

2.3.17. Quyidagi aynan chin mulohazalar sistemani teng kuchli soddaroq sistemaga keltiring:

1) $C\to(A\vee B),\ (B\wedge C)\to A,\ (A\wedge B)\to C$;

2) $A\to(B\vee C),\ B\to(A\vee C),\ (A\wedge B)\to C$;

3) $A\to B,\ A\to(B\vee C),\ B\to C$;

4) $P\to(Q\vee R),\ W\to(S\vee T),\ R\to(Q\vee\neg P),\ (W\wedge T)\to\neg S$;

5) $W \to (M \vee S),\ R \to T,\ \neg Q \to T,\ M \to (S \vee W),\ P \to (T \vee R);$

6) $\neg A \to (B \vee C),\ B \to \neg(A \wedge C), C \to (A \vee \neg B),\ A \to (B \vee C),$

$(A \wedge C) \to B,\ (\neg A \wedge \neg B) \to C;$

7) $P \to Q,\ P \wedge Q,\ Q \to P;$

8) $P \to Q,\quad Q \to \neg P\ \ R \to P;$

9) $A \vee C,\ B \to C,\ B \wedge C \to A;$

10) $A \wedge B,\ B \to C,\ C \to (A \vee B);$

11) $A \to B,\ C \to B,\ B \wedge C \to A;$

Yechim: 11)

$$(A \to B) \wedge (C \to B) \wedge (B \wedge C \to A) = (\overline{A} \vee B) \wedge (\overline{C} \vee B) \wedge (\overline{B \wedge C} \vee A) = =$$

$$(\overline{A} \vee B) \wedge (\overline{C} \vee B) \wedge (\overline{B} \vee \overline{C} \vee A) = (\overline{A} \vee B) \wedge ((A \wedge \overline{A}) \vee \overline{C} \vee B) \wedge$$

$$\wedge (\overline{B} \vee \overline{C} \vee A) = (\overline{A} \vee B) \wedge (A \vee B \vee \overline{C}) \wedge (\overline{A} \vee B \vee \overline{C}) \wedge (\overline{B} \vee \overline{C} \vee A) =$$

$= (\overline{A} \vee B) \wedge (A \vee \overline{C}) = (A \to B) \wedge (C \to A).$ Bundan $A \to B,\ C \to A.$

2.3.18. Quyidagi mulohazalar sistemani teng kuchli soddaroq sistemaga keltiring (mulohazalardan kamida birtasi chin qiymat qabul qiladi):

1) $\neg(A \to B),\ \neg B \wedge A,\ \neg D \wedge C,\ \neg(C \to A);$

2) $A \wedge B \wedge C,\ \neg A \wedge \neg B \wedge \neg C, A \wedge B \wedge \neg C,\ \neg(A \vee B \vee \neg C);$

3) $P \to Q,\ \neg(Q \to P),\ P \wedge Q;$

4) $A \wedge \neg B \wedge C,\ A \wedge \neg(B \to \neg C), A \wedge \neg(B \vee C);$

5) $A \wedge \neg B \wedge C,\ \neg(A \to C) \wedge \neg B,\ \neg A \wedge \neg(C \to B);$

6) $\neg A \vee B,\ A \leftrightarrow B;$

7) $A \to B,\ B \vee C,\ A \wedge C;$

8) $A \to (C \vee B),\ B \vee (A \to C),\ C \vee (A \to B),\ B \vee C;$

9) $A \wedge B \wedge \neg C,\ \neg(A \to C) \wedge \neg A,\ A \wedge (B \vee C);$

10) $\neg A \wedge C,\ A \to B,\ B \vee (A \to C);$

11) $A\wedge B\wedge\neg C,\ \neg(A\to B)\wedge\neg C,\ \neg A\wedge\neg(B\to C).$

Yechim: 11)

$(A\wedge B\wedge\neg C)\vee(\neg(A\to B)\wedge\neg C)\vee(\neg A\wedge\neg(B\to C))=$

$=(A\wedge B\wedge\neg C)\vee(\neg(\neg A\vee B)\wedge\neg C)\vee(\neg A\wedge\neg(\neg B\vee C))=$

$=(A\wedge B\wedge\neg C)\vee(A\wedge\neg B\wedge\neg C)\vee(\neg A\wedge B\wedge\neg C)=$

$=(A\wedge B\wedge\neg C)\vee(A\wedge\neg B\wedge\neg C)\vee(A\wedge B\wedge\neg C)\vee(\neg A\wedge B\wedge\neg C)=$

$=(A\wedge\neg C)\vee(B\wedge\neg C).$ Bundan, $A\wedge\neg C,\ B\wedge\neg C.$

2.4. Mukammal diz'yunktiv va kon'yunktiv normal shakllar.

Elementar diz'yunkstiya (elementar kon'yunkstiya) ***to'g'ri elementar diz'yunkstiya (elementar kon'yunkstiya)*** deb aytiladi, shunda va faqat shundagina, qachonki elementar diz'yunkstiya (elementar kon'yunkstiya) ifodasida har bir elementar mulohaza x_i bir marta qatnashgan bo'lsa.

Masalan, $x_1\vee x_2\vee x_3$ va $\overline{x_1}\vee x_4\vee x_6$ elementar diz'yunkstiyalar va $x_1\wedge x_2\wedge x_3$ va $x_1\wedge\overline{x_3}\wedge x_6$ elementar kon'yunkstiyalar mos ravishda to'g'ri elementar diz'yunkstiyalar va elementar kon'yunkstiyalar deb aytiladi.

Elementar diz'yunkstiya (elementar kon'yunkstiya) $x_1, x_2,\dots, x_n$ mulohazalarga nisbatan ***to'liq elementar diz'yunkstiya (elementar kon'yunkstiya)*** deb aytiladi, qachonki ularning ifodasida $x_1, x_2,\dots, x_n$ mulohazalarning har bittasi bir matragina qatnashgan bo'lsa.

Masalan, $x_1\vee\overline{x_2}\vee x_3$ va $\overline{x_1}\vee\overline{x_2}\vee x_3$ elementar diz'yunkstiyalar va $\overline{x_1}\wedge\overline{x_2}\wedge\overline{x_3}$, $x_1\wedge x_2\wedge\overline{x_3}$ elementar kon'yunkstiyalar x_1, x_2, x_3 mulohazalarga nisbatan to'liq elementar diz'yunkstiyalar va elementar kon'yunkstiyalar bo'ladi.

Diz'yunktiv normal shakl (kon'yunktiv normal shakl) ***MDNSh (MKNSh)*** deb aytiladi, agar DNSh (KNSh) ifodasida bir xil elementar kon'yunkstiyalar (elementar diz’yunkstiyalar) bo’lmasa va hamma elementar kon’yunkstiyalar (elementar diz’yunkstiyalar) to’g’ri va to’liq bo’lsa.

Asosiy mantiqiy amallarning MDNSh va MKNSh ko’rinishlari quyidagicha bo’ladi:

a) MDNSh: $\bar{x}=\bar{x}$; $xy=xy$; $x\vee y=xy\vee \bar{x}\, y\vee \bar{y}$; $x\to y=xy\vee \bar{x}\, y\vee \bar{x}\, \bar{y}$; $x\leftrightarrow y=xy\vee \bar{x}\, \bar{y}$.

b) MKNSh: $\bar{x}=\bar{x}$; $xy=(\bar{x}\vee y)\,(x\vee \bar{y})\,(x\vee y)$;
$x\vee y=x\vee y$; $x\to y=\bar{x}\vee y$; $x\leftrightarrow y=(\bar{x}\vee y)\,(x\vee \bar{y})$.

Masalan, $x\wedge y\wedge z\vee x\wedge y\wedge \bar{z}\vee \bar{x}\wedge y\wedge z\vee x\wedge \bar{y}\wedge z$ DNSh x, y, z mulohazalarga nisbatan MDNSh bo’ladi. $(x\vee y)\wedge(x\vee \bar{y})\wedge(\bar{x}\vee y)$ KNSh x, y mulohazalarga nisbatan MKNSh bo’ladi.

Misol. 1. $A=(\overline{x\vee z})\wedge(x\to y)$;

$$A=(\overline{x\vee z})\wedge(x\to y)=\bar{x}\wedge\bar{z}\wedge(\bar{x}\vee y)=(\bar{x}\vee(\bar{y}\wedge y)\vee(\bar{z}\wedge z))\wedge$$
$$\wedge(\bar{z}\vee(\bar{y}\wedge y)\vee(\bar{x}\wedge x))\wedge(\bar{x}\vee y\vee(\bar{z}\wedge z))=\{(\bar{x}\vee\bar{y}\vee\bar{z})\wedge(\bar{x}\vee\bar{y}\vee z)\wedge$$
$$\wedge(\bar{x}\vee y\vee\bar{z})\wedge(\bar{x}\vee y\vee z)\}\wedge\{(x\vee y\vee\bar{z})\wedge(\bar{x}\vee y\vee\bar{z})\wedge(x\vee\bar{y}\vee\bar{z})\wedge$$
$$\wedge(\bar{x}\vee\bar{y}\vee\bar{z})\}\wedge\{(x\vee\bar{y}\vee\bar{z})\wedge(x\vee\bar{y}\vee z)\}=(\bar{x}\vee\bar{y}\vee\bar{z})\wedge(\bar{x}\vee\bar{y}\vee z)\wedge$$
$$\wedge(\bar{x}\vee y\vee\bar{z})\wedge(\bar{x}\vee y\vee z)\wedge(x\vee y\vee\bar{z})\wedge(x\vee\bar{y}\vee\bar{z})\wedge(x\vee\bar{y}\vee z).$$

2. $A=(x\vee y)\wedge(y\vee z)\wedge(z\vee t)$

$$A=[z\vee y\vee(z\wedge\bar{z})\vee(t\wedge\bar{t})]\wedge[(x\wedge\bar{x})\vee y\vee z\vee(t\wedge\bar{t})]\wedge[(x\wedge\bar{x}$$
$$)\vee(y\wedge\bar{y})\vee\ \vee z\vee t)=[(x\vee y\vee z\vee t)\wedge(x\vee y\vee\bar{z}\vee t)\wedge$$
$$\wedge(x\vee y\vee z\vee\bar{t})\wedge(x\vee y\vee\bar{z}\vee\bar{t})]\wedge[(x\vee y\vee z\vee t)\wedge$$
$$\wedge(\bar{x}\vee y\vee z\vee t)\wedge(x\vee y\vee z\vee\bar{t})\wedge(\bar{x}\vee y\vee z\vee\bar{t})]\wedge$$
$$\wedge[(x\vee y\vee z\vee t)\wedge x\vee\bar{y}\vee z\vee t)\wedge(\bar{x}\vee y\vee z\vee t)\wedge(\bar{x}\vee\bar{y}\vee z\vee t)]$$

Muammoli masala va topshiriqlar:

2.4.1. Quyidagi formulalarni DNSH ga keltiring:

1) $(X \leftrightarrow Y) \wedge \neg(Z \to T)$;

2) $((X \to Y) \to (Z \to \neg X)) \to (Y \to \neg Z)$;

3) $(X \to (Y \to Z)) \to ((X \to \neg Z) \to (X \to \neg Y))$;

4) $((X \to Y) \to \neg Z) \to (X \vee (X \leftrightarrow Z))$;

5) $(X \to Y) \to Z$;

6) $X \to (Y \to Z)$;

7) $(\neg X \wedge \neg Y) \vee (X \leftrightarrow Z)$;

8) $(X \leftrightarrow Z) \to (X \wedge Z)$;

9) $(X \leftrightarrow Y) \to ((\neg X \to Z) \to \neg Y)$;

10) $(X \vee \neg(Y \to Z)) \wedge (X \vee Z)$;

11) $\neg(X \vee Z) \wedge (X \to Y)$.

Yechim:11) $\neg(X \vee Z) \wedge (X \to Y) = \neg X \wedge \neg Z \wedge (\neg X \vee Y) = (\neg X \wedge \neg Z \wedge \neg X) \vee$
$\vee (\neg X \wedge \neg Z \wedge Y) = (\neg X \wedge \neg Z) \vee (\neg X \wedge \neg Z \wedge Y)$.

2.4.2. 2.4.1. misoldagi formulalarni KNSH ga keltiring.

Yechim:11 $\neg(X \vee Z) \wedge (X \to Y) = \neg X \wedge \neg Z \wedge (\neg X \vee Y) = (\neg X \wedge \neg Z \wedge \neg X) \vee$
$\vee (\neg X \wedge \neg Z \wedge Y) = (\neg X \wedge \neg Z) \vee (\neg X \wedge \neg Z \wedge Y) = (\neg X \wedge \neg Z) \wedge (1 \vee Y) =$
$= \neg X \wedge \neg Z$.

2.4.3. 2.4.1. misoldagi formulalarni MDNSH ga keltiring.

Yechim:11 $\neg(X \vee Z) \wedge (X \to Y) = \neg X \wedge \neg Z \wedge (\neg X \vee Y) = (\neg X \wedge \neg Z \wedge \neg X) \vee$
$\vee (\neg X \wedge \neg Z \wedge Y) = (\neg X \wedge (Y \vee \neg Y) \neg Z) \vee (\neg X \wedge \neg Z \wedge Y) =$
$(\neg X \wedge Y \wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge \neg Z) \vee (\neg X \wedge \neg Z \wedge Y)$.

2.4.4. 2.4.1. misoldagi formulalarni MKNSH ga keltiring.

Yechim:11 $\neg(X \vee Z) \wedge (X \to Y) = \neg X \wedge \neg Z \wedge (\neg X \vee Y) = (\neg X \wedge \neg Z \wedge \neg X) \vee$
$\vee (\neg X \wedge \neg Z \wedge Y) = (\neg X \wedge \neg Z) \vee (\neg X \wedge \neg Z \wedge Y) = (\neg X \wedge \neg Z) \wedge (1 \vee Y) =$
$= \neg X \wedge \neg Z = \neg X \wedge (Y \vee \neg Y) \wedge \neg Z = (\neg X \wedge Y \wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge \neg Z).$

2.4.5. Quyidagi qiymatlarni qabul qiladigan o'zgaruvchilar yordamida shunday elementar kon'yunksiya tuzingki, natijada formula 1 ga teng bo'lsin:

1) (0,0);	5) (0,0,1);	9) (1, 0,1);
2) (1,0);	6) (1,0,0,1);	10) (1,1,1,0);
3) (1,1);	7) (0,1,0,0);	11) (0,1,1).
4) (1,0,0);	8) (0,0,0,1);	

2.4.6. Quyidagi qiymatlar satrida *F* funksiya 1 qiymatni qabul qilsa, MKNSh dan foydalanib, *F* ni toping.

1) *F*(0,0)= *F*(0,0)=1; 2) *F*(1,0)=1;

3) *F*(0,1,0)= *F*(1,0,1)= *F*(1,1,1)=1; 4) *F*(0,1,1)=) *F*(1,1,0)=1;

5) *F*(1,0,0)= *F*(0,1,0)= *F*(0,0,1)=1;

6) *F*(0,1,1)= *F*(1,0,1)= *F*(1,1,0)= *F*(1,1,1)=1;

7) *F*(1,0,1)= *F*(0,1,0)= *F*(0,0,0)=1;

8) *F*(0,1)= *F*(1,0)= *F*(1,1)=1;

9) *F*(1,1,0,0)= *F*(0,0,1,1)= 1;

10) *F*(0,1,0,1)= *F*(1,0,1,0)= *F*(1,0,0,0)= *F*(1,1,1,1)=1;

11) *F*(0,0,0)=*F*(0,1,0)= *F*(1,1,1)=1;

2.4.7. Quyidagi qiymatlarni qabul qiladigan o'zgaruvchilar yordamida shunday elementar diz'yunksiya tuzingki, natijada formula 0 ga teng bo'lsin:

1) (0,0);	5) (0,0,1);	9) (0,1,1,0);
2) (1,0);	6) (1,0,0,1);	10) (1,1,1);
3) (1,1);	7) (0,1,0,0);	11) (0,1,1).

4) (1,0,1); 8) (1,0,1,1);

Yechim: 11) Elementar diz'yunksiya $Y_1 \vee Y_2 \vee Y_3$ ko'rinishda bo'lib, 0 ga teng bo'lishi uchun har biri 0 ga teng bo'lishi kerak: $Y_1 = 0, Y_2 = 0, Y_3 = 0$. Demak, (0,1,1) uchun $X_1 \vee \neg X_2 \vee \neg X_3$ bo'lsa, $X_1 = 0, \neg X_2 = 0, \neg X_3 = 0$ bo'ladi. Bundan $X_1 = 0, X_2 = 1, X_3 = 1$.

2.4.8. Quyidagi qiymatlar satrida F funksiya 0 qiymatni qabul qilsa, MKNSh dan foydalanib, F ni toping.

1) $F(0,1) = F(1,1) = 0;$ 2) $F(0,1) = 0;$

3) $F(0,1,1) = F(1,1,1) = 0;$ 4) $F(1,0,0) = F(1,0,1) = 0;$

5) $F(0,1,1) = F(0,0,0) = F(0,1,0) = 0;$

6) $F(1,1,1) = F(0,0,1) = F(1,1,0) = F(1,0,0) = 0;$

7) $F(1,1,0,1) = F(0,0,1,0) = F(1,0,1,0) = F(0,0,1,1) = F(0,0,0,0) = 0;$

8) $F(0,1) = F(1,0) = F(1,1) = 0;$

9) $F(1,0,0,0) = F(0,1,1,1) = 0;$

10) $F(0,1,0,1) = F(1,0,1,0) = F(1,0,0,1) = F(0,1,01,0);$

11) $F(0,0,0) = F(0,1,0) = F(1,1,1) = 0;$

Yechim: 11) Mukammal kon'yunktiv normal shaklning elementar diz'yunksiyalarini topish uchun birinchi shartdan $(X, Y, Z) = (0,0,0)$, ikkinchi shartdan $(X, Y, Z) = (0,1,0)$ va uchinchi shart bo'yicha $(X, Y, Z) = (1,1,1)$ bo'lishi kerak. Demak, $X \vee Y \vee Z$, $X \vee \neg Y \vee Z$ va $\neg X \vee \neg Y \vee \neg Z$ ekanligini aniqlatmiz. Bundan, $F(X, Y, Z)$ funksiya elementar diz'yunksiyalarining kon'yunksiyasi bo'ladi.

Ya'ni: $F(X,Y,Z) = (X \vee Y \vee Z) \wedge (X \vee \neg Y \vee Z) \wedge (\neg X \vee \neg Y \vee \neg Z).$

2.4.9. Quyidagi mulohazalar algebrasi formulalarining har biri uchun chinlik jadvalini tuzib, MDN shaklini toping:

1) $X \to Y$;   2) $(X \wedge Y) \wedge Z$;

3) $(X \leftrightarrow Z) \to (X \wedge \neg Y)$;   4) $X \vee (Y \to (Z \leftrightarrow (X \wedge Y)))$;

5) $((X \wedge \neg Y) \vee Z) \wedge T$;   6) $(X \leftrightarrow Y) \wedge (Y \leftrightarrow Z) \wedge (Z \leftrightarrow T)$;

7) $((X \vee Y) \to Z) \leftrightarrow \neg X$;   8) $(\neg Z \to \neg Y) \to ((X \wedge \neg Z) \wedge Y)$;

9) $(X \leftrightarrow Y) \wedge (\neg Z \to (T \wedge \neg X))$;   10) $((X \vee \neg Z) \wedge Y) \leftrightarrow ((Y \vee \neg X) \wedge Z)$;

11) $\neg(X \wedge Y) \to \neg(X \vee Z)$.

Yechim: 7) Berilgan formula uchun chinlik jadvalini tuzamiz:

X	Y	Z	$X \vee Y$	$X \vee Y \to Z$	$\neg X$	$(X \vee Y \to Z) \leftrightarrow \neg X$
0	0	0	0	1	1	1
0	0	1	0	1	1	1
0	1	0	1	0	1	0
0	1	1	1	1	1	1
1	0	0	1	0	0	1
1	0	1	1	1	0	0
1	1	0	1	0	0	1
1	1	1	1	1	0	0

Endi, formulaning qiymati 1 ga teng bo’ladigan satrdagi o’zgaruvchilarning qiymatini tanlab olamiz: $F(0,0,0)=F(0,0,1)=F(0,1,1)=F(1,0,0)=F(1,1,0)=1$. Har biri uchun elementar kon’yunksiya tuzamiz: $\neg X \wedge \neg Y \wedge \neg Z$, $\neg X \wedge \neg Y \wedge Z$, $\neg X \wedge Y \wedge Z$, $X \wedge \neg Y \wedge \neg Z$ va $X \wedge Y \wedge \neg Z$. Nihoyat, elementar kon’yunksiyalarning diz’yunksiyasini tuzib, quyidagi formulaga ega bo’lamiz:

$$F(X,Y,Z) = (\neg X \wedge \neg Y \wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge Z) \vee (\neg X \wedge Y \wedge Z) \vee$$
$$\vee (X \wedge \neg Y \wedge \neg Z) \vee (X \wedge Y \wedge \neg Z).$$

2.4.10. Quyidagi mulohazalar algebrasi formulalarining har biri uchun chinlik jadvalini tuzib, MKN shaklini toping:

1) $X \to Y$;
2) $(X \wedge Y) \wedge Z$;
3) $(X \leftrightarrow Z) \to (X \wedge \neg Y)$;
4) $X \vee (Y \to (Z \leftrightarrow (X \wedge Y)))$;
5) $((X \wedge \neg Y) \vee Z) \wedge T$;
6) $(X \leftrightarrow Y) \wedge (Y \leftrightarrow Z) \wedge (Z \leftrightarrow T)$;
7) $((X \vee Y) \to Z) \leftrightarrow \neg X$;
8) $(\neg Z \to \neg Y) \to ((X \wedge \neg Z) \wedge Y)$;
9) $(X \leftrightarrow Y) \wedge (\neg Z \to (T \wedge \neg X))$;
10) $((X \vee \neg Z) \wedge Y) \leftrightarrow ((Y \vee \neg X) \wedge Z)$;
11) $\neg (X \wedge Y) \to \neg (X \vee Z)$.

Yechim: 7) Berilgan formula uchun chinlik jadvalini tuzamiz:

X	Y	Z	$X \vee Y$	$X \vee Y \to Z$	$\neg X$	$(X \vee Y \to Z) \leftrightarrow \neg X$
0	0	0	0	1	1	1
0	0	1	0	1	1	1
0	1	0	1	0	1	0
0	1	1	1	1	1	1
1	0	0	1	0	0	1
1	0	1	1	1	0	0
1	1	0	1	0	0	1
1	1	1	1	1	0	0

Endi, formulaning qiymati 0 ga teng bo’ladigan satrdagi o’zgaruvchilarning qiymatini tanlab olamiz: $F(0,1,0)=F(1,0,1)=F(1,1,1)=0$. Har biri uchun elementar diz’yunksiya tuzib olamiz: $X \vee \neg Y \vee Z$, $\neg X \vee Y \vee \neg Z$ va $X \vee Y \vee Z$. Nihoyat, elementar diz’yunksiyalarning kon’yunksiyasini tuzib, quyidagi formulaga ega bo’lamiz:

$$F(X,Y,Z) = (X \vee \neg Y \vee Z) \wedge (\neg X \vee Y \vee \neg Z) \wedge (X \vee Y \vee Z).$$

2.4.11. **1.**Quyidagi berilgan formulalarning chinlik to’plamini toping:

1) $A = xy \vee x\bar{y} \vee \bar{x}y$;

2) $B = (x \vee \bar{y})(\bar{x} \vee y)(\bar{x} \vee \bar{y})$;

3) $C = xyz \vee \bar{x}yz \vee x\overline{yz}$;

4) $D = (x \vee y \vee \bar{z})(\bar{x} \vee y \vee z)(x \vee \bar{y} \vee \bar{z})$.

5) $E = \overline{x\bar{y}} \leftrightarrow \bar{x} \vee xy$;

6) $F = (x \leftrightarrow y) \wedge (x\bar{y} \vee \bar{x}y)$;

7) $G = xy \to (x \to \bar{y})$;

8) $J = x \vee y \to (x \leftrightarrow y)$;

9) $L = x \vee y \to z$;

10) $M = (x \to z)(y \to z) \to (x \to y)$.

2. Yuqorida keltirilgan formulalardan tuzilgan $A \vee B$, $A \vee C$, $A \vee D$, $A \vee F$, $A \wedge B$, $A \wedge C$, $A \wedge D$, $A \wedge F$, $A \to B$, $A \to C$, $A \leftrightarrow D$, $A \leftrightarrow F$, $C \to D$, $C \to F$, $C \leftrightarrow D$, $C \leftrightarrow B$, $C \leftrightarrow F$, $F \leftrightarrow E$, $A \to B \to C$, $A \to F \to C$, $(A \leftrightarrow F) \to C$, $(A \leftrightarrow F) \to D$, $A \leftrightarrow F \leftrightarrow E$, $E \to B$, $E \to C$, $E \leftrightarrow D$, $E \leftrightarrow F$, $G \to B$, $G \to C$, $G \leftrightarrow D$, $G \leftrightarrow F$, $J \to B$, $J \to C$, $J \leftrightarrow D$, $J \leftrightarrow F$, $L \to B$, $L \to C$, $L \leftrightarrow D$, $L \leftrightarrow F$, $M \to B$, $M \to C$, $M \leftrightarrow D$, $M \leftrightarrow F$, $E \to F \to L$, $M \to J \to G$, $(L \leftrightarrow E) \to M$, $(A \leftrightarrow G) \to F$, $A \leftrightarrow M \leftrightarrow J$ murakkab formulalarning chinlik to’plamini toping.

2.4.12. Quyidagi formulalarni mukammal kon’yunktiv normal shaklga keltiring:

1. $A \equiv (\bar{x} \to \bar{y}) \to ((y \leftrightarrow z) \to (x \& z))$;

2. $A \equiv ((x \leftrightarrow y) \to \bar{x}) \downarrow (x \to (y \& x))$;

3. $A \equiv ((\overline{x \& y}) \to \bar{x}) \leftrightarrow ((\overline{x \& y}) \to \bar{y})$;

4. $A \equiv ((\overline{x \& y}) \leftrightarrow \bar{x}) \& ((\overline{x \& y}) \to \bar{y})$;

5. $A\equiv(x\leftrightarrow\bar{z})\rightarrow y\&z;$

6. $A\equiv(\bar{x}\rightarrow z)\rightarrow(\bar{y}\leftrightarrow\bar{x});$

7. $A\equiv(x\leftrightarrow y\rightarrow y\&z)\rightarrow((x\rightarrow y)\rightarrow(z\rightarrow y)).$

2.4.13. Quyidagi formulalarni mukammal diz’yunktiv normal shaklga keltiring:

1 $A\equiv(z\leftrightarrow x)\rightarrow((\overline{x\vee z})\rightarrow x);$

2 $A\equiv((\overline{x\&y})\rightarrow x)\vee(x\&(y\vee z));$

3 $A\equiv\overline{x\&(x\vee z)}\leftrightarrow(x\&y\vee z);$

4 $A\equiv(x\&y\mid y\&z)\rightarrow((x\leftrightarrow y)\rightarrow(z\rightarrow y));$

5 $A\equiv((\overline{x\&y})\leftrightarrow\bar{x})\downarrow((\overline{x\&y})\rightarrow\bar{y});$

6 $A\equiv(x|\bar{z})\rightarrow y\&z;$

7 $A\equiv(\bar{x}\rightarrow z)|(\bar{y}\leftrightarrow\bar{x}).$

2.4.14. Formulalarning aynan chinlik yoki aynan yolg’onlik alomatlaridan foydalanib, quyidagi formulalarning aynan chin, aynan yolg’on yoki bajariluvchi ekanligini ko’rsating:

1) $A\equiv(\overline{x\&y})\leftrightarrow(\bar{x}\vee(x\&y));$

2) $A\equiv(x\leftrightarrow y)\leftrightarrow((x\&\bar{y})\vee(\bar{x}\&y));$

3) $A\equiv(x\&y)\rightarrow(x\rightarrow\bar{y});$

4) $A\equiv(x\rightarrow y)\downarrow(\bar{y}\rightarrow\bar{x});$

5) $A\equiv(\bar{y}\rightarrow\bar{x})|(x\rightarrow y);$

6) $A\equiv(x\rightarrow y)\&(x\rightarrow\bar{y})\leftrightarrow\bar{x};$

7) $A\equiv(x\rightarrow(y\rightarrow z))\rightarrow((x\rightarrow y)\rightarrow(x\rightarrow z));$

8) $A\equiv(z\rightarrow x)\rightarrow((z\rightarrow y)\rightarrow(z\rightarrow(x\&y)));$

9) $A\equiv(x\rightarrow z)\rightarrow((y\rightarrow z)\rightarrow((x\vee y)\rightarrow z));$

10) $A\equiv(x\rightarrow(y\rightarrow z))\rightarrow(x\&y\rightarrow z);$

11) $A\equiv(x\&y\rightarrow z)\rightarrow(x\rightarrow(y\rightarrow z)).$

2.4.15. $f(x,y,z)$ funkstiya shunda va faqat shunda chin qiymat oladiki, qachon o’zgaruvchilarning faqat bittasi chin qiymat olsa. $f(x,y,z)$ funkstiyaning chinlik jadvalini tuzing va uni formula orqali ifodalang.

2.4.16. Chinlik jadvalidan $f_1(x,y,z)$, $f_2(x,y,z)$, $f_3(x,y,z)$, $f_4(x,y,z)$, $f_5(x,y,z)$ funksiyalarni ifodalovchi formulalarni toping va ularni soddalashtiring:

x	y	z	$f_1(x,y,z)$	$f_2(x,y,z)$	$f_3(x,y,z)$	$f_4(x,y,z)$	$f_5(x,y,z)$
1	1	1	0	0	1	1	0
1	1	0	1	1	1	0	0
1	0	1	0	0	0	1	0
1	0	0	1	1	1	1	1
0	1	1	0	0	0	0	0
0	1	0	0	1	1	1	0
0	0	1	1	0	1	0	1
0	0	0	0	0	0	0	0

2.4.17. Quyidagi mukammal normal shakldagi formulalarning chinlik jadvalini tuzing:

1) $xy \vee x\bar{y} \vee \bar{x}y$;

2) $(x \vee \bar{y})(\bar{x} \vee y)(\bar{x} \vee \bar{y})$;

3) $xyz \vee \bar{x}yz \vee x\bar{y}z$;

4) $(x \vee y \vee \bar{z})(\bar{x} \vee y \vee z)(x \vee \bar{y} \vee \bar{z})$;

5) $x\bar{y}z \vee \bar{x}yz \vee xy\bar{z} \vee x\bar{y}\bar{z}$;

6) $(x \vee y \vee z)(\bar{x} \vee y \vee \bar{z})(\bar{x} \vee \bar{y} \vee z)(x \vee \bar{y} \vee \bar{z})(\bar{x} \vee \bar{y} \vee \bar{z})$;

7) $(x \vee y \vee z \vee t)(\bar{x} \vee y \vee \bar{z} \vee \bar{t})(\bar{x} \vee \bar{y} \vee z \vee t)$.

2.4.18. Bir, ikki va uch argumentli har qanday aynan chin bo'lgan formulalarning MDNSh ko'rinishini toping.

2.4.19. Bir, ikki va uch argumentli har qanday aynan yolg'on bo'lgan formulalarning MKNSh ko'rinishini toping.

2.4.20. Mulohazalar mantiqi formulasi tavtologiya bo'lishi uchun uning MKNSh.dagi har bir elementar diz'yunksiyasida kamida birta mulohaza inkori bilan qatnashishi kerakligini isbotlang.

2.4.21. Mulohazalar mantiqi formulasi aynan yolg'on bo'lishi uchun uning MDNSh.dagi har bir elementar kon'yunksiyasida kamida birta mulohaza inkori bilan qatnashishi kerakligini isbotlang.

2.4.22. Yuqoridagi 2.4.20. va 2.4.21. shartlardan foydalanib, quyidagi formulalarning tavtologiya yoki aynan yolg'on formula ekanligini ko'rsating:

1) $X \to (Y \to Z)) \to ((X \to \neg Z) \to (X \to \neg Y))$;

2) $\neg(P \to Q) \wedge \neg(Q \to P)$;

3) $(X \to Y) \wedge (X \to \neg Y) \wedge X$;

4) $(P \wedge Q) \to (P \leftrightarrow Q)$;

5) $(P \wedge Q) \to (P \to Q)$;

6) $(\neg P \to Q) \vee \neg P$;

7) $\neg(P \to (Q \to P))$;

8) $P \to (Q \to (P \wedge Q))$;

9) $(P \to Q) \vee (Q \to P)$;

10) $(X \leftrightarrow Y) \wedge ((X \wedge \neg Y) \vee (\neg X \wedge Y))$;

11) $(X \vee Z) \vee \neg Z) \to (\neg(X \vee Z) \wedge X \wedge Y)$.

Yechim: 11) Quyidagi teng kuchli almashtirishlar bajaramiz:

$$((X \vee Z) \vee \neg Z) \to (\neg(X \vee Z) \wedge X \wedge Y) \cong \neg((X \vee Z) \vee \neg Z) \vee (\neg(X \vee Z) \wedge X \wedge Y) \cong$$

$$\cong \neg((X \vee Z) \vee \neg Z) \vee (\neg(X \vee Z) \wedge X \wedge Y) \cong (\neg X \wedge \neg Z \wedge Z) \vee (\neg X \wedge \neg Z \wedge X \wedge Y) =$$

$$(\neg X \wedge \underline{\underline{\neg Z \wedge Z}}) \vee (\underline{\underline{\neg X \wedge X}} \wedge Y \wedge Z) = 0 \vee 0 = 0.$$

2.4.23. Tavtologiya MKNSh.ga ega emasligini isbotlang.

2.4.24. Aynan yolg'on formulalar MDNSh.ga ega emasligini isbotlang.

2.4.25. Tengkuchli formulalar yagona mukammal shaklga ega ekanligidan foydalanib, quyidagi formulalar uchun MKN yoki MDN shaklini toping va tengkuchli yoki tengkuchli emasligini aniqlang:

1) $\neg(X \wedge \neg Y) \to (\neg Y \to X)$ va $\neg(X \to Y) \vee X \vee Y$

2) $(X \vee (X \to Y) \wedge Y$ va $Y \wedge (\neg(Y \to X) \vee X)$

3) $\neg((P \to Q) \wedge (Q \to \neg P))$ va $Q \to \neg P$

4) $(X \leftrightarrow Z) \to (X \wedge \neg Y)$ va $((X \wedge \neg Z) \vee (Z \wedge \neg X)) \vee (X \wedge \neg Y)$.

5) $(X \vee Y) \wedge (X \vee Z) \wedge (Y \vee Z)$ va $(X \wedge Y) \vee (X \wedge Z) \vee (Y \wedge Z)$

6) $(X \wedge Y) \wedge (X \to (Y \wedge Z))$ va $((X \to Y) \wedge \neg((X \to Z) \to (X \to \neg Y))$.

7) $(X \wedge Z) \to Y$ va $X \wedge (Z \to Y)$

8) $(X \to \neg Y) \to (Y \wedge Z)$. va $(X \wedge Y) \vee (Y \wedge Z)$

9) $(X \to Y) \to (Y \wedge Z)$ va $(Z \to \neg Y) \to (X \wedge \neg Y)$

10) $(X \to Y) \to Z$ va $X \to (Y \to Z)$

11) $(X \leftrightarrow Y) \leftrightarrow Z$ va $X \leftrightarrow (Y \leftrightarrow Z)$

12) $(P \vee Q) \wedge R$ va $P \vee (Q \wedge R)$

Yechim: 11) Quyidagi teng kuchli almashtirishlar bajaramiz:

Formulaning chap tomonini MKNSh.ga keltiramiz:

$$(X \leftrightarrow Y) \leftrightarrow Z \cong$$

$$= (\neg((\neg X \vee Y) \wedge (X \vee \neg Y)) \vee Z) \wedge (((\neg X \vee Y) \wedge (X \vee \neg Y)) \vee \neg Z) \cong$$

$$\cong (((X \wedge \neg Y) \vee (\neg X \wedge Y)) \vee Z) \wedge (\neg X \vee Y \vee \neg Z) \wedge (X \vee \neg Y \vee Z) \cong$$

$$\cong (((X \vee Y) \wedge (\neg X \wedge \neg Y)) \vee Z) \wedge (\neg X \vee Y \vee \neg Z) \wedge (X \vee \neg Y \vee \neg Z) \cong$$

$$(X\vee Y\vee Z)\wedge(\neg X\vee\neg Y\vee Z)\wedge(\neg X\vee Y\vee\neg Z)\wedge(X\vee\neg Y\vee\neg Z).$$

Endi formulaning o’ng tomonini MKNSh.ga keltiramiz

$$X\leftrightarrow(Y\leftrightarrow Z)=$$
$$=(X\vee Y\vee Z)\wedge(\neg X\vee\neg Y\vee Z)\wedge(\neg X\vee Y\vee\neg Z)\wedge(X\vee\neg Y\vee\neg Z).$$

Ko’rinib turibdiki formulalarning chap va o’ng tomoni bir xil MKNSh.ga ega. Demak, formulalar tengkuchli.

12) Formulaning chap tomonini MDNSh.ga keltiramiz:

$$(P\vee Q)\wedge R\cong(P\wedge R)\vee(Q\wedge R)\cong$$
$$=(P\wedge(Q\vee\neg Q)\wedge R)\vee((P\vee\neg P)\wedge Q\wedge R)\cong$$
$$\cong(P\wedge Q\wedge R)\vee(P\wedge\neg Q\wedge R)\vee(\neg P\wedge Q\wedge R).$$

Formulaning o’ng tomonini MDNSh.ga keltiramiz:

$$P\vee(Q\wedge R)\cong P\vee(P\wedge Q\wedge R)\vee(\neg P\wedge Q\wedge R\cong$$
$$(P\wedge Q)\vee(P\wedge\neg Q)\vee(P\wedge Q\wedge R)\vee(\neg P\wedge Q\wedge R)\cong$$
$$\cong(P\wedge Q\wedge\neg R)\vee(P\wedge\neg Q\wedge R)\vee(P\wedge\neg Q\wedge\neg R)\vee(P\wedge Q\wedge R)\vee(\neg P\wedge Q\wedge R).$$

Ko’rinib turibdiki formulalarning chap va o’ng tomoni bir xil MDNSh.ga ega emas. Demak, formulalar tengkuchli emas.

2.4.26. Shunday ikki o’zgaruvchili $F(X,Y)$ formula topingki, natijada quyidagi formulalar tavtologiya bo’lsin:

1) $((F\wedge Y)\to\neg X)\to((X\to\neg Y)\to F)$;

2) $((F\wedge Y)\to\neg X)\to((X\to Y)\to F)$;

3) $((\neg X\wedge F)\to\neg Y)\to((\neg Y\to X)\to F)$;

4) $(X\to(F\wedge Y))\to((X\to((X\wedge Y)\vee F)$;

5) $(F\to(X\vee Y))\to((F\to X)\to Y)$;

6) $((X\to Y)\vee F)\to(F\wedge(Y\vee\neg X))$;

7) $((F\to Y)\wedge\neg X)\to((X\wedge\neg Y)\to F)$;

8) $((F\vee Y)\wedge\neg X)\to((X\vee Y)\to F)$;

9) $(F \to (X \vee Y)) \to (\neg(X \wedge F) \to Y);$

10) $((\neg X \wedge Y) \vee F) \to ((F \vee (X \to Y));$

11) $((\neg X \vee \neg Y) \wedge F) \to (F \to (X \wedge Y));$

12) $((F \vee Y) \to X) \to ((F \to (Y \to X));$

Yechim: 11) Formula uchun chinlik jadvalini tuzib olamiz:

X	Y	$\neg X$	$\neg Y$	$\neg X \vee \neg Y$	$(\neg X \vee \neg Y) \wedge F$	$F \to (X \wedge Y)$	For.
0	0	1	1	1	F(0,0)	$\neg F(0,0)$	$\neg F(0,0)$
0	1	1	0	1	F(0,1)	$\neg F(0,1)$	$\neg F(0,1)$
1	0	0	1	1	F(1,0)	$\neg F(1,0)$	$\neg F(1,0)$
1	1	0	0	0	0	1	1

$F(X,Y)$ funksiyani shunday tanlash kerakki, $\neg F(1,0) = \neg F(0,1) =$ $= \neg F(0,0) = 1$

Bundan, F(1,0)= F(0,1)= F(0,0)=0. Agar F(1,1)=1 desak, $F(X,Y) = X \wedge Y.$

Agar F(1,1)=0 desak, u holda ixtiyoriy atnan yolg'on formula bo'lishi mumkin ekan.

Yechim: 12) Formula uchun chinlik jadvalini tuzib olamiz:

$((F \vee Y) \to X) \to ((F \to (Y \to X));$

X	Y	$F \vee Y$	$(F \vee Y) \to X$	$Y \to X$	$F \to (Y \to X)$	For.
0	0	F(0,0)	$\neg F(0,0)$	1	1	1
0	1	1	0	0	$\neg F(0,1)$	1
1	0	F(1,0)	1	1	1	1
1	1	1	1	1	1	1

Chinlik jadvalidan ko'rinib turibdiki, formula tavtologiya bo'lib, F(X,Y) ixtiyoriy formula bo'lishi mumkin.

2.4.27. Shunday uch o'zgaruvchili $F(X,Y,Z)$ formula topingki, natijada quyidagi formulalar tavtologiya bo'lsin:

1) $((Z \to (\neg Y \to \wedge X)) \to F) \to ((X \to Y) \wedge F \wedge Z);$

2) $((F \wedge Y) \to \neg X) \to ((X \to Z) \to F);$

3) $(((X \wedge Z) \to \neg Y) \vee F) \to (((Z \vee X) \wedge Y) \vee F);$

4) $(((\neg Y \wedge X) \to Z) \to F) \to (F \wedge (X \to Y) \wedge Z);$

5) $(((\neg X \vee Y) \to (Z \wedge \neg Y) \vee F);$

6) $((F \wedge (X \to Y)) \vee ((Y \to Z) \wedge F);$

7) $(((X \vee Y) \to Z) \to F) \to (F \wedge ((\neg X \to Y) \wedge \neg Z));$

8) $((X \to Y) \to F) \vee ((X \vee Y) \wedge F \wedge Z);$

9) $(F \to (\neg Z \wedge Y) \to X)) \to (X \wedge (Z \to Y) \wedge F);$

10) $((F \to (Z \wedge Y)) \wedge Z) \vee (Z \to ((X \wedge \neg Y) \vee F);$

11) $(X \to Y) \vee Z \vee F)) \to (((Y \to X) \to Z) \to F);$

Yechim: 11) Formula uchun chinlik jadvalini tuzib olamiz:

Quyidagi belgilash kiritamiz: (1) - $(Y \to X) \to Z);$ (2)- $((Y \to X) \to Z) \to F;$

(3)- $((X \to Y) \vee Z \vee F) \to (((Y \to X) \to Z) \to F));$

X	Y	Z	$X \to Y$	$(X \to Y) \vee Z$	$(X \to Y) \vee Z \vee F$	$Y \to X$	(1)	(2)	(3)
0	0	0	1	1	1	1	0	1	1
0	0	1	1	1	1	1	1	F	F
0	1	0	1	1	1	0	1	F	F
0	1	1	1	1	1	0	1	F	F
1	0	0	0	0	F	1	0	1	1
1	0	1	0	1	1	1	1	F	F
1	1	0	1	1	1	1	0	1	1
1	1	1	1	1	1	1	1	F	F

$(F \to (Z \wedge Y)) \wedge Z) \vee (Z \to ((X \wedge \neg Y) \vee F)$ formula tavtologiya bo'lishi uchin barcha *F(X,Y,Z)* funksiyalarni ko'rib chiqamiz va buning uchun quyidagi jadvalni tuzib olamiz so'ng MKNSh.dan foydalanib bu formulalarni yozib olamiz :

X	Y	Z	F	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8
0	0	0	*	0	0	0	0	1	1	1	1
0	0	1	1	1	1	1	1	1	1	1	1
0	1	0	1	1	1	1	1	1	1	1	1
0	1	1	1	1	1	1	1	1	1	1	1
1	0	0	*	0	0	1	1	0	0	1	1
1	0	1	1	1	1	1	1	1	1	1	1
1	1	0	*	0	1	0	1	0	1	0	1
1	1	1	1	1	1	1	1	1	1	1	1

$$F_1(X,Y,Z) = (X \vee Y \vee Z) \wedge (\neg X \vee Y \vee Z) \wedge (\neg X \vee \neg Y \vee Z) =$$
$$= (Y \vee Z) \wedge (\neg X \vee \neg Y \vee Z) = Z \vee (Y \wedge (\neg X \vee \neg Y)) = Z \vee (Y \wedge \neg X);$$

$$F_2(X,Y,Z) = (X \vee Y \vee Z) \wedge (\neg X \vee Y \vee Z) = Y \wedge Z;$$

$$F_3(X,Y,Z) = (X \vee Y \vee Z) \wedge (\neg X \vee \neg Y \vee Z);$$

$$F_4(X,Y,Z) = X \vee Y \vee Z;$$

$$F_5(X,Y,Z) = (\neg X \vee Y \vee Z) \wedge (\neg X \vee \neg Y \vee Z) = \neg X \vee Z;$$

$$F_6(X,Y,Z) = \neg X \vee Y \vee Z = X \to Y \vee Z;$$

$$F_7(X,Y,Z) = \neg X \vee \neg Y \vee Z;$$

$F_8(X,Y,Z)$ - ixtiyoriy tavtologiya.

3-BOB. BUL FUNKSIYALARI

3.1. Bul funksiyalari va ularning berilish usullari.

x_1	x_2	x_3	...	x_{n-1}	x_n	$f(x_1,...,x_n)$
0	0	0	...	0	0	f(0,0,...,0,0)
1	0	0	...	0	0	f(1,0,...,0,0)
...	...	...	...	...	...	....
1	1	1	...	1	0	f(1,1,...,1,0)
1	1	1	...	1	0	f(1,1,...,1,1)

Bu jadvalning har bir satrida avval o’zgaruvchilarning $(\alpha_1,...,\alpha_n)$ qiymatlari va shu qiymatlar satrida f funksiyaning $f(\alpha_1,...,\alpha_n)$ qiymati beriladi. n ta o’zgaruvchi uchun qiymatlar satrlarining soni 2^n va ***bul funksiyalarning soni*** 2^{2^n} ga teng bo’ladi.

арг		0	•	→'	x	'←	y	+	∨
x	y	g_0	g_1	g_2	g_3	g_4	g_5	g_6	g_7
0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1

арг		↓	↔	y'	←	x'	→	|	1
x	y	g_8	g_9	g_{10}	g_{11}	g_{12}	g_{13}	g_{14}	g_{15}
0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1

Bu jadvaldan ko’rinib turibdiki, barcha ikki argumentli bul funksiyalari 16 ta bo’ladi.

$$\begin{cases} g_0(xy)=0 \text{ aynan nol} \\ g_{15}(xy)=1 \text{ aynan bir} \end{cases}$$

$$\begin{cases} g_2(x,y)=(x\to y)' - \text{implikasiya inkori} \\ g_{13}(x,y)=x\to y - \text{implikasiya} \end{cases}$$

$$\begin{cases} g_1(x,y)=xy \quad \text{kon' yunksiya} \\ g_{14}(x,y)=x|y \text{ Sheffer shtrixi} \end{cases}$$

$$\begin{cases} g_3(x,y)=x - \text{aynan } x \text{ ga teng funksiya} \\ g_{12}(x,y)=x' - x \text{ ning inkori} \end{cases}$$

$$\begin{cases} g_4(x,y)=(x\leftarrow y)'=(y\to x)' \\ g_{11}(x,y)=x\leftarrow y=y\to x - \text{antiimplikasiya} \end{cases}$$

$$\begin{cases} g_5(x,y)=y - \text{aynan } y \text{ ga teng funksiya} \\ g_{10}(x,y)=y' - y \text{ ning inkori} \end{cases}$$

$$\begin{cases} g_6(x,y)=x+y \text{ Jegalkin yig'indi} \\ g_9(x,y)=x\leftrightarrow y - \text{ekvivalentlik} \end{cases}$$

$$\begin{cases} g_7(x,y)=x\vee y - \text{diz' yunksiya} \\ g_8(x,y)=x\downarrow y - \text{Pirs strelkasi} \end{cases}$$

Asosiy elementar bul funksiyalari:

$$f_1(x)=x,\quad f_2(x)=\bar{x},\quad f_3(x,y)=xy,\quad f_4(x,y)=x\vee y,\ f_5(x,y)=x\to y,$$

$$f_6(x,y)=x\leftrightarrow y,\quad f_7(x_1,.....,x_n)=1,\quad f_8(x_1,......,x_n)=0.$$

Agar f (0,0,.....,0)=0 bo'lsa, u holda f $(x_1,x_2,.....,x_n)$ funksiyaga ***0 saqlovchi funksiya (*** P_0 ***)*** deb aytiladi. Agar $f(1,1,...,1)=1$ bo'lsa, u vaqtda $f(x_1,x_2,...,x_n)$ funksiyaga ***1 saqlovchi funksiya (*** P_1 ***)*** deb aytamiz.

n argumentli ***0 saqlovchi funksiyalarning soni*** 2^{2^n-1} ga va ***1 saqlovchi funksiyalarning soni ham*** 2^{2^n-1} ga teng bo'ladi

$$f(x_1,x_2,....,x_{i-1},1,x_{i+1},....,x_n)=f(x_1,x_2,....,x_{i-1},0,x_{i+1},....,x_n)\ ,$$

bajarilsa, x_i argumentga $f(x_1, x_2, ..., x_n)$ funksiyaning ***soxta argumenti*** deb aytiladi.

$f(x_1, x_2, ..., x_{i-1}, 1, x_{i+1}, ..., x_n) \neq f(x_1, x_2, ..., x_{i-1}, 0, x_{i+1}, ..., x_n)$ bo’lsa, u holda x_i argumentga $f(x_1, x_2, ..., x_n)$ funksiyaning ***muhim argumenti*** deb aytiladi.

Misol. $f(x, y) = x \cdot (x \vee y)$ funksiya uchun u argumenti soxta argument bo’ladi, chunki $f(1,0) = f(0,1)$.

Muammoli masala va topshiriqlar:

3.1.1. $f_1 = \overline{x\, y \vee \bar{z}}$ va $f_2 = x\,(x\, y \vee \overline{yz} \vee \overline{(y \vee \bar{t}\, z)})$ funksiyalarga tengkuchli bo’lgan funksiyalarni toping.

3.1.2. Yolg’on qiymat saqlovchi ($f(0,0,...,0) = 0$) n argumentli har xil funksiyalarning soni nechta?

3.1.3. Chin qiymat saqlovchi ($f(1,1,...,1) = 1$) n argumentli har xil funksiyalarning soni nechta?

3.1.4. Quyidagi $f_1(x, y, z, t) = (x \vee y)(z \vee t)$ va $f_2(x, y, z, t) =$ $= xz \vee yz \vee xt \vee yt$ hamda $f_3(x, y, z, t) = xy \vee zt$ va $f_4(x, y, z, t) = (x \vee z)(y \vee z)(x \vee t)(y \vee t)$ funksiyalarning tengkuchliligini isbotlang.

3.1.5. Quyidagi bul funksiyalari uchun qiymatlar jadvalini tuzing:

1) f (x,y,z)=((x→z)y’)→x’;

2) f (x,y,z)=((x∨y’)→z)((x|y)↔z’);

3) f (x,y,z)=x’→(x↔(y+(xz)));

4) f (x,y,z)=((((x|y)↓z)|y)↓z);

5) f (x,y,z)=(x’y’z’)↓(xyz);

6) f (x,y,z)=x’y’+yz’+xy;

7) $f(x,y,z)=(x\downarrow y)'+zx+xy;$

8) $f(x,y,z)=((x|y)+(y|z))+xyz;$

9) $f(x,y,z)=((x+y)(y+z)')';$

10) $f(x,y,z)=(xyz)|(x'y'z');$

11) $f(x,y,z)=((x\to(y\vee z))(yz)')\to x;$

Yechim: 11) $f(x,y,z)=((x\to(y\vee z))((yz)')\to x$ bul funksiyasi uchun qiymatlar jadvalini tuzamiz:

x	y	z	$y\vee z$	$x\to(y\vee z)$	yz	$(yz)'$	$(x\to(y\vee z))(yz)'$	f
0	0	0	0	1	0	1	1	1
0	0	1	1	1	0	1	1	1
0	1	0	1	1	0	1	1	1
0	1	1	1	1	1	0	0	0
1	0	0	0	0	0	1	0	0
1	0	1	1	1	0	1	1	0
1	1	0	1	1	0	1	1	0
1	1	1	1	1	1	0	0	0

3.1.6. Tegishli qiymatlar jadvalini tuzib, bul funksiyalarining teng yoki teng emasligini tekshiring:

1) $f(x,y,z)=((x\vee y)\vee z)\to((x\vee y)(x\vee z)),$

$g(x,y,z)=x\vee(y\leftrightarrow z);$

2) $f(x,y,z)=(x'\vee y)(y\vee z),$

$g(x,y,z)=(x\vee y\vee z)(x'\vee y\vee z)(x'\vee y\vee z');$

3) $f(x,y,z)=(x\to y)\to z),$

$g(x,y,z)=x\to(y\to z);$

4) $f(x,y)=((x+y)\to(x\vee y))((x'\to y)\to(x+y)),$

$g(x,y)=x|y;$

5) $f(x,y,z)=((x\vee y')z)\vee(xz')\vee(z(y\vee z'))$,

$g(x,y,z)=x\vee z$;

6) $f(x,y,z)=xy\vee xz\vee yz$,

$g(x,y,z)=(x+y)z\vee zy$;

7) $f(x,y,z)=xy'\vee x'y\vee x'z'$,

$g(x,y,z)=(x'\vee y')(x\vee y\vee z')$;

8) $f(x,y,z)=x'y'z'\vee x'yz\vee xyz\vee xy'z$,

$g(x,y,z)=(x\rightarrow yz)(y\leftrightarrow z)\vee(y\rightarrow xz)(x\leftrightarrow z)$;

9) $f(x,y,z)=((y'+x)+z(x+y'))'$,

$g(x,y,z)=z'\rightarrow(y\rightarrow x)'$;

10) $f(x,y,z)=(x+z')'(y+x'z)$,

$g(x,y,z)=y+(z\rightarrow x)'$;

11) $f(x,y,z)=(x+y)'\vee(x+z)'$,

$g(x,y,z)=xyz+x'y'z$.

Yechim: 11) f va g funksiyalarining qiymatlar jadvalini tuzamiz:

$f(x,y,z)=(x+y)'\vee(x+z)'$

x	y	z	$x+y$	$(x+y)'$	$x+z$	$(x+z)'$	$f(x,y,z)$
0	0	0	0	1	0	1	1
0	0	1	0	1	1	0	1
0	1	0	1	0	0	1	1
0	1	1	1	0	1	0	0
1	0	0	1	0	1	0	0
1	0	1	1	0	0	1	1
1	1	0	0	1	1	0	1
1	1	1	0	1	0	1	1

$g(x,y,z)=xyz+x'y'z'$

x	y	z	xy	xyz	x'	y'	z'	$x'y'$	$x'y'z'$	$g(x,y,z)$
0	0	0	0	0	1	1	1	1	1	1
0	0	1	0	0	1	1	0	0	0	0
0	1	0	0	0	1	0	1	0	0	0
0	1	1	0	0	1	0	0	0	0	0
1	0	0	0	0	0	1	1	0	0	0
1	0	1	0	0	0	1	0	0	0	0
1	1	0	1	0	0	0	1	0	0	0
1	1	1	1	1	0	0	0	0	0	1

$f(0,0,1) \neq g(0,0,1)$, $f(0,1,0) \neq g(0,1,0)$, $f(1,0,1) \neq g(1,0,1)$, $f(1,1,0) \neq g(1,1,0)$. Demak, $f(x,y,z) \neq g(x,y,z)$.

3.1.7. Quyidagi tengliklarni tekshirib, distributivlik xossasini bir bul funksiyalasining boshqasiga nisbatan bajarilishini tekshiring:

1) $(x\to y)\vee z=(x\vee z) \to(y\vee z);$

2) $(x\leftrightarrow y)\vee z=(x\vee z) \leftrightarrow y\vee z);$

3) $x\to(y\to z)=(x\to y) \to(x\to z);$

4) $x\to yz=(x\to z)\ (x\to z);$

5) $x\to(y\vee z)=(x\to z) \vee (y\to z);$

6) $x\to(y\leftrightarrow z)=(x\to z)\leftrightarrow(y\to z);$

7) $x\ (yz)=(xy)(xz);$

8) $x\vee(y\vee z)=(x\vee z) \vee(x\vee z);$

9) $x\to xy=x\to y;$

10) $(x\vee y)\to y= x\to y;$

11) $x\to(x\to y) = x\to y;$

12) $x \to(x\leftrightarrow y)= x\to y;$

E’tibor bering 9-11 misollarda distributivlik xossasi bajarilsa, bir xil natijaga olib keladi.

Yechim: 9) $x\to xy=(x\to x)(x\to y)=1(x\to y)=x\to y$.

3.1.8. Agar 3.1.7. dagi 1),2) misollarda chap distributivlik xossasi ko’rilgandi, endi o’ng tomonli distributivlik xossasi bajarilishini tekshiring.

Ya’ni: $x\vee(y\to z)=(x\vee y)\to(x\vee z)$, $x\vee(y\leftrightarrow z)=(x\vee y)\leftrightarrow(x\vee z)$

3.1.9. Ikki modul bo’yicha yig’indi (Jegalkin yig’indi) quyidagi xossalarga ega ekanligini tekshiring.

1) $x+y=(x\leftrightarrow y)'$;

2) $x+y=y+x$;

3) $(x+y)+z=x+(y+z)$;

4) $x+0=x$;

5) agar $x=y+z$; unda $y=x+z$;

6) agar $x+z=y+z$; unda $y=x$;

7) $(x+y)z=xz+yz$;

8) $x+x=0$;

9) $xy\vee xz\vee yz=xy+xz+yz$;

10) $(x_1+x_2+\ldots+x_n)y=x_1y+x_2y+\ldots+x_ny$;

11) $x_1+x_2+\ldots+x_n=\begin{cases}0, & agar\ x_1,x_2,\ldots,x_n\ \ 1\ lar\ soni\ juft\ bo'lsa,\\ 1, & agar\ x_1,x_2,\ldots,x_n\ \ 1\ lar\ soni\,toq\ bo'lsa,\end{cases}$

3.1.10. Bul funksiyasidagi Sheffer shtrixi (|) quyidagi xossalarga ega ekanligini isbotlang:

1) $x|1=x|x=x'$;

2) $x|0=1$;

3) $x|x'=1$;

4) $x|y=y|x$;

5) $(x|y)'=x'|y'$;

6) $x|(x\vee y)=x'$;

7) $(x|y)'=(x'\vee y')'$;

8) $x\vee(x|y)=y\vee(x|y)$;

9) $(x|x)|y=y\to x$;

10) $(x|x)|(y|y)=x\vee y$;

11) $((x|x)'|(y|y)')'=xy$;

Yechim: 11) $((x|x)'|(y|y)')'=(x''|y'')'=(x|y)'=(x'\vee y')'=((xy)')'=xy$.

3.1.11. Bul funksiyasidagi Pirs strelkasi $(\downarrow)$ quyidagi xossalarga ega ekanligini isbotlang:

1) $x\downarrow x=x'$;

2) $x\downarrow y=y\downarrow x$;

3) $x\downarrow 1=0$;

4) $x\downarrow 0=1$;

5) $x\downarrow x'=0$;

6) $x(x\downarrow y)=y(x\downarrow y)$;

7) $(x\downarrow y)'=x'\downarrow y'$;

8) $x\downarrow (yx)=x'$;

9) $(x\downarrow y)'=(x'y')'$;

10) $(x\downarrow x')'(x\downarrow x)'=x$;

11) $x(x\downarrow y)=x(1\downarrow y)$.

Yechim: 11) $x(x\downarrow y)=x(1\downarrow y)$. Bu misolni chinlik jadvali yordamida osongina ko'rsatish mumkin. Tenglikning o'ng va chap tomonlari uchun quyidagicha tuzib olamiz.

x	y	$x\downarrow y$	$x(x\downarrow y)$	$1\downarrow y$	$x(1\downarrow y)$
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0
1	1	0	0	0	0

3.1.12. Bul funksiyalarini quyidagicha inkor (') va diz'yunksiya $(\vee)$ amallari yordamida ifodalash mumkinligini isbotlang:

1) $xy=(x'\vee y')'$;

2) $x\to y=x'\vee y$;

3) $x\leftrightarrow y=(x\to y)(y\to x)=(x'\vee y)(x\vee y')$;

4) $x+y=(x'\vee y)'\vee(x\vee y')'$;

5) $x|y=x'\vee y'$;

6) $x\downarrow y=(x\vee y)'$;

7) $(x\to y)'=(x'\vee y)'$;

8) $x\vee y=(x'\vee y)'\vee y$.

Yechim: 8) $(x'\vee y)'\vee y=(x''y')\vee y=(x\vee y)(y'\vee y)=(x\vee y)1=x\vee y$.

3.1.13. Amallarni kon'yunksiya (.) va inkor (')amallari yordamida ifodalang:

1) diz'yunksiya $(\vee)$;

2) implikasiya $(\to)$;

3) ekvivalentlik $(\leftrightarrow)$;

4) Jegalkin yig'ndisi (+);

5) Sheffer shtrixi (|) ; 6) Pirs strelrasi ($\downarrow$);

3.1.14. Bul funksiyalarini quyidagicha Jegalkin yig'ndisi (+) va konstant 1 amallari yordamida ifodalash mumkinligini isbotlang:

1) $x'=x+1$; *2)* $x\vee y=(x+1)(y+1)+1=xy+x+y$;

3) $x\to y=xy+x+1$; *4)* $x\leftrightarrow y=x+y+1$;

5) $x|y=xy+1$; *6)* $x\downarrow y=xy+x+y+1$;

7) $(x\to y)'=xy+x$; *8)* $x'y\vee xy'=x+y$;

9) $x'y'\vee xy=x+y+1$; *10)* $(x\leftrightarrow y)'=x+y$;

11) $(x\leftrightarrow y)\leftrightarrow z=x+y+z$;

Yechim: 10) $(x\leftrightarrow y)\leftrightarrow z=[x+y+1+z]+1=x+y+z+0=x+y+z$.

3.1.15. Bul funksiyalarini quyidagicha Sheffer shtrixi (|) yordamida ifodalash mumkinligini isbotlang:

1) $x'=x|x$; *2)* $xy=(x|y)|(x|y)$;

3) $x\vee y=(x|x)|(y|y)$ *4)* $x\to y=x|(y|y)$;

5) $(x\to y)'=(x|(y|y))|(x|(y|y))$;

6) $x\downarrow y=[(x|x)|(y|y)]|[(x|x)|(y|y)]$;

7) $x+y=(x|(y|y))|(y|(x|x))$;

8) $x\leftrightarrow y=[(x|(y|y))|(y|(x|x))]|[(x|(y|y))|(y|(x|x))]$;

Yechim: 7) $x+y=(x\leftrightarrow y)'=((x\to y)(y\to x))'=(x\to y)|(y\to x)=$

$=(x|(y|y))|(y|(x|x))$.

3.1.16. Amallarni Pirs strelrasi ($\downarrow$)yordamida ifodalang:

1) inkor (') 2) diz'yunksiya ($\vee$); 2) implikasiya ($\to$);

3) ekvivalentlik ($\leftrightarrow$); 4) Jegalkin yig'ndisi (+) ;

5) Sheffer shtrixi (|) ; 6) kon'yunksiya (.).

Yechim: 3) $x\leftrightarrow y=(x\to y)(y\to x)=((x\to y)'\vee(y\to x)')'=$

$=(x\to y)'\downarrow(y\to x)'=(x'\vee y')\downarrow(x\vee y)'=$

$(x'\downarrow y)\downarrow(x\downarrow y')=((x\downarrow x)\downarrow y)\downarrow(x\downarrow(y\downarrow y))$.

3.1.17. Tengkuchlilikni isbotlang:

1) $x \vee y = (x \to y) \to y$;

2) $x \leftrightarrow y = (x \to y) \& (y \to x)$;

3) $x \downarrow y = ((x|x)|(y|y))|((x|x)(y|y))$;

4) $x \vee (y \leftrightarrow z) = (x \vee y) \leftrightarrow (x \vee z)$;

5) $x \& (y \leftrightarrow z) = ((x \& y) \leftrightarrow (x \& z)) \leftrightarrow x$;

6) $x \to (y \leftrightarrow z) = (x \to y) \leftrightarrow (x \to z)$;

7) $x \vee (y \to z) = (x \vee y) \to (x \vee z)$;

8) $x \& (y \to z) = (x \to y) \to (x \& z)$;

9) $x \to (y \vee z) = (x \to y) \vee (x \to z)$;

10) $x \to (y \& z) = (x \to y) \& (x \to z)$;

11) $x \to (y \to z) = (x \to y) \to (x \to z)$.

Yechim: 11) $x \to (y \to z) = x' \vee y' \vee z$ endi o'ng tomonini soddalashtiramiz:

$$(x \to y) \to (x \to z) = (x' \vee y)' \vee (x' \vee z) = xy' \vee x' \vee z =$$
$$= (x \vee x' \vee z)(y' \vee x' \vee z) = x' \vee y' \vee z$$

3.1.18. *A* va *B* formulalar ekvivalentligini tekshiring:

1) $A = \overline{x \oplus y \cdot z} \cdot \overline{\overline{y} \to x \cdot z} \cdot (\overline{x} \downarrow y)$, $B = \overline{(x \cdot y \to (y \downarrow z)) \vee x \cdot z} \cdot z$;

2) $A = (x \oplus y \cdot z) \to (\overline{x} \to (y \to z))$, $B = x \to ((y \to z) \to x)$;

3) $A = (x \cdot y \to z) \vee ((x \downarrow y)|z)$, $B = ((x \to y \cdot z) \oplus (x \leftrightarrow y)) \vee (y \to x \cdot z)$;

4) $A = (\overline{x} \to (\overline{y} \to (x \leftrightarrow z))) \cdot (x \leftrightarrow (y \to (z \vee (x \to y))))$,
$B = (x \to (y \to z)) \to x$;

5) $A = (((x|y) \downarrow \overline{z})|y) \downarrow (\overline{y} \to z)$, $B = ((x|y) \downarrow (y|\overline{z})) \cdot (x \to (y \to z))$;

6) $A = (\overline{x} \vee \overline{y \cdot z}) \to ((x \to y) \to ((y \vee z) \to \overline{x}))$, $B = (x \to y) \to (\overline{y} \to \overline{x})$;

7) $A = (x|\overline{y}) \to ((y \downarrow \overline{z}) \to (x \oplus z))$, $B = x \cdot y \cdot z \oplus (\overline{x} \to z)$;

8) $A = (x \cdot \overline{y} \vee \overline{x} \cdot z) \oplus ((y \to z) \to \overline{x} \cdot y)$, $B = (x \cdot \overline{y} \cdot \overline{z} \oplus y) \oplus z$;

9) $A=(\bar{x}\vee y)\to((y|\bar{z})\to(x\leftrightarrow x\cdot z)),\ B=x\cdot y\vee(\overline{x\to x\cdot\bar{y}}\to z);$

10) $A=x\to((\bar{x}\cdot\bar{y}\to(\bar{x}\cdot\bar{z}\to y))\to y)\cdot z,\ B=\overline{x\cdot(y\to\bar{z})};$

11) $A=((x\vee y)\cdot\bar{z}\to((x\leftrightarrow\bar{z})\oplus\bar{y}))\cdot((x\oplus y)\cdot\bar{z}),\ B=(x\to y\cdot z)\cdot\overline{x\to y};$

12) $A=\overline{((x\leftrightarrow y)\to(x\to z))}\vee(x\oplus\bar{y}\cdot z),\ B=x\leftrightarrow(z\to y);$

13) $A=\overline{(x\downarrow y)\vee(x\leftrightarrow z)}|(x\oplus y\cdot z),\ B=\bar{x}\cdot y\cdot z\vee\overline{x\to z};$

14) $A=x\to((y\to z)\to y\cdot z),\ B=(x\vee(x\cdot y\to z))\cdot(x\oplus y\cdot z);$

15) $A=\overline{((x\vee y)\to y\cdot z)\vee(y\to x\cdot z)}\vee(x\to(\bar{y}\to z)),\ B=(x\to y)\vee z.$

Yechim: 15) $A=\overline{((x\vee y)\to y\cdot z)\vee(y\to x\cdot z)}\vee(x\to(\bar{y}\to z))=$

$=\overline{((x\vee y)'\vee y\cdot z)\vee(y'\vee x\cdot z)}\vee(x'\vee(y\vee z))=$

$=(x\vee y)(y'\vee z')y(x'\vee z')\vee x'\vee y\vee z=$

$=(x\vee y)z'y(x'\vee z')\vee x'\vee y\vee z=$

$=(xz'y\vee z'y)(x'\vee z')\vee x'\vee y\vee z=$

$=xz'y\vee z'yx'\vee z'y\vee x'\vee y\vee z=x'\vee y\vee z.$

$B=(x\to y)\vee z=x'\vee y\vee z$ *A* va *B* formulalar ekvivalent

3.1.19. Barcha muhim va soxta argumentlarni aniqlang:

1) $f(\tilde{x}^3)=(10101010);$

2) $f(\tilde{x}^3)=(10011001);$

3) $f(\tilde{x}^3)=(00111100);$

4) $f(\tilde{x}^3)=(11110011);$

5) $f(\tilde{x}^4)=(0101111101\ 011111);$

6) $f(\tilde{x}^4)=(1100110000\ 110011);$

7) $f(\tilde{x}^4)=(1011010110\ 110101);$

8) $f(\tilde{x}^2)=((x_1\vee x_2)\to x_1\cdot x_2)\oplus(x_1\to x_2)\cdot(x_2\to x_1);$

9) $f(\tilde{x}^2)=(x_1\cdot x_2\oplus(x_1\to x_2))\to(x_1\leftrightarrow x_1\cdot x_2);$

10) $f(\tilde{x}^3) = ((x_1 \to \overline{x_2}) \oplus (x_2 \to \overline{x_3})) \oplus (x_2 \to x_3);$

11) $f(\tilde{x}^3) = ((x_1 \vee x_2 \cdot \overline{x_3})(x_2 \to x_1 \cdot x_3)) \to (x_1 \vee x_3);$

12) $f(\tilde{x}^3) = ((x_1 \downarrow (x_2 \mid x_3)) \downarrow (x_2 \downarrow (x_1 \mid x_3))) \downarrow (x_1 \mid x_2).$

Yechim: 4) Berilgan $f(\tilde{x}^3)$ funksiyaning qiymatlar jadvalini tuzamiz:

x_1	x_2	x_3	$f(\tilde{x}^3)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Funksiya qabul qilgan qiymatlarini taqqoslab, x_3 o'zgaruvchiga nisbatan qo'shni funksiyalar orasidagi quyidagi munosabatni ko'ramiz:

$f(0,0,0)=f(0,0,1)=1$,

$f(0,1,0)=f(0,1,1)=1$,

$f(1,0,0)=f(1,0,1)=0$,

$f(1,1,0)=f(1,1,1)=1$.

Bundan $f(x_1, x_2, 1) = f(x_1, x_2, 0)$. Ya'ni, f funksiyasi uchun x_3 argument soxta ekan.

Endi $f(\tilde{x}^3)$ funksiyaga tengkuchli bo'lgan $g(\tilde{x}^2)$ funksiyani quramiz. Buning uchun qiymatlar jadvalidan x_3 soxta argument ustunini va $(x_1, x_2, 1)$ satrlarni o'chiramiz. Natijada quyidagi jadvalga ega bo'lamiz:

x_1	x_2	$g(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

Funksiya qabul qilgan qiymatlarini taqqoslaymiz: $g(1,0)=0$, $g(1,1)=1$ hamda $g(1,0)=0$ va $g(0,0)=1$. Bundan x_1, x_2 ikkala o'zgaruvchi ham muhim ekanligi kelib chiqadi. ($f(\widetilde{x}^3) = x_1 \to x_2$)

3.1.20. Quyidagi f funksiyalari uchun x_1 argument soxta ekanligini ko'rsating:

1) $f(\widetilde{x}^2) = (x_2 \to x_1) \cdot (x_2 \downarrow x_1)$; 2) $f(\widetilde{x}^2) = (x_1 \leftrightarrow x_2) \vee (x_1 | x_2)$;

3) $f(\widetilde{x}^3) = ((x_1 \oplus x_2) \to x_3) \cdot \overline{(x_3 \to x_2)}$;

4) $f(\widetilde{x}^3) = ((x_1 \vee x_2 \cdot \overline{x_3}) \leftrightarrow (\overline{x_1} \to \overline{x_2} \cdot x_3))(x_2 \downarrow x_3)$

5) $f(\widetilde{x}^3) = ((x_1 \vee x_2 \vee \overline{x_3}) \to (x_1 x_2 | x_3)) \oplus (x_2 \to x_1) \cdot x_3$.

Yechim: 1) $f(\widetilde{x}^2) = (x_2 \to x_1) \cdot (x_2 \downarrow x_1) = (x_2' \vee x_1)(x_2 \vee x_1)' =$

$= (x_2' \vee x_1)(x_2' \vee x_1') = x_2' \vee (x_1 \cdot x_1') = x_2'$.

f funksiyasini soddalashtirib, x_1 argument soxta ekanligi aniqlandi.

3.1.21. Quyidagi shartlarni qanoatlantiruvchi barcha bul funksiyalarini sanab o'ting:

1) 1 argumentli, 0 ni saqlovchi;

2) 1 argumentli, 1 ni saqlovchi;

3) 2 argumentli, 0 ni saqlovchi;

4) 2 argumentli, 1 ni saqlovchi;

5) 1 argumentli, ham 1 ni ham 0 ni saqlovchi;

6) 2 argumentli, ham 1 ni ham 0 ni saqlovchi;

7) 2 argumentli, 1 ni saqlovchi ammo 0 ni saqlamaydigan;

8) 2 argumentli, 0 ni saqlovchi ammo 1 ni saqlamaydigan;

3.1.22. Isbotlang:

1) 0 ni saqlovchi bul funksiyalar superpozisiyasidan hosil bo'lgan funksiya yana 0 ni saqlovchi funksiya bo'ladi.

2) 1 ni saqlovchi bul funksiyalar superpozisiyasidan hosil bo'lgan funksiya yana 1 ni saqlovchi funksiya bo'ladi.

3.1.23. n argumentli barcha bul funksiyalari orasida 1 ni saqlovchi funksiyalar soni 0 ni saqlovchi funksiyalar soniga tengligini isbotlang.

3.1.24. Amallarni implikasiya ($\to$) va inkor (')amallari yordamida ifodalang:

1) diz'yunksiya ($\vee$); 2) kon'yunksiya ($.$) implikasiya ($\to$);

3) ekvivalentlik ($\leftrightarrow$); 4) Jegalkin yig'ndisi (+) ;

5) Sheffer shtrixi (|) ; 6) Pirs strelrasi ($\downarrow$).

3.2. Ikkilamchi funksiyalar. Ikkilamchilik prinstipi.

$f(x_1, x_2,..., x_n)$ funksiyaga ikki taraflama bo'lgan funksiyani topish uchun f funksiyaning chinlik jadvalida hamma o'zgaruvchilarni ularning inkoriga almashtirib, simmetrik mos ravishda joyini o'zgartirish kerak.

$f^*(x_1, x_2,.....,x_n) = \overline{f}(\overline{x}_1, \overline{x}_2,.....,\overline{x}_n)$ funksiyaga $f(x_1, x_2,..., x_n)$ funksiyaning ***ikki taraflama funksiyasi*** deb aytiladi.

$f\ (x_1, x_2,.....,x_n) = f^*(x_1, x_2,.....,x_n) = \overline{f}(\overline{x}_1, \overline{x}_2,.....,\overline{x}_n)$ munosabat bajarilsa, u holda $f(x_1, x_2,..., x_n)$ *ga* ***o'z-o'ziga ikki taraflama funksiya*** deb aytiladi.

Misollar. 1. Mulohazalar algebrasining asosiy elementar funksiyalariga ikki taraflama bo'lgan funksiyalarni toping.

1. $f_1(x) = x$ ga ikki taraflama funksiya $f_1^*(x) = x$ bo'ladi.
2. $f_2(x) = \overline{x}$ ga ikki taraflama funksiya $f_2^*(x) = \overline{x}$ bo'ladi.
3. $f_3(x, y) = xy$ ga ikki taraflama funksiya $f_3^* = x \vee y$ bo'ladi.
4. $f_4(x, y) = x \vee y$ ga ikki taraflama funksiya $f_4^* = x\,y$ bo'ladi.
5. $f_5(x, y) = x \to y$ ga ikki taraflama funksiya $f_5^* = \overline{y \to x}$ bo'ladi.
6. $f_6(x, y) = x \leftrightarrow y$ ga ikki taraflama funksiya $f_6^* = \overline{x \leftrightarrow y}$ bo'ladi.
7. f_7=1 ga f_7^*=0 va f_8=0 ga f_8^*=1 bo'ladi.

2. $f(x,y,z) = xy \vee yz \vee xz$ funksiyaning o'z-o'ziga ikki taraflama funksiya ekanligini isbot qiling.

$$f^*(x,y,z) = \overline{\bar{x}\,\bar{y} \vee \bar{y}\,\bar{z} \vee \bar{x}\,\bar{z}} = \overline{\bar{x}\,\bar{y}} \wedge \overline{\bar{y}\,\bar{z}} \wedge \overline{\bar{x}\,\bar{z}} = (x \vee y)\,(y \vee z)\,(x \vee z) =$$

$$= [(x \vee y)\,y \vee (x \vee y)\,z]\,(x \vee z) = [y \vee yz \vee xz]\,(x \vee z) = (y \vee xz)\,(x \vee z) =$$

$$= xy \vee yz \vee x\,(x \vee z)\,z = xy \vee yz \vee xz\ .$$

Demak, $f(x,y,z) = f^*(x,y,z)$ ekanligi uchun f o'z-o'ziga ikki taraflama funksiyadir.

n ta argumentli o'z-o'ziga ikki taraflama funksiyalar soni $2^{2^{n-1}}$ taga teng.

Muammoli masala va topshiriqlar:

3.2.1. n ga shunday shart beringki, natijada $R_n(x_1, x_2, ..., x_n) = x_1 + x_2 + ... + x_n$ o'z-o'ziga ikki taraflama bo'lgan funksiya bo'lsin.

3.2.2. n argumentli o'z-o'ziga ikki taraflama bo'lgan funksiyalarning sonini toping.

3.2.3. $f = (\bar{x} \vee y\,\bar{z})\,(xy \vee x\bar{z})$ va $\varphi = (x \vee \bar{y})\,z\bar{t} \vee \bar{x}t$ funksiyalarga ikki taraflama bo'lgan funksiyalarni toping.

3.2.4. Bir argumentli har bir bul funksiyasi uchun ikki taraflama bul funksiyasini toping.

3.2.5. Ikki argumentli har bir bul funksiyasi uchun ikki taraflama bul funksiyasini toping.

***Yechish**:* $g_0^*(x,y) = (g_0(x', y'))' = 0' = 1 = g_{15}(x, y)$;

$g_1^*(x,y) = (g_1(x', y'))' = (x'y')' = x'' \vee y'' = x \vee y = g_7(x, y)$;

$g_2^*(x,y) = (g_2(x', y'))' = ((x' \rightarrow y')')' = x'' \vee y' = y' \vee x = (y \rightarrow x) = g_{11}(x, y)$;

$g_3^*(x,y) = (g_3(x', y'))' = (x')' = x = g_3(x, y)$;

3.2.6. Quyidagi bul funksiyalari uchun ikki taraflama funksiyasini toping. Ularni MDNSH ko'rinishda ifodalang:

1) $x'yz \vee xy'z' \vee xyz$; 2) $x'yz \vee x'yz' \vee xy'z \vee xy'z'$;

3) $x(y+z) \vee ((x' \vee y \vee z) \to (x'yz))$;

4) $(z \to (x \vee y))' \vee x(y+z)$;

5) $xyz + xy + xz + z$;

6) $x'y \vee (z \to x)'$;

7) $(z \to x')(x \to y)$;

8) $(x \to (z \to y))(x \to (z \to y'))$;

9) $x'y'z \vee x'yz' \vee xy'z' \vee xy'z \vee xyz$;

10) $x'y' \vee yz \vee y'z'$;

11) $(x' \to z)' \vee (x \to z)y'$.

Yechim: 11) $f(x, y, z) = (x' \to z)' \vee (x \to z)y'$ deb belgilash kiritib, $f(x, y, z)$ funksiyaga ikki taraflama funksiyani topamiz:

$f^*(x, y, z) = (f(x', y', z'))' = ((x \to z')' \vee (x' \to z')y)' =$

$(x \to z')''((x' \to z')y)' = (x \to z')((x' \to z')' \vee y') =$

$(x' \vee z')((x \vee z')' \vee y') = (x' \vee y')(x'z \vee y') =$

$= x'(x'z \vee y') \vee z'(x'z \vee y') = x'x'z \vee x'y' \vee z'x'z \vee z'y' =$

$= x'y' \vee x'z \vee y'z'$.

3.2.7. Quyidagi bul funksiyalari bir-biriga ikki taraflama funksiya ekanligini isbotlang:

1) $xyz + yz + y + 1$, $xyz + xy + xz + x + y + 1$;

2) $xy + xz + x + y + z$, $xy + xz + x$;

3) $xyz + xy + z$, $xyz + xz + yz$;

4) $xy + yz + x + y + z + 1$, $xy + yz + y + 1$;

5) $xyz + xy + x + y$, $xyz + xz + yz + x + y + z + 1$;

6) $xy + x + y + z + 1$, $xy + z$;

7) $xyz + x + 1$, $xyz + xy + xz + yz + y + z$;

8) $xy + yz + x + 1,\ xy + yz + z + 1$;

9) $xyz + xy + xz + y + 1,\ xyz + yz + x + y$;

10) $xyz + x + z,\ xyz + xy + xz + yz + y$;

11) $yz + x + y,\ yz + x + z$.

Yechim: 11) Berilgan funksiyaga ikki taraflama funksiyani topamiz $f(x, y, z) = yz + x + y$ va $g(x, y, z) = yz + x + z$ deb olamiz:

$f^*(x, y, z) = (y'z' + x' + y')' = y'z' + x' + y' + 1 =$

$= (y + 1)(z + 1) + (x + 1) + (y + 1) + 1 = yz + y + z + 1 + x + 1 + y +$

$+ 1 + 1 = yz + x + z.$

Demak, bul funksiyalari bir-biriga ikki taraflama funksiya ekan.

3.2.8. Quyidagi bul funksiya ikki taraflama funksiyasining inkori bilan ustma-ust tushishini isbotlang:

1) $x(y + z + 1) + yz$;

2) $x'yz' \vee xy'z$;

3) $x'yz' \vee x'yz \vee xy'z \vee xy'z$;

4) $(x + z)(y + 1) + xy + yz$;

5) $xy + xz + yz + x + y + z$;

6) $xyz \vee x'y'z'$;

7) $xyz \vee xy'z \vee x'yz' \vee x'y'z'$

8) $((x \vee y \vee z) \rightarrow yz) \vee xyz' \vee x'y'z$;

9) $(x + z + 1)y + xz + 1$;

10) $xy + xz + yz + x + 1$;

11) $xy + xz + yz + z$.

Yechim: 11) $f(x, y, z) = xy + xz + yz + z$. $f(x, y, z)$ funksiya ikki taraflama funksiyasini topamiz:

$f^*(x, y, z) = (x'y' + x'z' + y'z' + z')' = x'y' + x'z' + y'z' + z' + 1 =$

$=(x + 1)(y + 1) + (x + 1)(z + 1)(z + 1) + z + 1 + 1 =$

$=xy+x+y+1+xz+x+z+1+yz+y+z+1+z=xy+xz+yz+z+1=$

$=(xy+xz+yz+z)' =f '(x, y, z).$

Demak, $f(x, y, z)$ funksiyaning inkori ikki taraflama funksiyaga teng kuchli ekanligi isbotlandi. $f^*(x, y, z) =f ' (x, y, z).$

3.2.9. Barcha ikki argumentli o'z-o'ziga ikki taraflama bul funksiyalarini toping. Ulardan nechtasi barcha argumentlarga bog'liq bo'ladi?

3.2.10. Quyidagi bul funksiyalari o'z-o'ziga ikki taraflama ekanligini isbotlang:

1) $x(z\to y)\vee(y\to z)'$;

2) $x'yz\vee xy'z\vee x'y'z\vee x'y'z'$;

3) $xy\vee xz\vee yz$;

4) $x'y\vee x'z'\vee yz'$;

5) $xy + xz + yz + y + z$;

6) $x'yz'\vee x'yz\vee xyz'\vee xyz$;

7) $x'y'z\vee x'yz'\vee xy'z'\vee xyz$;

8) $xz + (x + z)(y+ 1)$;

9) $(x \to y)'(z \to z) \vee (x \vee y)'(z \to z)$;

10) $(x'\vee y\vee z)(x'\vee y\vee z')(x'\vee y'\vee z)(x'\vee y'\vee z')$;

11) $xy'\vee xz'\vee y'z'$.

Yechim: 11) $f(x, y, z) = xy'\vee xz'\vee y'z'.$

$f^*(x, y, z) = (x'y\vee x'z\vee yz)' = (x\vee y')(x\vee z')(y'\vee z') = (x\vee y'z') \vee (y'\vee$

$\vee z') = xy'\vee xz'\vee y'z'\vee y'z' = xy'\vee xz'\vee y 'z' =f(x, y, z).$

3.2.11. Quyidagi bul funksiyalaridan qaysilari o'z-o'ziga ikki taraflama ekanligini aniqlang:

1) $x'y'\vee x'z'\vee y'z'$;

2) $x'yz\vee xy'z\vee xyz'$;

3) $xy + xz + yz$;

4) $(x'\vee z)y'\vee (x'\vee y)z'\vee x'yz$;

5) $((x \to y)+1)(z+1) \vee xy'z$;

6) $xyz' \vee xy'z \vee x'yz \vee xyz$;

7) $x'yz' \vee x'yz \vee xy'z' \vee xyz' \vee xyz$;

8) $((x \vee y \vee z) \to x'yz) \vee x(y + z + 1)$;

9) $xyz \vee x'yz' \vee xy'z' \vee x'y'z'$;

10) $xyz' \vee xy'z' \vee x'yz' \vee x'y'z'$;

11) $(x + 1)(y \to z)' \vee (x + y)z$.

Yechim: 11) $f(x, y,z) = (x + 1)(y \to z)' \vee (x + y)z$.

$f(x, y, z) =(x +1)(y \to z)' \vee (x +y)z = x'(y' \vee z) \vee (x \leftrightarrow y)'z =$

$=x'yz' \vee ((x' \vee y)(x \vee y'))'z = x'yz' \vee (xy' \vee x'y)z = x'yz' \vee xy'z \vee x'yz$.

$f^* = (xy'z \vee x'yz' \vee xy'z')' = (x' \vee y \vee z')(x \vee y' \vee z)(x' \vee y \vee z) =$

$=(x' \vee y)(x \vee y' \vee z) = x'y' \vee x'z \vee xy \vee yz = x'y'z' \vee x'y'z \vee x'y'z \vee x'yz \vee xyz' \vee$

$\vee xyz \vee xyz \vee x'yz = x'y'z' \vee x'y'z \vee x'yz \vee xyz' \vee xyz$.

3.2.12. Funksiyalarning qiymatlar jadvali berilgan bo'lsa, ularga ikki taraflama funksiyalarning qiymatlarini toping:

x	y	z	1)	2)	3)	4)	5)	6)	7)	8)	9)	10)	11)
0	0	0	0	1	1	0	1	1	1	0	0	0	0
0	0	1	1	1	0	0	0	1	1	0	0	1	1
0	1	0	1	0	1	0	0	0	1	0	1	1	1
0	1	1	0	1	1	1	0	1	0	0	0	1	1
1	0	0	0	1	0	1	1	0	1	1	1	0	0
1	0	1	0	0	0	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	0	1	1	1	0	0
1	1	1	1	0	0	0	1	0	1	0	1	1	1

Yechim: 11) $f(x_1, x_2, ..., x_n)$ funksiyaga ikki taraflama bo'lgan funksiyani topish uchun f funksiyaning chinlik jadvalida hamma o'zgaruvchilarni ularning inkoriga almashtirib, simmetrik mos ravishda joyini o'zgartirish kerak.

$f_{11}(01110101)$ funksiyaga ikki taraflama funksiya $f_{11}^* = (01010001)$ ga teng.

3.2.13. 3.2.12 misolda berilgan funksiyalarning qaysi biri o'z-o'ziga ikki taraflama funksiya bo'lishini aniqlang:

Yechim: 11) $f_{11}(01110101) \neq f_{11}^* = (01010001)$. Demak, o'z-o'ziga ikki taraflama emas.

3.2.14. O'z-o'ziga ikki taraflama uch argumentli bul funksiyalar soni nechta? Bunday funksiyalarni toping va nechtasi barcha argumentlariga bog'liq bo'ladi?

3.2.15. Asosiy tengkuchlilik va ikki taraflama funksiyaning ta'rifidan foydalanib, f funksiya g funksiyaga ikki taraflama yoki ikki taraflama emasligini aniqlang:

1) $f = x \oplus y,\ g = x \leftrightarrow y;$ 2) $f = x | y,\ g = x \downarrow y;$

3) $f = x \rightarrow y,\ g = \overline{x} \cdot y;$ 4) $f = x \cdot y \rightarrow z,\ g = \overline{x} \cdot \overline{y} \cdot z;$

5) $f = (\overline{x} \rightarrow \overline{y}) \rightarrow (y \rightarrow x),\ g = (x \rightarrow y) \cdot (\overline{y} \rightarrow \overline{x});$

6) $f = x \cdot y \vee z,\ g = x \cdot (y \vee z).$

3.2.16. Quyidagi bul funksiyalariga ikki taraflama funksiyani toping va uni soddalashtiring:

1) $f = (x \vee y \vee z) \cdot (y \oplus z) \vee x \cdot y \cdot z;$

2) $f = (x \vee (1 \rightarrow y)) \vee y \cdot \overline{z} \vee (\overline{x} | \overline{y \downarrow \overline{z}});$

3) $f = (x \downarrow y) \oplus ((x | y) \downarrow (\overline{x} \leftrightarrow y \cdot z));$

4) $f = (\overline{x} \vee \overline{y} \vee (y \cdot \overline{z} \oplus 1)) \downarrow z;$

5) $f = (x \cdot (y \cdot z \vee 0) \leftrightarrow (z \cdot 1 \vee \overline{x} \cdot y)) \vee \overline{y} \cdot z;$

6) $f = (x \downarrow z) \oplus ((x \vee y) \leftrightarrow (\overline{x} \downarrow (y \vee \overline{z})));$

3.2.17. To'rt argumentli o'z-o'ziga ikki taraflama funksiyalarga misol keltiring.

3.2.18. n argumentli o'z-o'ziga ikki taraflama funksiyalar sonini toping.

3.2.19. n argumentli funksiyaning ikki taraflama funksiyasi uning inkoriga teng bo'lgan funksiyalar sonini toping. Ya'ni f uchun $f^{*} = f'$.

3.2.20. n argumentli o'z-o'ziga ikki taraflama funksiyalardan barcha argumentlari muhim bo'lganlarining sonini toping.

3.2.21. n argumentli o'z-o'ziga ikki taraflama funksiyalarning qiymatlar jadvalida 0 va 1 lar soni o'zaro teng bo'lishini isbotlang.

3.2.22. n argumentli o'z-o'ziga ikki taraflama funksiyalarda 0 ni saqlovchi funksiyalar va 1 ni saqlovchi funksiyalar soni o'zaro teng bo'lishini isbotlang.

3.3. Bul funksiyalarining o'zgaruvchilar bo'yicha yoyilmasi. Mukammal diz'yunktiv normal shakl. Mukammal kon'yunktiv normal shakl.

$f(x_1, x_2, ..., x_n)$ funksiyani x_1, x_2 ***o'zgaruvchilari bo'yicha yoyilmasi*** quyidagicha:

$$f(x_1,x_2,...,x_n) = x_1 f(1,x_2,...,x_n) \vee \overline{x_1} f(0,x_2,...,x_n) = x_1\{x_2 f(1,1,x_3,...,x_n) \vee$$
$$\vee \overline{x_2} f(1,0,x_3,...,x_n)\} \vee \overline{x_1}\{x_2 f(0,1,x_3,...,x_n) \vee \overline{x_2} f(0,0,x_3,...,x_n)\} =$$

| distributivlik qonunini qo'llab qavslarni ochamiz|

$$= x_1 x_2 f(1,1,x_3,...,x_n) \vee x_1 \overline{x_2} f(1,0,x_3,...,x_n) \vee \overline{x_1} x_2 f(0,1,x_3,...,x_n) \vee$$
$$\vee \overline{x_1}\,\overline{x_2} f(0,0,x_3,...,x_n). \quad x = x^1,\ \overline{x} = x^0 \ \Rightarrow x^e -$$ x ning e –darajasi deb,

$$f(x_1,x_2,...,x_n) = \bigvee_{e_1 e_2 \in E^2} x_1^{e_1} x_2^{e_2} f(e_1,e_2,x_3,...,x_n)$$ natija olinadi.

Misol: $f(x,y,z) = (\overline{y} \leftrightarrow x\overline{z}) \to yz$ $x,\ y$ o'zgaruvchilari bo'yicha yoyilmasini toping. Yoyilma koeffisiyentlarini topamiz:

$$f(1,1,z) = (\overline{1} \leftrightarrow 1\overline{z}) \to 1z = (0 \leftrightarrow \overline{z}) \to z = z \to z;$$

$$f(1,0,z)=(\bar{0}\leftrightarrow 1\bar{z})\to 0z=(1\leftrightarrow \bar{z})\to 0=\bar{z}\to 0=z;$$

$$f(0,1,z)=(\bar{1}\leftrightarrow 0\bar{z})\to 1z=(0\leftrightarrow 0)\to z=1\to z=z;$$

$$f(0,0,z)=(\bar{0}\leftrightarrow 0\bar{z})\to 0z=(1\leftrightarrow 0)\to 0=1.$$

Yoyilma koeffisiyentlarini formulaga qo'yamiz:

$$f(x,y,z)=(\bar{y}\leftrightarrow x\bar{z})\to yz=xyf(1,1,z)\vee x\bar{y}f(1,0,z)\vee \bar{x}yf(0,1,z)\vee$$

$$\vee\bar{x}\bar{y}f(0,0,z)=xy(z\to z)\vee x\bar{y}z\vee \bar{x}yz\vee \bar{x}\bar{y}.$$

$f(x_1,x_2,...,x_n)$ funksiyani k ta $(k\le n)$ ***o'zgaruvchilari bo'yicha yoyilmasi*** quyidagicha:

$$f(x_1,x_2,...,x_n)=\bigvee_{e_1e_2...e_k\in E^k} x_1^{e_1}x_2^{e_2}...x_2^{e_k}\, f(e_1,e_2,...,e^k,x_{k+1},...,x_n).$$

$f(x_1,x_2,...,x_n)=\bigvee_{e_1e_2...e_n\in E^n} x_1^{e_1}x_2^{e_2}...x_2^{e_n}$ - ***Mukammal diz'yunktiv normal shakl. (MDNSh)***

$f(x_1,x_2,...,x_n)=\bigwedge_{e_1e_2...e_n\in E^n} (x_1^{e_1}\vee x_2^{e_2}\vee ...\vee x_2^{e_n})$ - ***Mukammal kon'yunktiv normal shakl. (MKNSh)***

Muammoli masala va topshiriqlar:

3.3.1. Berilgan bul funksiyalarini a) x_1 o'zgaruvchisi bo'yicha yoyilmasini toping; b) x_1, x_2 o'zgaruvchilari bo'yicha yoyilmasini toping:

1) $f(\tilde{x}^2)=(x_2\to x_1)\cdot(x_1\downarrow x_2);$

2) $f(\tilde{x}^2)=(x_1\leftrightarrow x_2)\vee(x_1\,|\,x_2);$

3) $f(\tilde{x}^3)=((x_1\oplus x_2)\to x_3)\cdot\overline{(x_3\to x_2)};$

4) $f(\tilde{x}^3)=((x_1\vee x_2\cdot\overline{x_3})\leftrightarrow(\overline{x_1}\to\overline{x_2}\cdot x_3))(x_2\downarrow x_3)$

5) $f(\tilde{x}^3)=((x_1\vee x_2\vee\overline{x_3})\to(x_1x_2\,|\,x_3))\oplus(x_2\to x_1)\cdot x_3$;

6) $A=(\bar{x}\vee y\cdot\bar{z})\to((x\to y)\to((y\vee z)\to\bar{x}))$;

7) $A=(x|\bar{y})\to((y\downarrow\bar{z})\to(x\oplus z))$;

8) $A=(x\cdot\bar{y}\vee\bar{x}\cdot z)\oplus((y\to z)\to\bar{x}\cdot y)$;

9) $A=(\bar{x}\vee y)\to((y|\bar{z})\to(x\leftrightarrow x\cdot z))$;

10) $A=x\to((\bar{x}\cdot\bar{y}\to(\bar{x}\cdot\bar{z}\to y))\to y)\cdot z$;

11) $B=(x\to y\cdot z)\cdot\overline{x\to y}$.

3.3.2. Berilgan bul funksiyalarini MDNSh ga keltirib, ikki taraflama funksiyasini toping:

1) $f=(x\vee y\vee z)\cdot(y\oplus z)\vee x\cdot y\cdot z$;

2) $f=(x\vee(1\to y))\vee y\cdot\bar{z}\vee(\bar{x}\,|\,\overline{y\downarrow\bar{z}})$;

3) $f=(x\downarrow y)\oplus((x|y)\downarrow(\bar{x}\leftrightarrow y\cdot z))$;

4) $f=(\bar{x}\vee\bar{y}\vee(y\cdot\bar{z}\oplus 1))\downarrow z$;

5) $f=(x\cdot(y\cdot z\vee 0)\leftrightarrow(z\cdot 1\vee\bar{x}\cdot y))\vee\bar{y}\cdot z$;

6) $f=(x\downarrow z)\oplus((x\vee y)\leftrightarrow(\bar{x}\downarrow(y\vee\bar{z})))$;

7) $A=\overline{x\oplus y\cdot z}\cdot\overline{\bar{y}\to x\cdot z}\cdot(\bar{x}\downarrow y)$;

8) $A=(x\oplus y\cdot z)\to(\bar{x}\to(y\to z))$;

9) $A=((x\to y\cdot z)\oplus(x\leftrightarrow y))\vee(y\to x\cdot z)$;

10) $A=((x|y)\downarrow(y|\bar{z}))\cdot(x\to(y\to z))$.

11) $A=(x\to(y\to z))\to x$.

3.3.3. Berilgan bul funksiyalarini MKNSh ga keltirib, ikki taraflama funksiyasini toping:

1) $f(\tilde{x}^3)=x_1\overline{x_2}\vee\overline{x_2}x_3\vee(x_1\to x_2x_3);$

2) $f(\tilde{x}^3)=(x_1\leftrightarrow\overline{x_2})\vee(x_1x_3\oplus(x_2\to x_3));$

3) $f(\tilde{x}^3)=(x_1\cdot x_2\oplus x_3)\cdot(x_1\cdot x_3\to x_2);$

4) $f(\tilde{x}^3)=(\overline{x_1}\vee\overline{x_2}\vee\overline{x_3})\cdot(x_1\cdot x_2\vee x_3);$

5) $f(\tilde{x}^3)=(x_1\to x_2)\oplus(x_1|x_2\cdot x_3);$

6) $A=(\bar{x}\to(\bar{y}\to(x\leftrightarrow z)))\cdot(x\leftrightarrow(y\to(z\vee(x\to y)))),$

7) $A=((x\vee y)\cdot\bar{z}\to((x\leftrightarrow\bar{z})\oplus\bar{y}))\cdot((x\oplus y)\cdot\bar{z});$

8) $A=\overline{((x\leftrightarrow y)\to(x\to z))}\vee(x\oplus\bar{y}\cdot z);$

9) $A=\overline{(x\downarrow y)\vee(x\leftrightarrow z)}|(x\oplus y\cdot z);$

10) $A=x\to((y\to z)\to y\cdot z);$

11) $f(\tilde{x}^3)=(\overline{x_1}\cdot x_2\to x_3)\cdot((x_1\to x_3)\to x_2).$

3.3.4. Berilgan bul funksiyalarining MKNSh va MDNSh ni toping.

1) $f(\tilde{x}^3)=(10101010);$ 2) $f(\tilde{x}^3)=(10011001);$

3) $f(\tilde{x}^3)=(00111100);$ 4) $f(\tilde{x}^4)=(0101111101\ 011111);$

5) $f(\tilde{x}^4)=(1100110000\ 110011);$ 6) $f(\tilde{x}^4)=(1011010\ 110110101);$

7) $f(\tilde{x}^2)=((x_1\vee x_2)\to x_1\cdot x_2)\oplus(x_1\to x_2)\cdot(x_2\to x_1);$

8) $f(\tilde{x}^2)=(x_1\cdot x_2\oplus(x_1\to x_2))\to(x_1\leftrightarrow x_1\cdot x_2);$

9) $f(\tilde{x}^3)=((x_1\to\overline{x_2})\oplus(x_2\to\overline{x_3}))\oplus(x_2\to x_3);$

10) $f(\tilde{x}^3)=((x_1\vee x_2\cdot\overline{x_3})(x_2\to x_1\cdot x_3))\to(x_1\vee x_3);$

11) $f(\tilde{x}^3)=((x_1\downarrow(x_2|x_3))\downarrow(x_2\downarrow(x_1|x_3)))\downarrow(x_1|x_2).$

3.3.5. Berilgan diz'yunktiv normal shakldagi funksiyalarni mukammal diz'yunktiv normal shaklga keltiring:

1) $f(\tilde{x}^3)=\overline{x_1}\cdot x_2\vee\overline{x_3};$

2) $f(\tilde{x}^3)=\overline{x_1}\cdot\overline{x_2}\vee x_2\cdot\overline{x_3}\vee x_1\cdot\overline{x_3};$

3) $f(\tilde{x}^3)=x_1\cdot x_2\cdot\overline{x_3}\vee\overline{x_1}\cdot x_2\vee\overline{x_3}\,;$

4) $f(\tilde{x}^3)=\overline{x_1}\cdot\overline{x_2}\vee x_1\cdot x_2\cdot\overline{x_3}\vee\overline{x_3};$

5) $f(\tilde{x}^3)=x_1\vee x_2\cdot x_3\vee\overline{x_2}\cdot\overline{x_3}.$

3.3.6. Berilgan kon’yunktiv normal shakldagi funksiyalarni mukammal kon’yunktiv normal shaklga keltiring:

1) $f(\tilde{x}^3)=\overline{x_1}\cdot(\overline{x_2}\vee x_2);$

2) $f(\tilde{x}^3)=(x_1\vee x_2)\cdot(\overline{x}_2\vee x_3)\cdot\overline{x_3};$

3) $f(\tilde{x}^3)=(\overline{x_1}\vee\overline{x_2}\vee x_3)\cdot(\overline{x_1}\vee\overline{x}_3)\cdot(\overline{x_2}\vee x_3);$

4) $f(\tilde{x}^3)=(\overline{x_1}\vee x_2\vee\overline{x}_3)\cdot(\overline{x_2}\vee x_3);$

5) $f(\tilde{x}^3)=(\overline{x_1}\vee x_2)\cdot(x_1\vee\overline{x}_3)\cdot(x_2\vee x_3).$

3.3.7. 3.3.5. misolda berilgan diz’yunktiv normal shakldagi funksiyalarni mukammal kon’yunktiv normal shaklga keltiring.

3.3.8. 3.3.6. misolda berilgan kon’yunktiv normal shakldagi funksiyalarni mukammal diz’yunktiv normal shaklga keltiring.

3.3.9. 3.3.5. misolda berilgan diz’yunktiv normal shakldagi funksiyalarni x_1, x_2 o’zgaruvchilari bo’yicha yoyilmasini toping.

3.3.10. 3.3.6. misolda berilgan kon’yunktiv normal shakldagi funksiyalarni x_1, x_2 o’zgaruvchilari bo’yicha yoyilmasini toping.

3.4. Jegalkin ko'phadi. Funksiyalar sistemasining to'liqligi va yopiqligi. Muhim yopiq sinflar.

$x+y=\overline{x\leftrightarrow y}$. Mantiq algebrasidagi ko'paytma va 2 moduli bo'yicha qo'shish mantiq amallari uchun kommutativ, assostiativ va distributiv arifmetik qonunlar o'z kuchini saqlaydi.

Bul algebrasidagi asosiy mantiqiy amallarni kiritilgan arifmetik amallar orqali quyidagicha ifodalash mumkin:

1. $\bar{x}=x+1$; 2. $x\wedge y=xy$; 3. $x\vee y=xy+x+y$;

4. $x\to y=xy+x+1$; 5. $x\leftrightarrow y=x+y+1$.

2 moduli bo'yicha qo'shish amalining ta'rifiga asosan $x+x=0$ va $xx=x$ $(x^n=x)$.

$\sum x_{i_1}x_{i_2}.....x_{i_k}+a$ ko'rinishidagi ko'phad - ***Jegalkin ko'phadi***

$x_{i_1}+x_{i_2}+.....+x_{i_k}+a$ ko'rinishidagi funksiya - ***chiziqli funksiya***

n argumentli ***chiziqli funksiyalar soni*** 2^{n+1} ga teng va bir argumentli funksiyalar doimo chiziqli funksiya bo'ladi.

0<1 munosabati orqali {0,1} to'plamini tartiblashtiramiz. $\alpha=(\alpha_1,...,\alpha_n)$ va $\beta=(\beta_1,...,\beta_n)$ qiymatlar satri bo'lsin. α qiymatlar satri β qiymatlar satridan shunda va faqat shundagina oldin keladi deb aytamiz, qachon $\alpha\prec\beta$ yoki α va β qiymatlar satri ustma-ust tushsa, u holda $\alpha\prec\beta$ shaklida yozamiz.

$\alpha=(\alpha_1,...,\alpha_n)$ va $\beta=(\beta_1,...,\beta_n)$ ixtiyoriy qiymatlar satri bo'lsin. $\alpha\prec\beta$ dan $f(\alpha_1,...,\alpha_n)\le f(\beta_1,...,\beta_n)$ bajarilishi kelib chiqsa, u holda $f(x_1,...,x_n)$ funksiya ***monoton funksiya*** deb aytiladi.

$\alpha\prec\beta$ dan $f(\alpha_1,...,\alpha_n)>f(\beta_1,...,\beta_n)$ munosabat kelib chiqsa, u holda $f(x_1,...,x_n)$ ***nomonoton funksiya*** deb aytiladi.

Asosiy elementar mantiqiy funksiyalardan 0, 1, x, xy, $x\vee y$ funkstiyalar monoton funkstiyalar bo'lib, $\bar{x}$, $x\to y$, $x\leftrightarrow y$, $x+y$ funksiyalar nomonoton funkstiyalardir.

Monoton funkstiyalarning superpozistiyasidan hosil qilingan funksiya yana ***monoton funksiya*** bo'ladi.

Agar mantiq algebrasining istalgan funksiyasini $\Phi = \{\varphi_1,...,\varphi_n\}$ sistemadagi funksiyalar superpozistiyasi orqali ifodalash mumkin bo'lsa, u holda F ga ***to'liq funksiyalar sistemasi*** deb aytiladi.

Istalgan funksiyani MKNSh yoki MDNSh ko'rinishida ifodalash mumkinligidan $\{xy, x \vee y, \bar{x}\}$ funksiyalar sistemasining to'liqligi kelib chiqadi. $\{xy, x+y, 1\}$ funksiyalar sistemasi ham to'liq bo'ladi, chunki istalgan funksiyani Jegalkin ko'phadi ko'rinishiga keltirish mumkin.

Misol. Ushbi funksiyani Jegalkin ko'phadi ko'rinishida ifodalang:

$$F(x_1, x_2, x_3) = (x_1 \mid x_2) + (x_1 \wedge x_3)$$

***Yechish*:** Berilgan funksiya uchun noma'lum koeffisientli ko'phad ko'rinishidagi ifodasini izlaymiz:

$$F(x_1, x_2, x_3) = (x_1 \mid x_2) + (x_1 \wedge x_3) = a \cdot x_1x_2x_3 + b \cdot x_1x_2 + c \cdot x_1x_3 + \\ + d \cdot x_2x_3 + e \cdot x_1 + f \cdot x_2 + g \cdot x_3 + h$$

Funksiyaning qiymatlar jadvalida noma'lum koeffisientlarni aniqlaymiz:

x_1	x_2	x_3	$(x_1 \mid x_2) + (x_1 \wedge x_3)$	$a \cdot x_1x_2x_3 + b \cdot x_1x_2 + c \cdot x_1x_3 + d \cdot x_2x_3 + e \cdot x_1 + f \cdot x_2 + g \cdot x_3 + h$	
0	0	0	1	*h*	*h=1*
0	0	1	1	*g+h*	*g=0*
0	1	0	1	*f+h*	*f=0*
0	1	1	1	*d+f+g+h*	*d=0*
1	0	0	1	*e+h*	*e=0*
1	0	1	0	*c+e+g+h*	*c=1*
1	1	0	0	*b+e+f+h*	*b=1*
1	1	1	1	*a+b+c+d+e+f+g+h*	*a=0*

Jadvalning 4 va 5- ustunlarini tenglashtirishdan hosil bo'lgan tenglamalar (noma'lum koeffisientlarga nisbatan) sistemasini yechib, 6- ustunni hosil qilamiz. Demak, $F(x_1,x_2,x_3)=(x_1 \mid x_2)+(x_1 \wedge x_3)=x_1x_2+x_1x_3+1$

Misol. Quyidagi funksiyalar sistemasining to'liqligini isbotlang:

a) $xy, \bar{x}$; b) $x\vee y, \bar{x}$; v) $xy, x+y, 1$;

g) $\overline{x\vee y}$; d) $\bar{x}\bar{y}$; i) $x+y, x\vee y, 1$;

j) $x+y+z, xy, 0, 1$; z) $x\to y, \bar{x}$; e) $x\to y, 0$.

Isbot. a). $x\vee y=\overline{\bar{x}\bar{y}}$, ya'ni diz'yunkstiya amalini kon'yunkstiya va inkor amallari orqali ifodalash mumkin. Demak, $\{xy, \bar{x}\}$ funksiyalar sistemasi to'liq bo'ladi.

b) $xy=\overline{\bar{x}\bar{y}}=\overline{\bar{x}\vee\bar{y}}$ ekanligi ma'lum. Demak, istalgan mantiqiy funksiyani diz'yunkstiya va inkor amallari orqali ifodalasa bo'ladi. Shuning uchun $\{x\vee y, \bar{x}\}$ funksiyalar sistemasi to'liqdir.

v) Ixtiyoriy mantiq algebrasining funksiyasini yagona Jegalkin ko'phadi ko'rinishiga keltirish mumkinligidan $\{xy, x+y, 1\}$ funksiyalar sistemasining to'liqligi kelib chiqadi.

g) va d). Mantiq algebrasidagi istalgan funksiyani $\psi(x,y)=\overline{xy}$ va $\varphi(x,y)=\overline{x\vee y}$ Sheffer funksiyalari orqali ifodalash mumkin. Haqiqatan ham,

$\bar{x}=\varphi(x,x)$

$x\vee y=\overline{\overline{x\vee y}}=\overline{\varphi(x,y)}=\varphi(\varphi(x,y),\varphi(x,y))$ $xy=\varphi(\bar{x},\bar{y})=\varphi(\varphi(x,x),\varphi(y,y))$

asosiy mantiqiy amallarni Sheffer funksiyasi orqali ifodalash mumkin. Demak, $\{\overline{xy}\}$ va $\{\overline{x\vee y}\}$ funksiyalar sistemasi to'liq bo'ladi.

i). $x \vee y = xy + x + y$ bo'lganligi uchun $x \vee y + (x + y) = xy$ bo'ladi. $\{xy, x + y, 1\}$ to'liq sistema ekanligi v) punktida isbot qilingan edi, demak, $\{x + y, x \vee y, 1\}$ sistema to'liqdir.

Misol. Quyidagi funksiyalar sistemasining to'liq emasligini isbotlaylik:

a) $\bar{x}$, 1; b) $xy, x \vee y$; v) $x + y, \bar{x}$; g) $xy \vee yz \vee xz, \bar{x}$;

d) $xy \vee yz \vee xz, 0, 1$.

a). $\bar{x} = x + 1$ ga teng. Demak, $\{\bar{x}, 1\}$ sistemasidagi funksiyalar bir argumentli funksiyalar bo'ladi. Bizga ma'lumki, bir argumentli funksiyalarning superpozistiyasi natijasida hosil qilingan funksiya yana bir argumentli funksiya bo'ladi. Natijada, bu sistemadagi funksiyalar orqali ko'p argumentli funksiyalarni ifodalab bo'lmaydi. Shuning uchun $\{\bar{x}, 1\}\}$ to'liq sistema emas.

b). $\{xy, x \vee y\}$ sistemasidagi funksiyalarning ikkalasi ham monotondir. Monoton funksiyalarning superpozistiyasi orqali hosil qilingan funksiya yana monoton bo'lishini isbot qilgan edik. Demak, bu ikkala funksiyaning superpozistiyasi orqali monoton bo'lmagan funksiyalarni ifodalash mumkin emas va natijada, $\{xy, x \vee y\}$ sistema to'liqmas sistema bo'ladi.

v). $\{x + y, \bar{x}\}$ cistemasidagi funksiyalar chiziqli funksiyalardir. Shuning uchun bu funksiyalar orqali chiziqlimas funksiyalarni ifodalab bo'lmaydi. Demak, $\{x + y, \bar{x}\}$ funksiyalar sistemasi to'liq emas.

g). $\{xy \vee yz \vee xz, \bar{x}\}$ sistemasidagi funksiyalar o'z-o'ziga ikkitaraflama funksiyalardir. Bu funksiyalarning superpozistiyasidan hosil qilingan har qanday funksiya ham o'z-o'ziga ikkitaraflama funksiya bo'ladi.

Demak, $\{xy \vee yz \vee xz, \bar{x}\}$ funksiyalar sistemasi to'liq emas.

d). $\{xy \vee yz \vee xz,\ 0, 1\}$ sistemadagi funksiyalarning hammasi monoton funksiyalar bo'ladi. Monoton emas funksiyalar bu sistemadagi funksiyalar orqali ifodalanmaydi. Demak, $\{xy \vee yz \vee xz,\ 0, 1\}$ sistema to'liq emas.

Funkstional yopiq sinflarga misollar:

1) bir argumentli funksiyalar;

2) hamma mantiq algebrasining funksiyalari;

3) L - chiziqli funksiyalar;

4) S - o'z-o'ziga ikkitaraflama funksiyalar;

5) M - monoton funksiyalar;

6) P_0 - nol qiymatni saqlovchi funksiyalar;

7) P_1 - bir qiymatni saqlovchi funksiyalar.

maksimal funkstional yopiq sinflar: P_0 - nol saqlovchi funksiyalar sinfi, P_1 - bir saqlovchi funksiyalar sinfi, M - monoton funksiyalar sinfi, S - o'z-o'ziga ikkitaraflama funksiyalar sinfi, L - chiziqli funksiyalar sinfi.

Muammoli masala va topshiriqlar:

3.4.1. 1) $x \to y \leftrightarrow z$; 2) $x \vee y \vee z \vee t$; 3) $x \leftrightarrow y \leftrightarrow z$ 4) $x \vee y \vee z$;

5) $xy \vee yz \vee xz$; 6) $xy\bar{z} \vee x\bar{y}z \vee \bar{x}yz \vee \bar{x}\bar{y}\bar{z}$ formulalarni Jegalkin ko'phadi ko'rinishiga keltiring.

3.4.2. Funksiyaning Jegalkin ko'phadi ko'rinishidagi ifodasi yagona ekanligini isbotlang.

3.4.3. Chiziqli funksiyalarning qaysi birlari o'z-o'ziga ikkitaraflama funksiya bo'ladi?

3.4.4. $xy \vee xz \vee yz = xy + xz + yz$ ekanligini isbot eting.

3.4.5. Jegalkin ko'phadi ko'rinishidagi funksiyaning hamma argumentlari soxta argumentlar emasligini isbotlang.

3.4.6 Quyidagi bul funksiyalariga teng kuchli bo'lgan Jegalkin ko'pxadini noma'lum koeffisientli ko'phad usuli bilan toping:

1) $x'(yz'\vee y'z)$;
2) $(x\to(y\to z'))(yz'\to x)$;
3) $(x+1)(y+1)z'\vee yz$;
4) $x'z'\vee(x'y\vee xy')z$;
5) $x'y'z\vee xz'$;
6) $(x'\vee y\vee z)(x\vee y\vee z')(x'\vee y'\vee z')$;
7) $x(y\to z)\vee(x'y\vee xy')(z+1)$;
8) $x'\vee y\vee z'$;
9) $x'y'z'\vee x'yz'\vee x'yz\vee xy'z\vee xyz'$;
10) $xz\vee(x+z)y\vee x'z'$;
11) $x'z'\vee(x'z\vee xz')y\vee xy'z'$.

3.4.7. Tenglikning o'ng va chap tomoni Jegalkin ko'phadi ko'rinishiga keltirib, quyidagilarning to'g'rilarini aniqlang:

1) $x\to(y\to z)=(x\to y)\to(x\to z)$;
2) $(xy\to z)\to(x\to z)=x'yz$;
3) $xy\to z=(x\to z)(y\to z)$;
4) $(x\leftrightarrow y)(xy'\vee y)=xy$;
5) $(x\leftrightarrow y')'=x\leftrightarrow y$;
6) $z\to(x\vee y)=(z\to x)\to(x\to y)$;
7) $x\leftrightarrow y=(xz\leftrightarrow yz)((xvz)\leftrightarrow(y\vee z))$;
8) $xy\vee(z\to x)=x'\to z'$,
9) $((x\to z)(y\to z))=((x\vee y)\to z)$
10) $x\leftrightarrow y=xy\vee x'y'$
11) $(x\to y)\to z=x\to(y\to z)$.

3.4.8. Quyidagi bul funksiyalarini Jegalkin ko'phadi ko'rinishini topib, qaysi biri aynan chin -1, yoki aynan yolg'onligini -0 aniqlang:

1) $(y\to z)\to((x\to y)\to(x\to z))$;
2) $x\to(x'\to y)$;
3) $(x\to y)\to((x\to(y\to z))\to(x\to z)$;
4) $((x\to y)\to x)\to x$;
5) $(x\to(y\to x))\to((y'\to x')'(x\to y))$;
6) $(x\to yz)\to((x\to y)\to(x\to z))$;
7) $x'\to(x\to y)$;

8) $(x\to(y\to z))(x\to y)(x\to z)'$;

9) $((x\to y)\to(y'\to x'))\to((y'\to x')\to(x\to y))$;

10) $(x'\vee(y\to x))'$;

11) $(x\vee y)'\vee x'y\vee x$.

3.4.9. Bir argumentli har bir bul funksiyasi chiziqli bo'lishini tekshiring.

3.4.10. Barcha ikki argumentli chiziqli bul funksiyalarini toping.

3.4.11. n argumentli chiziqli bul funksiyalari soni 2^{n+1} ta ekanligini isbotlang.

3.4.12. Quyidagi barcha bul funksiyalari chiziqli ekanligini isbotlang:

1) $x'y'z'\vee xy'z'\vee x'yz\vee xyz$;

2) $((x\vee y\vee z)\to xyz')\vee(x+y)z$;

3) $(x'y\vee xy')z'\vee(x+y)z$;

4) $(x+y)z\vee(x'y'\vee xy)z$;

5) $x'yz\vee(x+z+1)y'\vee xyz'$;

6) $((x'\vee y\vee z')\to xy'z')\vee(x'z\vee xz')y$;

7) $x'yz'\vee x'yz\vee xyz'\vee xyz$;

8) $x'(y'z\vee yz')\vee(y+z)x$;

9) $(x\vee y'\vee z)(x\vee y'\vee z')(x'\vee y'\vee z)(x'\vee y'\vee z')$

10) $xyz'\vee(x'\vee y\vee z)\to xyz)\vee x'y'z$;

11) $xyz\vee xy'z\vee x'yz'\vee x'y'z'$.

Yechim: 11) $xyz\vee xy'z\vee x'yz'\vee x'y'z' = xz(y\vee y')\vee x'z'\vee(y\vee y') = xz\vee x'z' =$

$= xz\vee(x+1)(z+1) = xz\vee(xz+x+z+1) = xz(xz+x+z+1)+xz+$

$xz+x+z+1 = xz+xz+xz+xz+x+z+1 = x+z+1$.

3.4.13. Berilgan funksiyalardan qaysi chiziqli ekanligini tekshiring:

1) $x'y'z'\vee x'y'z\vee x'yz\vee xyz'$;

2) $x'y'z'\vee x'yz'\vee xyz'\vee xyz$;

3) $(x\vee y\vee z')(x\vee y'\vee z')(x'\vee y\vee z')(x'\vee y'\vee z)$;

4) $(xy\to z)'\vee(x'\vee y')(x\vee y)z$;

5) $x'(y + z) \vee ((x \vee y \vee z) \to x'yz)$;

6) $x' \to (y \vee z')$;

7) $xyz \vee xyz' \vee xy'z$;

8) $((x' \to y)' \to z)'$;

9) $x'(yz' \vee y'z) \vee (y+z+1)x$;

10) $(yz' + x) \to (x \leftrightarrow xy')'$;

11) $((x \vee y \vee z) \to xy'z) \vee (x'z \vee xz')y \vee xy'z'$.

Yechish:11)$((x \vee y \vee z) \to xy'z) \vee (x'z \vee xz')y \vee xy'z' = ((x \vee y \vee z)' \vee xy'z) \vee ((x+ +1)z \vee x(z+1))y \vee xyz' = x'y'z' \vee xy'z \vee xy'z' \vee ((xz+z) \vee (xz+x))y = (x' \vee x)y'z' \vee xy'z \vee ((xz + z)(xz + x)+(xz + z) + (xz + x))y = y'z' \vee xy'z \vee (xz + xz+ + xz + xz + z + x))y = y'(z' \vee xz) \vee (x + z)y = y'((z' \vee x)(z' \vee z)) \vee (xy + yz)+(x \vee z')y' \vee (xy + yz) = (xz' + x + z')y' \vee (xy + yz) = (xy'z' + xy' + y'z') \vee (xy + yz) = =(xyz + xy + xz + x + xy + x + yz + y + z + 1) \vee (xy + yz) = (xyz + xz + yz + y + z + 1) \vee (xy + yz) = xyz + +xz+ yz + y + z +1+xy+yz = xyz + xyz + xyz + xy + xyz + xy + xyz + xzy + yz+yz + yz + yz + xyz + xz + y+z+1+xy = xyz + xy + xz + y + +z+1$ - chiziqli emas.

3.4.14. Chiziqli funksiyalar superpozisiyasidan hosil qilingan funksiya yana chiziqli bo'lishini isbotlang.

3.4.15. Quyidagilarni taqqoslang.

1) (0, 1, 0), (1, 0, 0);

2) (0, 1, 1, 0), (0, 1, 0, 1);

3) (1, 0, 0, 1), (1, 0, 1, 1);

4) (1, 1, 0), (1, 0, 0);

5) (0, 1), (1, 0);

6) (1, 0, 0), (1, 0, 1);

7) (0, 1, 0, 0), (0, 1,0,1);

8) (1,0,0,1,0) (1,0,1,1,0);

9) (0,1,0,0,1)(0,1,0,1,0);

10) (1, 0, 1), (1, 0, 0);

11) (0, 1, 1, 1) (0,1,0, 0).

Yechim: 11) Berilgan to'plamlarning birinchi elementlarini taqqoslaymiz: 0=0.

Ikkinchi elementlarni taqqoslaymiz : 1=1. Uchinchisini $1 \succ 0$ va nihoyat to'rtinchisi $1 \succ 0$. Shunday qilib, birinchi to'plamdagi elementlar yoki teng, yoki katta bo'lganligi sabali $(0, 1, 1, 1) \succ (0,1,0, 0)$.

3.4.16. Bul funksiyalarining monotonligini isbotlang.

1) $xyz \vee x'yz \vee xyz'$;

2) $xyz + xy + xz$;

3) $xy \vee xz \vee yz$;

4) $(x \vee y \vee z)(x \vee y' \vee z)(x \vee y \vee z')$;

5) $xyz + x + yz$;

6) $xyz \vee (xyz + xy)$;

7) $(x \vee y \vee z)(x' \vee y \vee z)(x \vee y' \vee z)(x \vee y \vee z')(x' \vee y' \vee z)(x \vee y' \vee z')$;

8) $x'y'z \vee xy'z' \vee x'yz \vee xyz$;

9) $xyz \vee xz \vee yz$;

10) $xy + (x + y)$;

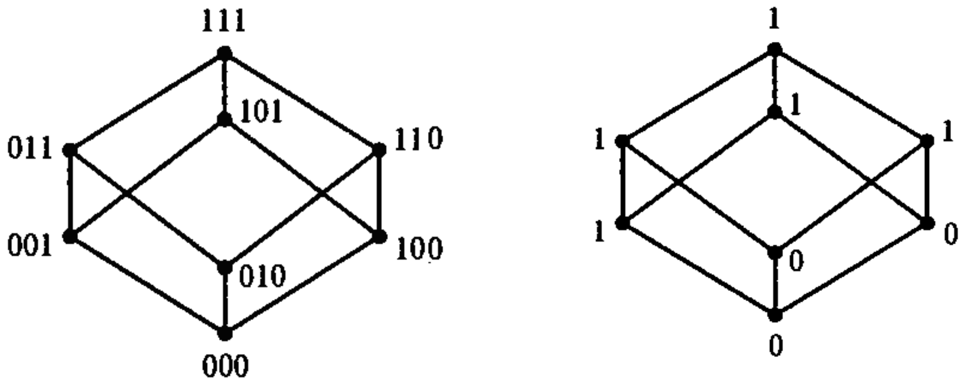
11) $(x \vee y \vee z)(x' \vee y \vee z)(x \vee y' \vee z)$.

Yechim: 11) avval qiymatlar jadvalini tuzamiz:

x	*y*	*z*	*f(x,y,z)*
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Endi har bir qiymatlar satrini va natijasini taqqoslaymiz:000 < 001, 000 < 101, 000 < 010, 000 <110, 000 < 011, 000 < 100, 000 < 111, 001 < 011, 001 < 101, 001 < 111, 010 < 110, 010 < 111, 010 < 011, 011 < 111, 100 < 101, 100 < 110, 100< 111, 101<111, 110< 111, 011 < 111. Demak, berilgan funksiyamiz monoton funksiya.

Berilgan funksiyani taqqoslashning yana bir usuli diagrammada tasvirlash yordamida bajariladi. (chap tomonda o’zgaruvchilarning qiymatlari diagrammasi tasvirlangan bo’lsa, o’ng tomonda funksiyaning qabul qilgan qiymatlari diagrammasi tasvirlangan.) Ko’rinib turibdiki, o’zgaruvchilarning qiymatlari pastdan yuqoriga o’sib borayapti va xuddi shunday funksiya qabul qilgan qiymatlar ham pastdan yuqoriga o’sib boradi. Demak, berilgan funksiyamiz monoton funksiya.



3.4.17. Barcha uch o’zgaruvchili monoton funksiyalarni toping. Bunday funksiyalar sonini aniqlang.

3.4.18. To’rt o’zgaruvchili monoton funksiyaga misol keltiring.

3.4.19. Quyidagi bul funksiyalaridan qaysilari monoton bo’lishini aniqlang:

1) xyz;

2) $(x + y)z+(x + z)'$;

3) $(x\vee y\vee z)(x'\vee y\vee z)(x\vee y\vee z')$;

4) $xyz + xz$;

5) $x'y'z'\vee x'yz'\vee zy'z'\vee xyz'\vee xyz$;

6) $xy(z + 1) + z$;

7) $x'yz' \vee xy'z \vee (x'y' + x' + y)$;

8) $x' (y+z) \vee ((x' \vee y \vee z) \to xyz) \vee x ((y \to z) + 1)$;

9) $(x \vee y \vee z)(x' \vee y \vee z)(x \vee y' \vee z')(x' \vee y' \vee z)$;

10) $xyz + xy + xz + yz + x + y + z$;

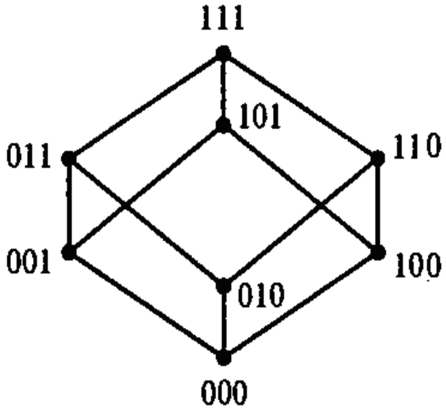
11) $(x \vee y)(x' \vee y)z \vee x'yz \vee xy'z'$.

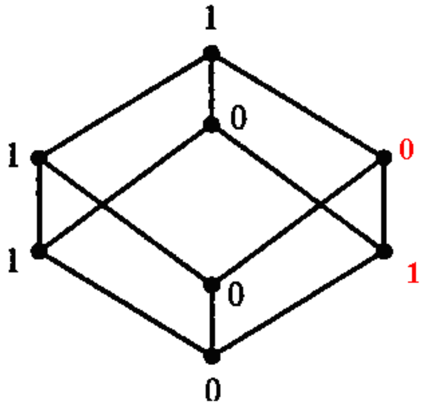
Yechim: 11) avval qiymatlar jadvalini tuzamiz:

x	y	z	*F(x,y,z)*
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Endi har bir qiymatlar satrini va natijasini taqqoslaymiz: 000 < 001, 000 < 101, 001 < 101, ammo, f (001)=1, f (101)=0 bundan f (001) > f (101). Demak, berilgan funksiyamiz nomonoton funksiya.

Diagramma usulida tekshirib ko'ramiz. (chap tomonda o'zgaruvchilarning qiymatlari diagrammasi tasvirlangan bo'lsa, o'ng tomonda funksiyaning qabul qilgan qiymatlari diagrammasi tasvirlangan.) Ko'rinib turibdiki, o'zgaruvchilarning qiymatlari pastdan yuqoriga o'sib borayapti, ammo, funksiya qabul qilgan qiymatlar o'zgaruvchan. Bu esa funksiya ta'rifga kora nomonoton funksiya bo'lishini anglatadi.





3.4.20. Asosiy elementar mantiqiy funksiyalardan monoton funksiyalarni toping.

3.4.21. Asosiy elementar mantiqiy funksiyalardan nomonoton funksiyalarni toping.

3.4.22. Monoton funkstiyalarning superpozistiyasidan hosil qilingan funksiya yana monoton funksiya bo'lishini isbotlang.

3.4.23. Monoton funksiyaning ikkitaraflama funksiyasi ham monoton bo'lishini isbotlang.

3.4.24. Keltirilgan tasdiq to'g'rimi? Kamida ikkita muhim argumentdan iborat bo'lgan chiziqli funksiya nomonoton funksiya bo'ladi.

3.4.25. Bir vaqtda ham chiziqli ham monoton funksiyalar mavjudmi? Agar mavjud bo'lsa, bunday funksiyalarga misollar keltiring va ta'riflang.

3.4.26. Barcha ikki o'zgaruvchili elementar funksiyalarni Jegalkin ko'phadi ko'rishida ifodalang.

3.4.27. Quyidagi funksiyalarni aniqmas koeffisiyentlar usuli bilan Jegalkin ko'phadi ko'rishiga keltiring:

1) $f(\tilde{x}^2) = (0100)$;

2) $f(\tilde{x}^3) = (10001110)$;

3) $f(\tilde{x}^3) = (01100110)$;

4) $f(\tilde{x}^3) = (01101001)$;

5) $f(\tilde{x}^3) = (00000111)$;

3.4.28. Jegalkin ko'phadi ko'rishiga keltiring:

1) $f(\tilde{x}^2) = x_1 \to (x_2 \to \overline{x_1} \cdot x_2)$;

2) $f(\tilde{x}^2) = x_1 \cdot (x_2 \leftrightarrow x_1 \cdot \overline{x_2})$;

3) $f(\tilde{x}^3)=(x_1\downarrow x_2)|(x_2\downarrow x_3);$

4) $f(\tilde{x}^3)=(x_1\vee x_2)\cdot(x_1|(x_3\to x_2));$

5) $f(\tilde{x}^3)=x_1\downarrow((x_1\to x_2)\vee\overline{x_3});$

6) $f(\tilde{x}^3)=(x_1\to(x_2\to x_3))\cdot((x_1\to x_2)\to x_3);$

7) $f(x,y,z)=x\vee\overline{y}\vee((x\oplus(y\downarrow z))\to\overline{y}\cdot z);$

8) $f(x,y,z)=((x\leftrightarrow y)\downarrow z)\cdot(y\cdot\overline{z}\to(x\oplus y\cdot z));$

9) $f(x,y,z)=(x\vee y\cdot\overline{z})\oplus(x\cdot\overline{y}\leftrightarrow(x\to y\cdot\overline{z}));$

10) $f(x,y,z)=(x\vee y\vee\overline{z})\to(x\cdot\overline{y}\leftrightarrow(x\oplus y\cdot\overline{z})).$

3.4.29. A va B bul funksiyalarini Jegalkin ko'phadi ko'rinishiga keltirib, ularning tengkuchliligini isbotlang:

1) $A=x_1\to(x_2\to x_3), B=(x_1\to x_2)\to(x_1\to x_3);$

2) $A=(x_1\cdot x_2\to x_3)\to(x_1\to x_3), B=\overline{x_1}\vee x_2\vee\overline{x_3};$

3) $A=x_1\cdot x_2\to x_3, B=(x_1\to x_3)(x_2\to x_3);$

4) $A=x_1\cdot x_2\vee(x_3\to x_1), B=\overline{x_1}\to\overline{x_3};$

5) $A=(x_1\to x_3)(x_2\to x_3), B=(x_1\vee x_2)\to x_3;$

6) $A=x_1\leftrightarrow x_2, B=(x_1x_3\leftrightarrow x_2x_3)((x_1\vee x_3)\leftrightarrow(x_2\vee x_3));$

3.4.30. Quyidagi funksiyalar sistemasi funkstional yopiq sinflar bo'lishini isbot qiling:

1) bir argumentli funksiyalar;

2) hamma mantiq algebrasining funksiyalari;

3) L - chiziqli funksiyalar;

4) S - o'z-o'ziga ikkitaraflama funksiyalar;

5) M - monoton funksiyalar;

6) P_0 - nul qiymatni saqlovchi funksiyalar;

7) P_1 - bir qiymatni saqlovchi funksiyalar.

3.4.31. Agar $\Phi=\{\varphi_1,...,\varphi_n\}$ va $F=\{f_1,...,f_n\}$ funkstional yopiq sinflar bo'lsa, u holda $\Phi\bigcap F$ va $\Phi^*=\{\varphi_1^*,...,\varphi_n^*\}$ lar ham funkstional yopiq sinflar va $\Phi\bigcup F$ ni funkstional yopiq sinf bo'lmasligini isbotlang.

3.4.32. Quyidagi maksimal funkstional yopiq P_0,P_1,S,L,M sinflarning birortasi ikkinchisining qism to'plami bo'lmasligini isbotlang.

3.4.33. Har qanday shaxsiy funkstional yopiq sinf P_0,P_1,S,L,M maksimal funkstional yopiq sinflarning birortasining qism to'plami ekanligini isbotlang.

3.4.34. Nol saqlamaydigan funksiya yoki nomonoton funksiya, yoki o'z-o'ziga ikkitaraflama bo'lmagan funksiya ekanligini isbotlang.

3.4.35. Agar birtadan ortiq muhim argumentga ega bo'lgan bul funksiyasi chiziqli finksiya bo'lsa, u holda u nomonoton funksiya bo'ladimi?

3.4.36. Ham chiziqli ham monoton bul funksiyalari mavjudmi? Agar mavjud bo'lsa, bunday funksiylarga misollar keltiring va tahlil qiling.

3.4.37. Agar birtadan ortiq muhim argumentga ega bo'lgan bul funksiyasi monoton bo'lsa, u holda u o'z-o'ziga ikki taraflama emasligi rostmi?

3.4.38. Ham o'z-o'ziga ikki taraflama ham monoton bul funksiyalari mavjudmi? Agar mavjud bo'lsa, bunday funksiylarga misollar keltiring va tahlil qiling.

3.4.39. n argumentli barcha chiziqli funksiyalarda 0 ni saqlovchi funksiyalar va 1 ni saqlovchi funksiyalar soni o'zaro teng bo'lishini isbotlang.

3.4.40. Agar bul funksiyasi chiziqli va monoton bo'lsa, unda u yoki 0 ni saqlamaydi yoki 1 ni saqlamaydi yoki o'z-o'ziga ikki taraflama bo'lishini isbotlang.

3.4.41. Kamida ikkita muhim argumentga ega bo'lgan $f(x_1,x_2,x_3,x_4)$ funksiaya 0 ni saqlovchi, o'z-o'ziga ikkki taraflama, chiziqli va $f(1,1,0,1)=1$ bo'lsa, bu funksiyani toping.

3.4.42. Quyidagi funksiyalar sistemasining to'liqligini isbotlang:

1) $\{\cdot,'\}$; 4) $\{(\to)',\,'\}$; 7) $\{|\}$; 10) $\{\leftrightarrow,\cdot,0\}$.

2) $\{\vee,'\}$; 5) $\{(\to)',1\}$; 8) $\{\downarrow\}$;

3) $\{\to,'\}$; 6) $\{\to,0\}$; 9) $\{+,\cdot,1\}$;

Yechim: 5) Agar mantiqiy amallarning barchasini $\{(\to)',1\}$ bilan ifodalab bo'lsa, yoki biror to'liq funksiyalar sistemasiga keltirib bo'lsa, $\{(\to)',1\}$ to'liq funksiyalar sistemasi bo'ladi. $x' = (0\vee x)' = (1\to x)'$; $x\vee y = (x'y')' =$ $(1\to x'y')' = (1\to((1\to x)'\to y)')'$; $x\cdot y = x\cdot y'' = (x\to y')' =$ $= (x\to(1\to y)')'$. $\{\cdot,\vee,'\}$ funksiyalar sistemasi to'liqligidan, $\{(\to)',1\}$ ning to'liqligi kelib chiqadi.

3.4.43. Quyidagi funksiyalar sistemasining to'liq emasligini isbotlang:

1) $\{\cdot,\vee\}$; 4) $\{+,'\}$; 7) $\{+,\to\}$; 10) $\{'\}$;

2) $\{\cdot,\to\}$; 5) $\{1,'\}$; 8) $\{\cdot,\vee,\to\}$; 11) $\{\leftrightarrow,'\}$;

3) $\{\to,\vee\}$; 6) $\{+,\vee\}$; 9) $\{\cdot,\vee,\to,\leftrightarrow\}$;

Yechim: 1) $\{\cdot,\vee\}$ funksiyalarning har biri 0 ni saqlaydi, yani $0\cdot 0=0$, $0\vee 0=0$. Bundan, 0 ni saqlovchi funksiyalar superpozisiyasidan hosil bo'lgan funksiya yana 0 ni saqlaydi. Biz bilamizki, hamma bul funksiyalar ham 0 ni saqlovchi emas. Demak, $\{\cdot,\vee\}$ funksiyalar sistemasi to'liq emas.

3.4.44. Quyidagi funksiyalar sistemasining to'liqligini tekshiring:

1) $\{\cdot,+\}$; 4) $\{+,1\}$; 7) $\{\cdot,0,1\}$; 10) $\{+,\cdot,\leftrightarrow\}$;

2) $\{\to,+\}$; 5) $\{\cdot,+,0\}$; 8) $\{\leftrightarrow,\vee,0\}$; 11) $\{+,\vee,\leftrightarrow\}$;

3) $\{\to,1\}$; 6) $\{+,0,1\}$; 9) $\{\to,\cdot,0\}$; 12) $\{\to,(\leftarrow)'\}$;

3.4.45. Quyidagi funksiyalar to'liq funksiyalar sistemasi bo'lishini isbotlang:

1) $x'y'z'$; 7) $(x+1)(y+1)(z+1)$;

2) $(x+y+1)(z+1)$; 8) $x'(y\to z')$;

3) $x'y'z'\vee xy'z$; 9) $z\leftrightarrow(y+xz)$;

4) $xy \to z'$; 10) $(1+x)(y'+z)'$;

5) $xyz+1$; 11) $x't' \vee yz'$.

6) $xy \to (x \to z')$;

3.4.46. Quyidagi funksiyalar sistemasining to'liqligini tekshiring:

1) $\{xy \vee y'z, 0, 1\}$; 7) $\{(y \to x)(y' \to z),\ 0,\ 1\}$;

2) $\{xy \vee xz \vee yz, x', 1\}$; 8) $\{x+y+z,\ x'\}$;

3) $\{xy \vee xz \vee yz, x \leftrightarrow y, x+y\}$; 9) $\{xy \vee xz \vee yz, x'\}$;

4) $\{y \to xz, 0, 1\}$; 10) $\{xy \vee xz \vee yz, 0, 1\}$;

5) $\{x+y+z, xy,\ x'\}$; 11) $\{x+y, 0, 1\}$;

6) $\{xy+z, (x \leftrightarrow y)+z,\ 1\}$; 12) $\{xy, 0,\ 1\}$;

Yechim: 1)-7) sitemalar to'liq. 8) sistemadagi ikkala funksiya ham chiziqli ekanligi va chiziqli funksiyalar superpozitsiyasidan hosil bo'lgan funksiyalar ham chiziqli bo'lishini inobatga olib, sistema to'liq emas degan xulosaga kelamiz; 9) sistemadagi ikkala funksiya ham o'z-o'ziga ikki taraflama funksiya, sistema to'liq emas degan xulosaga kelamiz; 10) sistemadagi uchala funksiya ham monoton, sistema to'liq emas degan xulosaga kelamiz.

3.4.47. Agar $F(f_1, f_2, ..., f_n)$ funksiyalar sistemasi to'liq bo'lsa, u holda shu funksiyalar sisitemasiga ikki taraflama funksiya $F^*(f^*_1, f^*_2, ..., f^*_n)$ ham to'liq funksiyalar sistemasi bo'lishini isbotlang.

3.4.48. Quyida berilgan funksiyalar sinflaridan qaysi yopiq funksiyalar sinfi bo'lishini aniqlang:

1) bir argumentli funksiyalar; 2) ikki argumentli funksiyalar; 3) hamma mantiq algebrasining funksiyalari; 4) o'z-o'ziga ikkitaraflama funksiyalar; 5) monoton funksiyalar; 6) nol qiymatni saqlovchi funksiyalar; 7) nolni hamda birni saqlovchi funksiyalar; 8) bir qiymatni saqlovchi funksiyalar; 9) nol qiymatni saqlovchi ammo birni saqlamaydigan funksiyalar; 10) chiziqli funksiyalar.

Yechim: 9) nol qiymatni saqlovchi ammo birni saqlamaydigan funksiyalar funksional yopiq sinf bo'lish bo'lmasligini ko'rsatish uchun shunday misol keltirish kerakki, shartni qanoatlantiruvchi funksiya superpozisiyasidan hosil bo'lgan funksiya funksional yopiq sinf bo'lmasin: $x+y$ 0 ni saqlaydi ammo 1 ni saqlamaydi. Endi uning superpozisiyasini ko'raylik : $(x+y)+z$ ni olsak, 0 ni saqlaydi va shu bilan birga 1 ni ham saqlaydi. Demak, funksional yopiq sinf emas ekan.

3.4.49. P_0, P_1, M, S, L funkstional yopiq sinflar maksimal funkstional yopiq sinflar bo'lishini isbotlang.

3.4.50. P_0, P_1, M, S, L sinflardan birortasi ham boshqasining tarkibiga kirmasligini isbotlang, ya'ni:

1) $P_0 - P_1$ ning tarkibiga kirmaydi, $P_1 - P_0$ ning tarkibiga kirmaydi;

2) $P_0 - S$ ning tarkibiga kirmaydi, $S - P_0$ ning tarkibiga kirmaydi;

3) $P_0 - \mathrm{L}$ ning tarkibiga kirmaydi, $L - P_0$ ning tarkibiga kirmaydi;

4) $P_0 - M$ ning tarkibiga kirmaydi, $M - P_0$ ning tarkibiga kirmaydi;

5) $P_1 - S$ ning tarkibiga kirmaydi, $S - P_1$ ning tarkibiga kirmaydi;

6) P_1 - L ning tarkibiga kirmaydi, $\mathrm{L} - P_1$ ning tarkibiga kirmaydi;

7) P_1 - M ning tarkibiga kirmaydi, $M - P_1$ ning tarkibiga kirmaydi;

8) S - M ning tarkibiga kirmaydi, $M - S$ ning tarkibiga kirmaydi;

9) $S - \mathrm{L}$ ning tarkibiga kirmaydi, $\mathrm{L} - S$ ning tarkibiga kirmaydi;

10) L - M ning tarkibiga kirmaydi, M - L ning tarkibiga kirmaydi.

Izoh: Har bir hol uchun shunday ikkita funksiya topish kerakki, bir funksiya birinchi sinfga tegishli bo'lib, ikkinchi sinfga tegishli bo'lmasin. Keyingi funksiya aksincha, ikkinchi sinfga tegishli bo'lib, birinchi sinfga tegishli bo'lmasin.

3.4.51. P_0, P_1, M, S, L sinflardan boshqa maksimal funkstional yopiq sinflar bo'lmasligini isbotlang.

3.5. Post teoremasi.

Post teoremasi. $\Phi=\{\varphi_1,...,\varphi_n\}$ funksiyalar sistemasining to'liqligi uchun bu sistemada P_0, P_1, M, S, L maksimal funkstional yopiq sinflarning har biriga kirmovchi kamida bitta funksiya mavjud bo'lishi etarli va zarur

Amalda birorta $\Phi=\{\varphi_1,...,\varphi_n\}$ sistemaning to'liq yoki to'liq emasligini aniqlash uchun Post jadvalidan foydalanadilar. Post jadvali quyidagi ko'rinishda bo'ladi:

	P_0	P_1	S	L	M
φ_1					
φ_2					
...	...	...	...	...	...
φ_{n-1}					
φ_n					

Jadvalning xonalariga o'sha satrdagi funksiya funkstional yopiq sinflarning elementi bo'lsa "+" ishora, bo'lmasa "-" ishorasi qo'yiladi.

$\Phi=\{\varphi_1,...,\varphi_n\}$ sistema to'liq funksiyalar sistemasi bo'lishi uchun, teoremaga asosan, jadvalning har bir ustunida kamida bitta "-" ishorasi bo'lishi etarli va zarur.

$\Phi=\{\varphi_1,...,\varphi_n\}$ funksiyalar sistemasi to'liq bo'lmasligi uchun P_0, P_1, M, S, L maksimal funkstional yopiq sinflarning birortasining qism to'plami bo'lishi, ya'ni Post jadvalining biror ustuni to'liq "+" ishoralaridan iborat bo'lishi kerak.

Misol. Quyidagi funksiyalar sistemalarining to'liq emasligini Post jadvali orqali isbot qilaylik:

a) $\Phi_1=\{0, xy, x+y+z\}$; b) $\Phi_2=\{1, xy, x+y+z\}$;

v) $\Phi_3=\{\bar{x}\bar{y}\vee\bar{x}\bar{z}\vee\bar{y}\bar{z}\}$; g) $\Phi_4=\{0, 1, x+y\}$;

d) $\Phi_5=\{0, 1, xy\}$

a)		P_0	P_1	S	L	M
	0	+	-	-	+	+
	xy	+	+	-	-	+
	$x+y+z$	+	+	+	+	-
b)	1	-	+	-	+	+
	xy	+	+	-	-	+
	$x+y+z$	+	+	+	+	-
v)	$\bar{x}\bar{y} \vee \bar{x}\bar{z} \vee \bar{y}\bar{z}$	-	-	+	-	-
g)	0	+	-	-	+	+
	1	-	+	-	+	+
	$x+y$	+	-	-	+	-
d)	0	+	-	-	+	+
	1	-	+	-	+	+
	xy	+	+	-	-	+

Muammoli masala va topshiriqlar:

3.5.1. Quyidagi funksiyalar sistemasining to'liq emasligini Post teoremasi yordamida isbotlang:

1) $\{\cdot, \vee\}$; 4) $\{+, '\}$; 7) $\{+, \rightarrow\}$; 10) $\{'\}$;

2) $\{\cdot, \rightarrow\}$; 5) $\{1, '\}$; 8) $\{\cdot, \vee, \rightarrow\}$; 11) $\{\leftrightarrow, '\}$;

3) $\{\rightarrow, \vee\}$; 6) $\{+, \vee\}$; 9) $\{\cdot, \vee, \rightarrow, \leftrightarrow\}$;

3.5.2. Quyidagi funksiyalar sistemasining to'liqligini Post teoremasi yordamida tekshiring:

1) $\{\cdot,+\}$; 4) $\{+,1\}$; 7) $\{\cdot,0,1\}$; 10) $\{+,\cdot,\leftrightarrow\}$;

2) $\{\rightarrow,+\}$; 5) $\{\cdot,+,0\}$; 8) $\{\leftrightarrow,\vee,0\}$; 11) $\{+,\vee,\leftrightarrow\}$;

3) $\{\rightarrow,1\}$; 6) $\{+,0,1\}$; 9) $\{\rightarrow,\cdot,0\}$; 12) $\{\rightarrow,(\leftarrow)'\}$;

3.5.3. Quyidagi funksiyalar to'liq funksiyalar sistemasi bo'lishini Post jadvali yordamida isbotlang:

1) $x'y'z'$;
2) $(x+y+1)(z+1)$;
3) $x'y'z'\vee xy'z$;
4) $xy\rightarrow z'$;
5) $xyz+1$;
6) $xy\rightarrow(x\rightarrow z')$;
7) $(x+1)(y+1)(z+1)$;
8) $x'(y\rightarrow z')$;
9) $z\leftrightarrow(y+xz)$;
10) $(1+x)(y'+z)'$;
11) $x't'\vee yz'$.

3.5.4. Quyidagi funksiyalar sistemasining to'liqligini Post jadvali yordamida tekshiring :

1) $\{xy\vee y'z,0,1\}$;
2) $\{xy\vee xz\vee yz,x',1\}$;
3) $\{xy\vee xz\vee yz,x\leftrightarrow y,x+y\}$;
4) $\{y\rightarrow xz,0,1\}$;
5) $\{x+y+z,xy,\ x'\}$;
6) $\{xy+z,(x\leftrightarrow y)+z,\ 1\}$;
7) $\{(y\rightarrow x)(y'\rightarrow z),\ 0,\ 1\}$;
8) $\{x+y+z,\ x'\}$;
9) $\{xy\vee xz\vee yz,x'\}$;
10) $\{xy\vee xz\vee yz,0,1\}$;
11) $\{x+y,0,1\}$;
12) $\{xy,0,\ 1\}$;

4-BOB. MULOHAZALAR HISOBI

4.1. Hisob tushunchasi. Mulohazalar hisobi. Keltirib chiqarish. Isbot tushunchasi. Teorema tushunchasi. Mulohazalar hisobining aksiomalari.

Mulohazalar hisobida uch kategoriyali simvollardan iborat alfavit qabul qilinadi:

Birinchi kategoriya simvollari: $x, y, z, \ldots, x_1, x_2, \ldots$. Bu simvollarni o’zgaruvchilar deb ataymiz.

Ikkinchi kategoriya simvollari: $\vee$, $\wedge$, $\rightarrow$, $-$. Bular mantiqiy bog’lovchilardir.

Uchinchi kategoriyaga qavs deb ataladigan (,) simvol kiritiladi.

Mulohazalar hisobida boshqa simvollar yo’q.

Mulohazalar hisobining aksiomalar tizimi XI aksiomadan iborat bo’lib, bular to’rt guruhga bo’linadi.

Birinchi guruh aksiomalari:

I_1 $\quad x \rightarrow (y \rightarrow x)$.

I_2 $\quad (x \rightarrow (y \rightarrow z)) \rightarrow ((x \rightarrow y) \rightarrow (x \rightarrow z))$.

Ikkinchi guruh aksiomalari:

II_1 $\quad x \wedge y \rightarrow x$.

II_2 $\quad x \wedge y \rightarrow y$.

II_3 $\quad (z \rightarrow x) \rightarrow ((z \rightarrow y) \rightarrow (z \rightarrow x \wedge y))$.

Uchinchi guruh aksiomalari:

III_1 $\quad x \rightarrow x \vee y$.

III_2 $\quad y \rightarrow x \vee y$.

III_3 $\quad (x \rightarrow z) \rightarrow ((y \rightarrow z) \rightarrow (x \vee y \rightarrow z))$.

To’rtinchi guruh aksiomalari:

IV_1 $\quad (x \rightarrow y) \rightarrow (\overline{y} \rightarrow \overline{x})$.

IV$_2$ $\quad x \to \overset{=}{x}$.

IV$_3$ $\quad \overset{=}{x} \to x$.

Keltirib chiqarish qoidasi

O'rniga qo'yish qoidasi. A formuladagi x o'zgaruvchilar o'rniga B formulani qo'yish operastiyasi (jarayoni)ni o'rniga qo'yish qoidasi deb aytamiz va uni quyidagi simvol bilan belgilaymiz: $\int\limits_x^B (A)$, $\dfrac{\vdash A}{\vdash \int\limits_{x_1,x_2,...,x_n}^{B_1,B_2,...,B_n} (A)}$.

$\dfrac{\vdash A}{\vdash \int\limits_x^B (A)}$ «agar A isbotlanuvchi formula bo'lsa, u vaqtda $\int\limits_x^B (A)$ ham isbotlanuvchi formula bo'ladi» deb o'qiladi.

Xulosa qoidasi (modus ponens-MP). Agar A va $A \to B$ lar mulohazalar hisobining isbotlanuvchi formulalari bo'lsa, u holda V ham isbotlanuvchi formula bo'ladi. Bu qoida quyidagicha sxematik ravishda yoziladi: $\dfrac{\vdash A; \ \vdash A \to B}{\vdash B}$.

$A_1, A_2, ..., A_n$ lar va $A_1 \to (A_2 \to (A_3 \to (...(A_n \to L)...)))$ isbotlanuvchi formulalar bo'lsa, u vaqtda L ham isbotlanuvchi formula bo'ladi.

$$\frac{\vdash A_1, \vdash A_2, ..., \vdash A_n, \vdash A_1 \to (A_2 \to (A_3 \to (...(A_n \to L)...)))}{\vdash L}$$

Isbotlanuvchi formulalarni hosil etish jarayoniga isbotlash deb aytiladi.

1-Misol. $\vdash A \to A$ implikastiyaning refleksivligini isbotlash uchun ushbu $\vdash (x \to (y \to z)) \to ((x \to y) \to (x \to z))$ - I$_2$ aksiomadan foydalanamiz. Bu erda $\int\limits_z^x (I_2)$ o'rniga qo'yishni bajarish natijasida

$\vdash (x \to (y \to x)) \to ((x \to y) \to (x \to x))$ (1)

kelib chiqadi. $\vdash (x \to (y \to z)) \to ((x \to y) \to (x \to z))$ - I_2 aksioma va (1) formulaga xulosa qoidasini qo'llab $\vdash (x \to y) \to (x \to x)$ (2)formulani hosil qilamiz. (2) formulaga nisbatan quyidagi o'rniga qo'yishni $\int_y^{\overline{\overline{x}}}$ (2)bajarish natijasida $\vdash (x \to \overline{\overline{x}}) \to (x \to x)$ (3) isbotlanuvchi formulaga ega bo'lamiz. $x \to \overline{\overline{x}}$ - IV_2 aksioma va (3) formulaga nisbatan xulosa qoidasini qo'llash natijasida $\vdash x \to x$ (4) isbotlanuvchi formulaga kelamiz. Nihoyat (4) formuladagi x o'zgaruvchi o'rniga A formulani qo'ysak $\vdash A \to A$ isbotlanishi kerak bo'lgan formula hosil bo'ladi.

2-misol. $\vdash \overline{x \vee y} \to \overline{x} \wedge \overline{y}$ ekanligini isbotlang.

$(z \to x) \to ((z \to y) \to (z \to x \wedge y))$ - II_3 aksiomaga nis-batan ketma-ket ikki marta o'rniga qo'yish usulini qo'llaymiz: avval x ni $\overline{x}$ ga va keyin y ni $\overline{y}$ ga almashtiramiz. Natijada quyidagi isbotlanuvchi formulaga ega bo'lamiz

$\vdash (z \to \overline{x}) \to ((z \to \overline{y}) \to (z \to \overline{x} \wedge \overline{y}))$. (5) $\int_z^{\overline{x \vee y}}$ (5) o'rniga qo'yishni bajarib, quyidagini hosil qilamiz $\vdash ((\overline{x \vee y}) \to \overline{x}) \to ((\overline{x \vee y} \to \overline{y}) \to (\overline{x \vee y} \to \overline{x} \wedge \overline{y}))$. (5a). Endi $\overline{x \vee y} \to \overline{x}$ (6) , $\overline{x \vee y} \to \overline{y}$ (7) formulalarning isbotlanuvchi ekanligini ko'rsatamiz. Buning uchun $(x \to y) \to (\overline{y} \to \overline{x})$ - IV_1 aksiomaga nisbatan $\int_y^{x \vee y} (IV_1)$ o'rniga qo'yishni bajaramiz.

Natijada $\vdash (x \to x \vee y) \to (\overline{x \vee y} \to \overline{x})$ (8) formulaga ega bo'lamiz. (8) formula va $x \to x \vee y$ - III_1 aksiomaga nisbatan xulosa qoidasini ishlatib, (6) ning isbotlanuvchi formula ekanligiga ishonch hosil qilamiz. Xuddi shunday (7) ning ham isbotlanuvchi formula ekanligini ko'rsatish mumkin.

(6) va (5) formulalarga xulosa qoidasini qo'llasak, $\vdash(\overline{x\vee y}\to\overline{y})\to(\overline{x\vee y}\to\overline{x}\wedge\overline{y})$ (9) isbotlanuvchi formula kelib chiqadi.

(7) va (9) formulalarga xulosa qoidasini qo'llab, $\vdash\overline{x\vee y}\to\overline{x}\wedge\overline{y}$

dastlabki formulaning isbotlanuvchi ekanligini hosil qilamiz.

$$\frac{\vdash A_1, \vdash A_2, \ldots, \vdash A_n, \vdash A_1\to(A_2\to(A_3\to(\ldots(A_n\to L)\ldots)))}{\vdash L}$$

Sillogizm qoidasi: *Agar* $A\to B$ *va* $B\to C$ *isbotlanuvchi formulalar bo'lsa, u vaqtda* $A\to C$ *formula ham isbotlanuvchi bo'ladi.* $\dfrac{\vdash A\to B,\ \vdash B\to C}{\vdash A\to C}$.

Kontrpozistiya qoidasi: Agar $A\to B$ isbotlanuvchi formula bo'lsa, u vaqtda $\overline{B}\to\overline{A}$ ham isbotlanuvchi formula, ya'ni $\dfrac{\vdash A\to B}{\vdash \overline{B}\to\overline{A}}$ bo'ladi.

Ikki karralik inkorni tushirish qoidasi

1) Agar $A\to\overline{\overline{B}}$ isbotlanuvchi formula bo'lsa, u vaqtda $A\to B$ ham isbotlanuvchi bo'ladi. $\dfrac{\vdash A\to\overline{\overline{B}}}{\vdash A\to B}$

2) Agar $\overline{\overline{A}}\to B$ isbotlanuvchi formula bo'lsa, u vaqtda $A\to B$ formula ham isbotlanuvchi, ya'ni $\dfrac{\vdash A\to\overline{\overline{B}}}{\vdash A\to B}$ bo'ladi.

Muammoli masala va topshiriqlar:

4.4.1. Quyidagi ifodalarning qaysi birlari mulohazalar hisobining formulalari bo'ladi:

1) $(\overline{p}_1\wedge\overline{p}_2)\to(p_1\vee p_2)$;

2) $((p_1 \vee p_2) \vee (p_1 p_2)) \to \overline{p}_3$;

3) $(p_1 \to (p_2 \vee p_3)) \to p_3$;

4) $(p_1 \to p_2) \to ((p_1 \to \overline{p}_2) \to p_1)$;

5) $(p_1 \wedge (\to p_2) \to (p_2 \to \overline{p}_1)$;

6) $(p_1 \to p_3) \to ((p_2 \to p_3) \to ((p_1 \vee p_2) \to p_3))$;

7) $((p_1 \to p_2) \wedge (p_1 \to p_3)) \to (p_1 \to (p_2 \wedge p_3))$;

8) $((p_1 \wedge \overline{p}_2) \to (\overline{\overline{p}}_1 \vee \vee p_2)) \leftrightarrow (\vee p_1 \vee p_2)$.

4.1.2. Quyidagi formulalarning hamma qism formulalarini yozib chiqing:

$A = \overline{x \to y} \wedge (\overline{x} \vee y)$, $B = (x \leftrightarrow y) \vee (\overline{x}y)$,

$C = (x \leftrightarrow y) \to (\overline{y} \to t)$, $D = xy \vee xz \vee yz$.

1) $x \to (y \to x)$; 2) $\overline{a \vee b} \to c$;

3) $a \wedge \overline{c \vee b}$; 4) $x \to y \wedge z$;

5) $x \vee yz \to x$; 6) $\overline{x \to y} \vee x \wedge y$;

7) $((x \to x) \wedge (y \to z)) \to (\overline{x} \vee z)$;

8) $(x \to y) \to ((x \to \overline{y}) \to \overline{y})$.

Yechim: 8) $(x \to y) \to ((x \to \overline{y}) \to \overline{y})$.

$(x \to y) \to ((x \to \overline{y}) \to \overline{y})$ - nolinchi chuqurlikdagi qismiy formula;

$x \to y,\ (x \to \overline{y}) \to \overline{y} -$ birinchi chuqurlikdagi qismiy formula;

$x, y,\ x \to \overline{y}, \overline{y} -$ ikkinchi chuqurlikdagi qismiy formula.

4.1.3. $L_1 = (A \to B) \to (\overline{B} \vee \overline{A})$, $L_2 = A \vee (A \to B)$, $L_3 = A \vee (B \to C)$ formulalar uchun quyidagi o’rniga qo’yishlarning natijalarini yozing:

1) $\int\limits_{A,B}^{B,C}(L_1)$; 2) $\int\limits_{A}^{A\to B}(L_2)$; 3) $\int\limits_{A,C}^{B\to A\wedge B,B}(L_3)$;

4) $\int\limits_{A,B}^{A\wedge B,A\vee B}(L_1)$; 5) $\int\limits_{A,B}^{B,A}(L_2)$; 6) $\int\limits_{A,B,C}^{A\wedge\overline{A},C,\overline{A}}(L_3)$.

4.1.4. O'rniga qo'yish qoidasini qo'llab, quyidagi formulalarning isbotlanuvchi ekanligini isbotlang:

1) $(A\to B)\wedge B\to B$;

2) $A\wedge B\to A\wedge B\vee C$;

3) $(\overline{A}\to B)\to((C\to B)\to(\overline{A}\vee C\to B))$;

4) $\overline{\overline{C\vee D}}\to C\vee D$;

5) $(A\wedge B\to(C\to B\wedge C))\to((A\wedge B\to C)\to(A\wedge B\to B\wedge C))$.

4.1.5. Quyidagi formulalardan qaysilari aksioma bo'lishini aniqlang:

1) $(F\to((F\to F)\to F))\to((F\to(F\to F))\to(F\to F))$;

2) $F\to((\neg F\to G)\to F)$;

3) $(G\to H)\to(F\to(G\to H))$;

4) $(\neg F\to G)\to(\neg G\to\neg\neg F)$;

5) $(\neg F\to\neg\neg G)\to((\neg F\to\neg G)\to F)$;

6) $\neg F\to(F\to\neg F)$;

7) $(G\to F)\to((G\to\neg F)\to G)$;

8) $(\neg G\to\neg\neg F)\to((\neg G\to\neg F))$;

9) $(F\to G)\to(H\to(G\to F))$;

10) $(G\to(\neg F\to F))\to((G\to\neg F)\to(G\to F))$;

11) $((\neg F\to G)\to(F\to G))\to(\neg F\to G)$;

12) $(\neg\neg\neg F\to\neg F)\to((\neg\neg\neg F\to F)\to\neg\neg F)$;

13) $(F\to((H\to F)\to(G\to(H\to F))))\to((F\to(H\to F)\to(F\to(G\to(H\to F))))$;

Yechim: 1) $I_2 = (x \to (y \to z)) \to ((x \to y) \to (x \to z))$ aksioma va o'rniga qo'yish usuli yordamida isbotlanuvchi yoki aksioma ekanligini ko'rstamiz:

$$\int_{x,y,z}^{F,F\to F,F} (I_2) = (F \to ((F \to \mathrm{F}) \to F)) \to ((F \to (F \to \mathrm{F})) \to (F \to \mathrm{F})).$$

2) $\mathrm{I}_1 = x \to (y \to x)$ aksioma va o'rniga qo'yish usuli yordamida isbotlanuvchi yoki aksioma ekanligini ko'rstamiz:

$$\int_{x,y}^{F,\neg F\to G} (I_1) = F \to ((\neg F \to G) \to F).$$

4.1.6. O'rniga qo'yish va xulosa qodalarini qo'llab, quyidagi formulalarning isbotlanuvchi ekanligini aniqlang:

1) $A \vee A \to A$; 2) $A \to A \wedge A$;

3) $A \wedge B \to B \wedge A$; 4) $A \vee B \to B \vee A$;

5) $(A \to B) \to (A \to A)$; 6) $\overline{\overline{\overline{A}}} \to \overline{A}$.

4.1.7. Keltirib chiqarishning hosilaviy qoidalaridan foydalanib, quyidagi formulalarning isbotlanuvchi ekanligini isbotlang:

1) $\overline{A} \vee \overline{B} \to \overline{A \wedge B}$; 2) $A \to R$;

3) $(A \to B) \to (A \to A \vee B)$; 4) $F \to A$;

5) $(A \to B) \to (A \to A)$; 6) $A \wedge \overline{A} \to F$;

7) $(A \to B) \wedge \overline{B} \to \overline{A}$; 8) $\overline{A} \wedge \overline{B} \to \overline{A \vee B}$.

4.1.8. Keltirib chiqarishning hosilaviy qoidalarini isbotlang:

1) $\dfrac{\vdash \overline{A}}{\vdash A \wedge B}$; 2) $\dfrac{\vdash A}{\vdash A \vee B}$; 3) $\dfrac{\vdash \overline{A}}{\vdash A \to B}$;

4) $\dfrac{\vdash B}{A \to B}$; 5) $\dfrac{\vdash A \wedge B}{\vdash A}$; 6) $\dfrac{\vdash \overline{B}}{\vdash \overline{A \wedge B}}$;

7) $\dfrac{\vdash A \to B, \vdash \overline{B}}{\vdash \overline{A}}$; 8) $\dfrac{\vdash A, \vdash B}{\vdash A \wedge B}$; 9) $\dfrac{\vdash \overline{A}, \vdash \overline{B}}{\vdash \overline{A \vee B}}$;

10) $\dfrac{\vdash A, \vdash \overline{B}}{\vdash \overline{A \to B}}$; 11) $\dfrac{\vdash A \to \overline{A}}{\vdash \overline{A}}$; 12) $\dfrac{\vdash \overline{A} \to A}{\vdash A}$;

13) $\dfrac{\vdash A \to B, \vdash \overline{A} \to B}{\vdash B}$; 14) $\dfrac{\vdash A \to B, \vdash A \to \overline{B}}{\vdash \overline{A}}$.

4.1.9. O’rniga qo’yish qoidasidan foydalanib quyidagi formulalarning chiqariluvchi ekanligini isbotlang:

1) $\vdash (A \to B) \to ((A \to B) \vee (B \& C))$;

2) $\vdash ((A \& B) \to (C \& D)) \to ((\overline{C \& D}) \to | \, (A \& B))$;

3) $\vdash (C \to A v B) \to ((C \to B \vee A) \to (C \to (A \vee B) \& (B \vee A))$;

4) $\vdash (A \to B) \to ((A \to B) \vee (C \to D))$;

5) $\vdash (C \to D) \to ((A \to B) \vee (C \to D))$;

6) $\vdash (A \to C) \& (B \to D) \to (A \to C)$;

7) $\vdash (A \to C) \& (B \to D) \to (B \to D)$;

8) $\vdash ((A \to B) \to C) \to ((C \to D) \to C) \to ((A \to B) \vee (C \to D) \to C)$;

4.1.10. O’rniga qo’yish qoidasidan va xulosaga kelish qoidasidan foydalanib quyidagi formulalarning chiqariluvchi ekanligini isbotlang:

1) $\vdash B \vee B \to B$;

2) $\vdash C \& D \to D \& C$;

3) $\vdash B \to B \& B$;

4) $\vdash C \vee D \to D \vee C$;

5) $\vdash (A \to B) \to (A \to A)$;

6) $\vdash \overline{\overline{A}} \to \overline{A}$;

7) $\vdash (\overline{A \vee B}) \to \overline{A}$;

8) $\vdash (\overline{A \vee B}) \rightarrow \overline{B}$;

4.1.11. Hosilaviy chiqarish qoidalaridan foydalanib, quyidagi formulalarning chiqariluvchi ekanligini isbotlang:

1) $\vdash \overline{A \vee B} \rightarrow (\overline{A \& B})$;
2) $\vdash (A \rightarrow B) \rightarrow (A \rightarrow A \vee B)$;
3) $\vdash (A \rightarrow B) \rightarrow (A \rightarrow A)$;
4) $\vdash (A \rightarrow B) \& \overline{B} \rightarrow \overline{A}$;
5) $\vdash A \& \overline{A} \rightarrow B$;
6) $\vdash \overline{A \& B} \rightarrow (\overline{A \vee B})$;
7) $\vdash (B \rightarrow B) \rightarrow (\overline{B} \rightarrow \overline{B})$;
8) $\vdash \overline{B} \rightarrow \overline{B}$;

4.1.12. Quyidagi misollar (MP) xulosa chiqarish qoidasi yordamida yechilgan bo'lsa, W formulani toping:

1) $F \rightarrow (H \rightarrow F),\ (F \rightarrow (H \rightarrow F)) \rightarrow (F \rightarrow (G \rightarrow (H \rightarrow F))),\quad W$;
2) $F \rightarrow (G \rightarrow H),\quad W,\quad (F \rightarrow G) \rightarrow (F \rightarrow H)$;
3) $W, (\neg G \rightarrow \neg G) \rightarrow ((\neg G \rightarrow G) \rightarrow G), (\neg G \rightarrow \neg G) \rightarrow G$;
4) $F \rightarrow G,\quad W,\quad H \rightarrow (F \rightarrow G)$;
5) $G,\ G \rightarrow (F \rightarrow G),\quad W$;
6) $W,\quad (\neg F \rightarrow \neg G) \rightarrow ((\neg F \rightarrow G) \rightarrow F),\quad (\neg F \rightarrow G) \rightarrow F$;
7) $G \rightarrow F,\quad W,\quad (G \rightarrow \neg F) \rightarrow (G \rightarrow F)$;
8) $W,\quad (G \rightarrow (\neg F \rightarrow F)) \rightarrow ((G \rightarrow \neg F) \rightarrow (G \rightarrow F)),\quad (G \rightarrow \neg F) \rightarrow (G \rightarrow F)$;
9) $\neg G \rightarrow (F \rightarrow \neg G),\quad (\neg G \rightarrow (F \rightarrow \neg G)) \rightarrow G,\quad W$;

Yechim:8) MP – $\dfrac{\vdash A;\ \vdash A \to B}{\vdash B}$ xulosa chiqarish qoidasi

$A = W, \quad A \to B = (G \to (\neg F \to F)) \to ((G \to \neg F) \to (G \to F))$ va $B = (G \to \neg F) \to (G \to F)$ ekanligi ko'rinib turibdi. Demak, $A = (G \to (\neg F \to F))$ bo'ladi.

9) MP – $\dfrac{\vdash A;\ \vdash A \to B}{\vdash B}$ xulosa chiqarish qoidasi

$A = \neg G \to (F \to \neg G), \quad A \to B = (\neg G \to (F \to \neg G)) \to G$ ekanligi ko'rinib turibdi. Demak, $B = \mathrm{W}$ dan, $B = \mathrm{W} = \mathrm{G}$ bo'ladi.

4.1.13. Quyidagi formulalar ketma-ketligi aksiomalardan keltirib chiqarish usuli bo'la oladimi? Agar bo'la olsa har bir ketma-ketlikni asoslang. Agar bo'la olmasa, buni isbotlang:

1) (1) $G \to (F \to G)$,

(2) $(G \to (F \to G)) \to (G \to (G \to (F \to G)))$,

(3) $G \to (G \to (F \to G))$.

2) (1) $(\neg G \to \neg F) \to ((\neg G \to F) \to G)$,

(2) $\neg F \to (\neg G \to \neg F)$,

(3) $\neg F \to ((\neg G \to F) \to G)$.

3) (1) $F \to (G \to F)$,

(2) $(F \to (G \to F)) \to ((F \to G) \to (F \to F))$,

(3) $(F \to G) \to (F \to F)$.

4) (1) $(F \to F) \to (G \to (F \to F))$,

(2) $F \to F$,

(3) $G \to (F \to F)$.

5) (1) $(H \to F) \to (G \to (H \to F))$,

(2) $((H \to F) \to (G \to (H \to F))) \to (F \to ((H \to F) \to (G \to (H \to F))))$,

(3) $F \to ((H \to F) \to (G \to (H \to F)))$,

(4) $(F \to ((H \to F) \to (G \to (H \to F)))) \to ((F \to (H \to F)) \to (F \to (G \to (H \to F))))$,

(5) $(F \to (H \to F)) \to (F \to (G \to (H \to F)))$,

(6) $F \to (H \to F)$,

(7) $F \to (G \to (H \to F))$.

6) (1) $\neg G \to \neg G$,

(2) $(\neg G \to \neg G) \to ((\neg G \to G) \to G)$,

(3) $(\neg G \to G) \to G$,

7) (1) $G \to (\neg F \to G)$.

4.2. Dedukstiya teoremasi. Mos keltirib chiqarish xaqida lemma. To'liqlik xaqida Gyodel teoremasi.

Keltirib chiqarish qoidasi H va W mulohazalar hisobining ikkita formulalar majmuasi bo'lsin. H,W orqali bu majmualarning yig'indisini (birlashmasini) belgilaymiz, ya'ni $H,W = H \cup W$. Agar W majmua bitta C formuladan iborat bo'lganda ham $H \cup \{C\}$ birlashmani H,C ko'rinishda yozamiz.

Keltirib chiqarishning asosiy qoidalari:

I. $$\frac{H \vdash A}{H,W \vdash A}$$ II. $$\frac{H,C \vdash A, \quad H \vdash C}{H \vdash A}$$

III. $$\frac{H,C \vdash A, W \vdash C}{H,W \vdash A}$$ IV. $$\frac{H \vdash C \to A}{H,C \vdash A}$$

V. *Dedukstiya teoremasi*: $$\frac{H,C \vdash A}{H \vdash C \to A}.$$

***Umumlashgan dedukstiya teoremasi*:**

$$\frac{\{C_1, C_2, ..., C_k\} \vdash A}{\vdash C_1 \to (C_2 (C_3 \to ...(C_k \to A)...))}.$$

VI. Kon'yunkstiyani kiritish qoidasi: $$\frac{H \vdash A, H \vdash B}{H \vdash A \wedge B}.$$

***VII. Diz'yunkstiyani kiritish qoidasi*:** $\dfrac{H, A \vdash C;\ H, B \vdash C}{H, A \vee B \vdash C}$.

Muammoli masala va topshiriqlar:

4.2.1. H formulalar majmuasidan ko'rsatilgan formulalarni keltirib chiqarish mumkinligini ko'rsating:

1) $H = \{A\} \vdash B \to A$;

2) $H = \{A \to B, B \to C\} \vdash A \to C$;

3) $H = \{A \to C\} \vdash \overline{C} \to \overline{A}$;

4) $H = \{A \to B, \overline{B}\} \vdash A$;

5) $H = \{A, \overline{\overline{A}} \to B\} \vdash B$;

6) $H = \{A \to B\} \vdash A \wedge C \to B \wedge C$;

7) $H = \{A \to B\} \vdash (C \to A) \to (C \to B)$;

8) $H = \{A \to B\} \vdash (B \to C) \to (A \to C)$;

9) $H = \{A \to (B \to C)\} \vdash B \to (A \to C)$;

10) $H = \{A \to B\} \vdash A \vee C \to B \vee C$.

4.2.2. Umumlashgan deduksiya teoremasidan foydalanib, formulalarning isbotlanuvchi ekanligini isbotlang:

1) $(x \to y) \to ((y \to z) \to (x \to z))$;

2) $(A \to B) \to (A \vee C \to B \vee C)$;

3) $(A \to B) \to ((C \to A) \to (C \to B))$.

4.2.3. Mantiq qonunlarining to'g'riligini ko'rsating:

1) $x \to (\overline{x} \to y)$; 2) $x \vee \overline{x}$; 3) $\overline{x} \wedge \overline{y} \to \overline{x \vee y}$.

4.2.4. Shartlarni o'rin almashtirish, shartlarni qo'shish va shartlarni ajratish qoidalaridan foydalanib, berilganlarning to'g'riligini isbotlang:

1) $\vdash x \to (y \to x \wedge y)$;

2) $\vdash (A \to B) \wedge \overline{B} \to \overline{A}$;

3) $\vdash \overline{A} \to (A \to B)$.

4.2.5. Quyidagi formula $A = x_1 \vee x_2 \to x_3$ va o'zgaruvchilarning 1) (0,0,1); 2) (1,0,0) qiymatlar satri berilgan. A formula va uning inkori $\overline{A}$ ni mos formulalar majmuasidan keltirib chiqaring.

4.2.6. Quyidagi formula $A = \overline{x}_1 \vee x_2 \to \overline{x}_3$ va o'zgaruvchilarning 1) (1,1,1); 2) (1,0,1); 3) (0,1,0) qiymatlar satri berilgan. A formula va uning inkori $\overline{A}$ ni mos formulalar majmuasidan keltirib chiqaring.

4.2.7. Quyidagi formula $A = (x \vee \overline{y}) \to \overline{x} \wedge \overline{z}$ va o'zgaruvchilarning 1) (1,0,0); 2) (0,1,1); 3) (0,1,0) qiymatlar satri berilgan. A formula va uning inkori $\overline{A}$ ni mos formulalar majmuasidan keltirib chiqaring.

4.2.8. Umumlashgan dedukstiya teoremasidan foydalanib, quyidagi formulalarni isbotlanuvchi ekanligini va ular mulohazalar algebrasida aynan chin(tavtalogiya) formulalar ekanligini isbotlang:

$(x \to y) \to ((y \to z) \to (x \to z))$,

$(x \to y) \to (x \vee z \to y \vee z)$

$(x \to y) \to ((z \to x) \to (z \to y))$

4.2.9. $x_1 \vee \overline{x}_4 \to \overline{x_2 \wedge \overline{x}_3}$ formula x_1, x_2, x_3, x_4 o'zgaruvchilarning (0,1,1,0) qiymatlar satrida $R_{0110}(x_1 \vee \overline{x_4} \to \overline{x_2 \wedge \overline{x_3}}) = 1$ qiymatga ega ekanligini isbotlang.

4.2.10. H formulalar to'plamidan berilgan formulani chiqariluvchi ekanligini isbotlang:

1. H={A→B} ├ A&C→B&C;
2. H={A→B} ├ (C→A)→(C→B);
3. H={A→B} ├ (B→C)→(A→C);
4. H={A→(B→C)} ├ B→(A→C);
5. H={A→B} ├ A ∨ C→B ∨ C;

4.2.11. Deduksiya va umumlashgan deduksiya teoremalaridan foydalanib, quyidagi qonunlarni isbotlang:

1. $\vdash (x\to(y\to z))\to(y\to(x\to z))$;
2. $\vdash (x\to(y\to z))\to(x\&y\to z))$;
3. $\vdash (x\&y\to z))\to(x\to(y\to z))$;
4. $\vdash x\to(\bar{x}\to y)$;
5. $\vdash x\vee\bar{x}$;
6. $\vdash \overline{x\,\&\,y}\to(\overline{x\vee y})$.

4.3. Mulohazalar hisobida yechilish, zidsizlik, to'liqlilik va erkinlik muammolari

Mulohazalar hisobi uchun ***yechilish muammosi*** hal qilinuvchidir (yechiluvchidir).

Mulohazalar hisobining zidsizlik muammosi Agar mulohazalar hisobining ixtiyoriy A va $\bar{A}$ formulalari bir paytda isbotlanuvchi formulalar bo'lolmasa, u holda bunday mulohazalar hisobi ziddiyatsiz aksiomatik nazariya, aks holda esa ziddiyatga ega bo'lgan aksiomatik nazariya deb ataladi.

Teorema. Mulohazalar hisobi ziddiyatsiz nazariyadir.

Mulohazalar hisobining to'liqlilik muammosi. Mulohazalar hisobining aksiomalar sistemasiga shu hisobning biror ixtiyoriy isbotlanmaydigan formulasini yangi aksioma sifatida qo'shishdan hosil bo'ladigan aksiomalar sistemasi ziddiyatga ega bo'lgan mulohazalar hisobiga olib kelsa, bunday mulohazalar hisobiga tor ma'nodagi to'liq aksiomatik nazariya deb aytiladi.

Har qanday aynan chin formulasi isbotlanuvchi formula bo'ladigan mulohazalar hisobiga keng ma'nodagi to'liq aksiomatik nazariya deb aytiladi.

Agar A aksiomani mulohazalar hisobining qolgan aksiomalaridan keltirib chiqarish mumkin bo'lmasa, u shu aksiomalar hisobining boshqa aksiomalaridan ***erkin aksioma*** deb ataladi.

Muammoli masala va topshiriqlar:

4.3.1. Har qanday aksiomatik nazariyani asoslash uchun nechta muammolarni ko'rib chiqishga to'g'ri keladi?

4.3.2. A(x) va B(x) ixtiyoriy predikatlar bo'lsin. Quyidagi formulalarning qaysi birlari $A(x) \to \overline{B(x)}$ formulaga tengkuchli formula bo'ladi:

1) $A(x) \vee B(x)$ 2) $\overline{A(x)} \vee \overline{B(x)}$

3) $\overline{A(x)} \to B(x)$ 4) $\overline{B(x)} \to A(x)$

5) $\overline{\overline{A(x)} \wedge B(x)}$ 6) $\overline{A(x) \wedge \overline{B(x)}}$

7) $B(x) \to \overline{A(x)}$

4.3.3. Quyidagi tasdiqlar (teoremalar)ning noto'g'riligini isbot qiling:

1) «Agar funkstiya x_0 nuqtada uzluksiz bo'lsa, u holda u shu nuqtada differenstiallanuvchi bo'ladi».

2) «Agar sonli qatorning n-hadi nolga teng bo'lsa, u holda bu qator yaqinlashuvchi bo'ladi».

3) «Agar to'rtburchakning diagonallari teng bo'lsa, u holda bu to'rtburchak to'g'ri burchakli bo'ladi».

SINOV TESTI

1.To’plamlar nazariyasi

1. Berilgan:U={0,1,2,3,4,5,6,7,8,9}, A={1,2,3}, B={2,3,4,5}. A∪B-toping.

A){6,7,8,9}

B){0,1,9}

C){1,4,5}

D){1,2,3,4,5}

2. Berilgan:U={0,1,2,3,4,5,6,7,8,9}, A={1,2,3}, B={2,3,4,5}. A∩B -toping.

A){2,3}

B){1,5,7}

C){4,5}

D){0,1,2}

3. Berilgan:U={0,1,2,3,4,5,6,7,8,9}, A={1,2,3}, B={2,3,4,5}. A\B -toping.

A)Ø

B){4,5}

C){1)

D){0,2,3,4,5,6,7,8,9}

4. Berilgan:U={0,1,2,3,4,5,6,7,8,9}, A={1,2,3}, B={2,3,4,5}. $A\setminus\overline{B}$ -toping.

A){2,3}

B){1}

C){1,2,3}

D){2,3,4,5}

5. Berilgan:U={0,1,2,3,4,5,6,7,8,9}, A={1,2,3}, B={2,3,4,5}. $\overline{A\setminus B}$ -toping.

A){0,1,2,3,4,5}

B){6,7,8,9,}

C){0,2,3,4,5,6,7,8,9}

D){1,2,3,4,5}

6. A va B bo'sh bo'lmagan to'plamlar bo'lsin A≠B u holda quyidagilardan qaysi bo'sh to'plam bo'ladi?

A) $A \cup B$

B) $A \cup \overline{B}$

C) $\overline{A} \cup B$

D) $\overline{A \cup \overline{A}}$

7. A va B bo'sh bo'lmagan to'plamlar bo'lsin A⊂B u holda quyidagilardan qaysi bo'sh to'plam bo'ladi?

A) $A \setminus B$

B) $A \cup B$

C) $A \cap B$

D) $A \cup \overline{B}$

8. A va B bo'sh bo'lmagan to'plamlar bo'lsin A⊂B u holda quyidagilardan qaysi universal to'plam bo'ladi?

A) $A \setminus B$

B) $A \cap B$

C) $\overline{A \setminus B}$

D) $\overline{A \cap B}$

9. A va B bo'sh bo'lmagan to'plamlar bo'lsin A⊂B u holda quyidagilardan qaysi universal to'plam bo'ladi?

A) $(\overline{A \cap B}) \setminus B$

B) $\overline{A} \setminus B$

C) $B \setminus A$

D) $(A \cap B) \cup \overline{A}$

10. A={a,b} va B={5,6}bo’lsa, A×B ni toping.

A) $\{(5,a),(6,a),(5,b),(6,b)\}$

B) $\{(a,5),(a,6),(b,5),(b,6)\}$

C) $\{5,6,a,b\}$

D) $\{a,b,5,6\}$

11. Berilgan:U={0,1,2,3,4,5,6,7,8,9}, A={0,2,3}, B={2,3,4,5}. A∪B-toping.

A){6,7,8,9}

B){0,1,9}

C){1,4,5}

D){0,2,3,4,5}

12. Berilgan:U={0,1,2,3,4,5,6,7,8,9}, A={0,2,3}, B={2,3,4,5}. A∩B -toping.

A){2,3}

B){1,5,7}

C){4,5}

D){0,1,2}

13. Berilgan:U={0,1,2,3,4,5,6,7,8,9}, A={1,2,3}, B={1,3,4,5}. A\B -toping.

A)∅

B){4,5}

C){2)

D){0,2,3,4,5,6,7,8,9}

14. Berilgan:U={0,1,2,3,4,5,6,7,8,9}, A={1,2,3}, B={3,4,5}. $A \setminus \overline{B}$ -toping.

A){3}

B){1}

C){1,2,3}

D){2,3,4,5}

15. Berilgan:U={0,1,2,3,4,5,6,7,8,9}, A={0,2,3}, B={2,3,4,5}. $\overline{A\setminus B}$ -toping.

A){0,1,2,3,4,5}

B){6,7,8,9,}

C){1,2,3,4,5,6,7,8,9}

D){1,2,3,4,5}

16. A va B bo’sh bo’lmagan to’plamlar bo’lsin A≠B u holda quyidagilardan qaysi bo’sh to’plam bo’ladi?

A) $A\cup B$

B) $A\cup\overline{B}$

C) $\overline{A}\cup B$

D) $\overline{B\cup\overline{B}}$

17. A va B bo’sh bo’lmagan to’plamlar bo’lsin A⊃B u holda quyidagilardan qaysi bo’sh to’plam bo’ladi?

A) $B\setminus A$

B) $A\cup B$

C) $A\cap B$

D) $A\cup\overline{B}$

18. A va B bo’sh bo’lmagan to’plamlar bo’lsin A⊃B u holda quyidagilardan qaysi universal to’plam bo’ladi?

A) $\overline{B\setminus A}$

B) $A\cap B$

C) $A\setminus B$

D) $\overline{A \cap B}$

19. A va B bo'sh bo'lmagan to'plamlar bo'lsin A⊃B u holda quyidagilardan qaysi universal to'plam bo'ladi?

A) $(\overline{A \cap B}) \setminus B$

B) $\overline{A} \setminus B$

C) $B \setminus A$

D) $(A \cap B) \cup \overline{B}$

20. Agar A={a,b} va B={5,6} bo'lsa, B×A ni toping.

A) $\{(a,5),(a,6),(b,5),(b,6)\}$

B) $\{(5,a),(6,a),(5,b),(6,b)\}$

C) $\{5,6,a,b\}$

D) $\{a,b,5,6\}$

21. Quyidagi tengliklardan qaysi biri noto‘g‘ri?

A) $A \setminus B = A \setminus (A \cap B)$

B) $A \setminus B = A \setminus (A \cap B)$

C) $A \cap (B \cap C) = (A \cap B) \cap C$

D) $(A \cap B) \cup A = A$

22. Quyidagi tengliklardan qaysi biri noto‘g‘ri?

A) $(A \cup B) \cap A = B$

B) $A \cup B = B \cup A$

C) $A \cap (B \setminus C) = (A \cap B) \setminus C$

D) $A \cup (B \cup C) = (A \cup B) \cup C$

23. To’rt elementli to’plamda nechta refleksiv binar munosabat mavjud

A)4096

B)512

C)32

D)64

24. Uch elementli to’plamda nechta simmetrik binar munosabat

A)64

B)32

C)48

D)16

25. To’rt elementli to’plamda nechta simmetrik binar munosabat mavjud

A)1024

B)2^{11}

C)2^{9}

D)2048

Mulohazalar mantiqi

26. Berilgan mulohazani formula ko’rinishida ifodalang:«Agar harorat noldan yuqori bo’lsa, unda muz eriydi va daraxt bo’lagi suzadi».

A)$A\rightarrow B$

B)$A\wedge B$

C)$A\leftrightarrow B$

D)$A\rightarrow(B\wedge C)$

27. Berilgan mulohazani formula ko’rinishida ifodalang:«Son juft bo’ladi faqat va faqat shunda, qachonki u ikkiga bo’linsa».

A)$A\leftrightarrow B$

B)$A \rightarrow B$

C)$A \wedge B$

D)$\neg A \wedge B$

28. Berilgan mulohazani formula ko’rinishida ifodalang:«Agar uchburchakning burchaklari har hil bolsa, u holda u na teng yonli va na teng tomonli bo’ladi».

A)$\neg A \leftrightarrow B$

B)$A \wedge B$

C)$A \rightarrow (\neg B \wedge \neg C)$

D)$\neg A \wedge \neg B$

29. Berilgan mulohazani formula ko’rinishida ifodalang:«Agar bugun imtihon olinmasa, unda kinoga yoki parka boraman».

A)$\neg (A \wedge B)$

B)$\neg (A \vee B)$

C)$\neg (A \rightarrow B)$

D)$A \rightarrow (B \vee C)$

30. Berilgan mulohazani formula ko’rinishida ifodalang:«Agar havo quyoshli va issiq bo’lsa, unda insonninig kayfiyati yaxshi bo’ladi».

A)$A \rightarrow (B \vee C)$

B)$\neg (A \vee B)$

C)$\neg (A \leftrightarrow B)$

D)$(A \wedge B) \rightarrow C$

31. Berilgan mulohazani formula ko’rinishida ifodalang:« Agar uchburchakning burchaklari orasida tenglari bolsa, u holda u teng yonli yoki teng tomonli bo’ladi».

A)$A \leftrightarrow B$

B)$\neg A \wedge B$

C)$A\to(\neg B\wedge\neg C)$

D)$A\to(B\vee C)$

32. Berilgan mulohazani formula ko'rinishida ifodalang:«Agar talaba biletning javobini bilmasa va ko'chirolmasa, u holda u ikki baho oladi».

A)$(\neg A\wedge\neg B)\to C$

B)$(A\wedge B)\to C$

C)$\neg A\wedge\neg B$

D)$A\to(\neg B\wedge\neg C)$

33. Berilgan mulohazani formula ko'rinishida ifodalang:«Agar dars bo'lmasa, unda uyga boraman va dush qabul qilaman».

A)$\neg(A\wedge B)$

B)$A\leftrightarrow\neg B$

C)$\neg A\to(B\wedge C)$

D)$\neg A\vee B$

34. Berilgan mulohazani formula ko'rinishida ifodalang:« Son juft bo'lmaydi faqat va faqat shunda, qachonki u ikkiga bo'linmasa».

A)$\neg A\leftrightarrow\neg B$

B)$\neg A\to B$

C)$\neg(A\wedge B)$

D)$\neg A\wedge B$

35. Berilgan mulohazani formula ko'rinishida ifodalang:«Agar to'rtburchakda teng burchaklar mavjud bo'lsa, unda u kvadrat yoki to'g'ri burchakli to'rtburchak yoki romb bo'ladi».

A)$\neg A\wedge B$

B)$A\to(B\vee C\vee D)$

C)$A\wedge\neg B$

D)A↔¬B

36. Berilgan mulohazani formula ko'rinishida ifodalang:«Agar imtihon qoldirilsa, unda bugun kinoga so'ng basseynga boraman».

A)(A↔B)

B)A∧B

C)A∨B

D)A→(B∧C)

37. Berilgan mulohazani formula ko'rinishida ifodalang:«Agar ishlar yaxshi ketsa, unda insonning kayfiyati yaxshi bo'ladi».

A)A∨B

B)¬A↔B

C)B→C

D)A→(B∧C)

38. Berilgan mulohazani formula ko'rinishida ifodalang:«Agar to'rtburchakning tomonlari teng bo'lsa, unda u kvadrat yoki romb».

A)¬A∧B

B)A→(¬B∧¬C)

C)A→(B∨C)

D)¬(A∧B)

39. Berilgan mulohazani formula ko'rinishida ifodalang:«Agar talaba bilet javoblarini bilsa va qo'shimcha savollarga javob bersa, unda u besh baho oladi».

A)(A∧B)→C

B)(A∧B)→C

C)¬A∧¬B

D)A→(¬B∧¬C)

40. Berilgan mulohazani formula ko'rinishida ifodalang: «Bugun tushlikda palov bo'lmasa, somsa yeyman».

A)$\neg A \wedge B$

B) $\neg A \rightarrow B$

C)$A \rightarrow (B \vee C)$

D)$\neg(A \wedge B)$

Mulohazalar algebrasi

41. Quyidagi mulohazalardan qaysi tavtologiya bo'ladi?

A)$A \vee B$

B)$A \vee \neg A$

C)$A \rightarrow B$

D)$A \leftrightarrow B$

42. Quyidagi mulohazalardan qaysi aynan yolg'on bo'ladi?

A)$\neg(A \vee \neg A)$

B)$A \leftrightarrow B$

C)$A \vee B$

D)$A \wedge B$

43. Quyidagi mulohazalardan qaysi bajariluvchi bo'ladi?

A)$A \wedge \neg A$

B)$A \rightarrow B$

C)$\neg(A \wedge \neg A)$

D)$\neg(A \vee \neg A)$

44. Quyidagi mulohazalardan qaysi tavtologiya bo'ladi?

A)$(A \vee B) \wedge A$

B)$\neg(A\vee\neg A)\wedge (A\vee\neg A)$

C)$\neg(A\wedge\neg A)$

D)$A\wedge\neg A$

45.Quyidagi mulohazalardan qaysi aynan yolg’on bo’ladi?

A)$A\vee B$

B)$A\wedge B$

C)$A\leftrightarrow B$

D)$A\wedge\neg A$

46. Quyidagi mulohazalardan qaysi tavtologiya bo’ladi?

A)$A\leftrightarrow B$

B)$A\vee B$

C)$A\wedge B$

D)$(B\rightarrow\neg A)\vee B$

47. Quyidagi mulohazalardan qaysi aynan yolg’on bo’ladi?

A)$\neg(A\vee\neg A)\wedge (A\vee\neg A)$

B)$A\vee\neg A$

C)$A\vee B$

D)$A\wedge B$

48. Quyidagi mulohazalardan qaysi bajariluvchi bo’ladi?

A)$A\wedge\neg A$

B)$A\vee B$

C)$\neg(A\vee\neg A)$

D)$A\leftrightarrow\neg A$

49. Quyidagi mulohazalardan qaysi tavtologiya bo’ladi?

A)$A\vee B$

B)$\neg(A\vee\neg A)$

C)$(A\to A)$

D)$A\leftrightarrow\neg A$

50.Quyidagi mulohazalardan qaysi aynan yolg'on bo'ladi?

A)$A\vee\neg A$

B)$(A\to A)$

C)$A\to A$

D)$A\leftrightarrow\neg A$

51.Quyidagi belgilar ketma-ketliklarining qaysi biri formula bo'ladi?

A) $\neg(A\vee B)\to\neg B\ )$

B) $(A\to B)\neg\vee B$

C) $((A\leftrightarrow B)\wedge\neg A)$

D) $(\neg B\to\vee A)$

52. Quyidagi belgilar ketma-ketliklarining qaysi biri formula bo'ladi?

A) $((B\to(A\wedge C))\wedge\neg(A\vee C))$

B) $(A\to\vee(B\wedge C))$

C) $\neg(\to B\vee C)\wedge A,D)$

D) $(\neg(A\to B)\vee\neg C))$

53. Quyidagi belgilar ketma-ketliklarining qaysi biri formula bo'lmaydi?

A) $\neg(\to B\vee C)\wedge A,D)$

B) $((A\leftrightarrow B)\wedge\neg A)$

C) $(\neg(B\vee C)\wedge(A\vee D))$

D) $((A \to B) \vee A)$

54. Quyidagi belgilar ketma-ketliklarining qaysi biri formula bo‘lmaydi?

A) $(B \vee C) \wedge AD)$

B) $(\neg(B \vee C) \wedge (A \vee D))$

C) $(\neg(B \vee C) \wedge A)$

D) $((B \vee C) \wedge (A \vee \neg D))$

55. $F \equiv (((A \vee B) \wedge \neg C) \to (A \wedge B))$ formulaning barcha qism formulalarini yozing.

A) $A, B, C, \neg C, (A \vee B),\ (A \wedge B),\ ((A \vee B) \wedge \neg C),\ F.$

B) $A, B, C, (A \vee B),\ ((A \vee B) \wedge \neg C),\ F.$

C) $A, B, C, \neg C, (A \vee B),\ (A \wedge B),\ ((A \vee B) \wedge \neg C).$

D) $A, B, C,\ (A \wedge B),\ ((A \vee B) \wedge \neg C),\ F.$

56. Quyidagi ikki o‘zgaruvchili formula o‘zgaruvchilar qiymatlarining nechta tanlanmasida 1 qiymat qabul qiladi? $(A \to (B \to (A \wedge B)))$

A)1

B)3

C)2

D)4

57. Quyidagi ikki o‘zgaruvchili formula o‘zgaruvchilar qiymatlarining nechta tanlanmasida 1 qiymat qabul qiladi? $(((A \vee \neg B) \to B) \wedge (\neg A \vee B))$

A)1

B)2

C)3

D)4

58. $(\neg((A \to \neg B) \vee C) \wedge B)$ uch o‘zgaruvchili qiymatlarining nechta tanlanmasida 1 qiymat qabul qiladi?

A)4

B)3

C)1

D)8

59. Quyidagi ikki o‘zgaruvchili formula o‘zgaruvchilar qiymatlarining nechta tanlamasida 0 qiymat qabul qiladi? $(P \to (Q \to (P \wedge Q)))$

A)2

B)0

C)4

D)1

60. $(((P \vee \neg Q) \wedge R) \to (P \wedge Q))$ uch o‘zgaruvchili formula o‘zgaruvchilar qiymatlarining nechta tanlamasida 0 qiymat qabul qiladi?

A)1

B)3

C)8

D)2

61. Quyidagi ikki o‘zgaruvchili formulalarning qaysi biri keltirilgan formula?

A) $(A \vee B)$

B) $(A \to B)$

C) $\neg(A \vee B)$

D) $\neg(A \wedge B)$

62. Quyidagi ikki o‘zgaruvchili formulalarning qaysi biri keltirilgan formula?

A) $(A \vee \neg B) \wedge \neg C$

B) $(A \to \neg B)$

C) $\neg(A \wedge B) \to C$

D) $\neg(A \wedge \neg B)$

63. Quyidagi uch o‘zgaruvchili formulalarning qaysi biri keltirilgan formula?

A) $(\neg(A \vee B) \vee C)$

B) $(\neg(A \wedge B) \vee C)$

C) $((A \to B) \vee C)$

D) $((\neg A \wedge B) \vee \neg C)$

64. Quyidagi uch o‘zgaruvchili formulalarning qaysi biri keltirilgan formula?

A) $((\neg A \wedge B) \vee (\neg C \wedge \neg B))$

B) $((\neg A \wedge B) \vee \neg(\neg C \wedge \neg B))$

C) $((\neg A \wedge B) \to (\neg C \wedge \neg B))$

D) $((\neg A \wedge B) \vee (\neg C \to \neg B))$

65. Quyidagi formulalarning qaysi biri DNF bo‘ladi?

A) $((\neg A \wedge B) \vee (\neg C \wedge \neg B))$

B) $((\neg A \wedge B) \vee (\neg C \to \neg B))$

C) $((\neg A \to B) \vee (\neg C \to \neg B))$

D) $(\neg(\neg A \wedge B) \vee \neg(\neg C \wedge \neg B))$

66. Quyidagi formulalarning qaysi biri DNF bo‘ladi?

A) $(B \to \neg A)$

B) $(A \wedge B) \vee (A \wedge \neg B)$

C) $(A \leftrightarrow B)$

D) $((A \vee B) \wedge C)$

67. Quyidagi formulalarning qaysi biri KNF bo‘ladi?

A) $(\neg A \vee B \vee C)$

B) $((\neg A \rightarrow B) \vee (\neg C \rightarrow \neg B))$

C) $((\neg A \wedge B) \vee (\neg C \rightarrow \neg B))$

D) $(\neg(\neg A \wedge B) \vee \neg C)$

68. Quyidagi ikki o‘zgaruvchili formulalarning qaysi biri KNF bo‘ladi?

A) $(B \rightarrow \neg A)$

B) $(A \vee \neg B) \wedge (\neg A \vee C)$

C) $\neg(A \wedge B)$

D) $((A \vee B) \wedge \neg(C \vee A))$

69. Quyidagi ikki o‘zgaruvchili formulaning MDNFida nechta xad bor?

$(\mathrm{A} \rightarrow (\mathrm{B} \rightarrow (\mathrm{A} \wedge \mathrm{B})))$

A)2

B)4

C)1

D)3

70. Quyidagi ikki o‘zgaruvchili formulaning MKNFida nechta xad bor?

$(((\mathrm{P} \vee \neg Q) \rightarrow Q) \wedge (\neg P \vee Q))$

A)5

B)4

C)2

D)3

71. $x \leftrightarrow y = 0$ tenglamani yeching

A) $(1,0),(0,1)$

B) $(0,1)$

C) $(1,0)$

D) $(1,1)$

72. $x \to \overline{y} = 0$ tenglamani yeching

A) $(0,1)$

B) $(1,1)$

C) $(1,0)$

D) $(0,0)$

73. $(1 \to \mathrm{x}) \to \mathrm{y} = 0$ tenglamani yeching

A) $(1,1)$

B) $(0,1)$

C) $(0,0)$

D) $(1,0)$

74. Qaysi funksiya bilan $\mathrm{x} \cdot \mathrm{y}$ funksiya ustma-ust tushadi?

A) $\overline{x \vee y}$

B) $x \vee \overline{y}$

C) $\overline{x} \vee y$

D) $\overline{\overline{x} \vee \overline{y}}$

75. $x \vee y = \overline{x}$ tenglamani yeching

A) $(0,1)$

B) $(0,0)$

C) $(1,0)$

D) $(1,1)$

Bul funksiyalari

76. Quyidagi bul funksiyalaridan qaysi kon'yunktiv normal shaklda berilgan (KNF)?

A) $(x \wedge y \wedge \bar{z}) \vee (\bar{x} \wedge \bar{z})$

B) $\overline{(x \vee y)} \wedge x$

C) $\overline{(x \wedge y)} \wedge x$

D) $(x \vee y \vee \bar{z}) \wedge (\bar{x} \vee \bar{z})$

77. Quyidagi bul funksiyalaridan qaysi kon'yunktiv normal shaklda berilgan (KNF)?

A) $(y \vee \bar{z}) \wedge (\bar{x} \vee \bar{z})$

B) $\overline{(y \vee \bar{z})} \wedge (\bar{x} \vee \bar{z})$

C) $\overline{(y \vee \bar{z}) \wedge (\bar{x} \vee \bar{z})}$

D) $(y \vee \bar{z}) \wedge \overline{(\bar{x} \vee \bar{z})}$

78. Quyidagi bul funksiyalaridan qaysi kon'yunktiv normal shaklda berilgan (KNF)?

A) $(\overline{(x \vee y)} \vee \bar{z}) \wedge (\bar{x} \vee \bar{z})$

B) $(y \vee \bar{z}) \wedge \overline{(\bar{x} \vee z)}$

C) $(y \vee \bar{z}) \wedge (\bar{x} \vee \overline{\bar{z} \vee y})$

D) $(y \vee z) \wedge (\bar{x} \vee \bar{z})$

79. Quyidagi bul funksiyalaridan qaysi kon'yunktiv normal shaklda berilgan (KNF)?

A) $(\overline{y \vee \bar{z}} \vee x) \wedge (\bar{x} \vee \bar{z})$

B) $(x \wedge y \vee \bar{z}) \wedge (\bar{x} \vee \bar{z})$

C) $\overline{(y \vee \bar{z}) \wedge (\bar{x} \vee \bar{z})}$

D) $(x \vee y \vee \bar{z}) \wedge (\bar{x} \vee \bar{z})$

80. Quyidagi bul funksiyalaridan qaysi kon'yunktiv normal shaklda berilgan (KNF)?

A) $(y \vee \bar{z} \wedge x) \wedge (\bar{x} \vee \bar{z})$

B) $(\bar{x} \vee y \vee \bar{z}) \wedge (\bar{x} \vee \bar{z})$

C) $(y \vee \bar{z}) \wedge (\bar{x} \vee y \wedge \bar{z})$

D) $(x \wedge y \vee \bar{z}) \wedge (\bar{x} \vee \bar{z})$

81. Quyidagi bul funksiyalaridan qaysi diz'yunktiv normal shaklda berilgan (DNF)?

A) $(x \vee y \wedge \bar{z}) \vee (\bar{x} \wedge \bar{z})$

B) $\overline{(x \vee y)} \wedge x$

C) $\overline{(x \wedge y)} \wedge x$

D) $(x \wedge y \wedge \bar{z}) \vee (\bar{x} \wedge \bar{z})$

82. Quyidagi bul funksiyalaridan qaysi diz'yunktiv normal shaklda berilgan (DNF)?

A) $\overline{(y \vee \bar{z})} \wedge (\bar{x} \vee \bar{z})$

B) $\overline{(y \vee \bar{z}) \wedge (\bar{x} \vee \bar{z})}$

C) $(y \vee \bar{z}) \wedge \overline{(\bar{x} \vee \bar{z})}$

D) $(y \wedge \bar{z}) \vee (\bar{x} \wedge \bar{z})$

83. Quyidagi bul funksiyalaridan qaysi diz'yunktiv normal shaklda berilgan?

A) $(y \vee \bar{z}) \wedge \overline{(\bar{x} \vee z)}$

B) $(y \vee \bar{z}) \wedge (\bar{x} \vee \overline{\bar{z} \vee y})$

C) $(y \wedge z) \vee (\bar{x} \wedge \bar{z})$

D) $(y \vee \bar{z}) \wedge \overline{(\bar{x} \vee \bar{z})}$

84. Quyidagi bul funksiyalaridan qaysi diz’yunktiv normal shaklda berilgan ?

A) $(\overline{y \vee \bar{z}} \vee x) \wedge (\bar{x} \vee \bar{z})$

B) $(x \wedge y \wedge \bar{z}) \vee (\bar{x} \wedge \bar{z})$

C) $\overline{(y \vee \bar{z}) \wedge (\bar{x} \vee \bar{z})}$

D) $(x \vee y \vee \bar{z}) \wedge (\bar{x} \vee \bar{z})$

85. Quyidagi bul funksiyalaridan qaysi diz’yunktiv normal shaklda berilgan?

A) $(y \wedge \bar{z} \wedge x) \vee (\bar{x} \wedge \bar{z})$

B) $\overline{(y \vee \bar{z} \vee x)} \wedge (\bar{x} \vee \bar{z})$

C) $(y \vee \bar{z}) \wedge (\bar{x} \vee y \wedge \bar{z})$

D) $(x \wedge y \vee \bar{z}) \wedge (\bar{x} \vee \bar{z})$

86. Bul funksiyalaridan qaysi YOKI-EMAS amallari yordamida berilgan?

A) $\overline{x_1 \wedge x_2} \vee (\bar{x}_2 \wedge x_3)$

B) $(x \vee y) \wedge (x \vee \bar{y})$

C) $(x \wedge y) \oplus 1$

D) $\overline{x_1 \vee x_2} \vee (\bar{x}_2 \vee x_3)$

87. Bul funksiyalaridan qaysi VA-EMAS amallari yordamida berilgan?

A) $x \wedge \overline{(\bar{x} \wedge \bar{y})}$

B) $x \oplus y \oplus z \oplus 1$

C) $(\bar{x}_1 \wedge x_2) \vee (\bar{x}_1 \wedge \bar{x}_2)$

D) $(\bar{x}_1 \vee x_2) \wedge (\bar{x}_1 \vee \bar{x}_2)$

88. Bul funksiyalaridan qaysi Jegalkin yig'indi ko'rinishida berilgan?

A) $\overline{(\bar{x} \vee \bar{y})} \vee x \wedge \bar{y}$

B) $\overline{(x \wedge y)} \vee \bar{x} \wedge \bar{y}$

C) $x \wedge (\bar{x} \vee \bar{y})$

D) $x \wedge y \oplus z \oplus 1$

89. Bul funksiyalaridan qaysi YOKI-EMAS amallari yordamida berilgan?

A) $x \wedge (\bar{y} \vee z) \vee y \wedge (z \vee \bar{x})$

B) $x \vee \overline{(\bar{y} \vee z)} \vee y$

C) $x \wedge y \oplus z \oplus 1$

D) $\overline{(x \wedge y)} \vee \bar{x} \wedge \bar{y}$

90. Bul funksiyalaridan qaysi VA-EMAS amallari yordamida berilgan?

A) $x \oplus y \oplus z \oplus 1$

B) $\overline{(\bar{x} \wedge z)} \wedge x \wedge y$

C) $\overline{(\bar{x} \vee \bar{y})} \vee x \wedge \bar{y}$

D) $(\bar{x}_1 \vee x_2) \wedge (\bar{x}_1 \vee \bar{x}_2)$

91. Bul funksiyalaridan qaysi Jegalkin yig'indi ko'rinishida berilgan?

A) $x \wedge y \wedge z \vee x \wedge \bar{y}$

B) $(x \vee y) \wedge (x \vee \bar{y}) \vee x$

C) $\overline{(\bar{x} \vee \bar{y})} \vee x \wedge \bar{y}$

D) $x \wedge y \wedge z \oplus x \wedge y \oplus 1$

92. Bul funksiyalaridan qaysi YOKI-EMAS amallari yordamida berilgan?

A) $\overline{x \vee (\bar{x} \vee \bar{y})}$

B) $x \wedge (\bar{x} \vee \bar{y})$

C) $(y \wedge z) \vee (\bar{x} \wedge y \wedge z)$

D) $(\bar{x} \vee y) \wedge (x \vee \bar{y})$

93. Bul funksiyalaridan qaysi VA-EMAS amallari yordamida berilgan?

A) $x_1 \wedge \bar{x}_3 \vee x_2$

B) $x \wedge (\bar{x} \wedge \bar{y})$

C) $x \wedge y \wedge z \oplus x \wedge y \oplus 1$

D) $x \wedge (\bar{x} \vee \bar{y})$

94. Bul funksiyalaridan qaysi Jegalkin yig'indi ko'rinishida berilgan?

A) $x \wedge y \oplus z \oplus 1$

B) $\overline{(x \wedge y)} \vee \bar{x} \wedge \bar{y}$

C) $\overline{(\bar{x} \vee \bar{y})}$

D) $y \wedge (x \vee \bar{y}) \vee x$

95. Bul funksiyalaridan qaysi YOKI-EMAS amallari yordamida berilgan?

A) $y \wedge z \vee x \wedge \bar{y}$

B) $(x \wedge y) \oplus y$

C) $\overline{x_1 \vee x_2}$

D) $x_1 \wedge \bar{x}_3 \vee x_2$

96. Bul funksiyalaridan qaysi YOKI-EMAS amallari yordamida berilgan?

A) $\overline{x_1 \vee x_2} \vee (\bar{x}_2 \wedge x_3)$

B) $(x \vee y) \wedge \overline{(x \vee \bar{y})}$

C) $(x \wedge y) \oplus 1$

D) $\overline{x_1 \vee x_2} \vee (\bar{x}_2 \vee \bar{x}_3)$

97. Bul funksiyalaridan qaysi VA-EMAS amallari yordamida berilgan?

A) $x \wedge \overline{(x \wedge y)}$

B) $x \oplus y \oplus z$

C) $(x_1 \wedge x_2) \vee (\bar{x}_1 \wedge \bar{x}_2)$

D) $(x_1 \vee x_2) \wedge (\bar{x}_1 \vee \bar{x}_2)$

98. Bul funksiyalaridan qaysi Jegalkin yig'indi ko'rinishida berilgan?

A) $(\overline{\bar{x} \vee \bar{y}}) \vee x \wedge \bar{y}$

B) $\overline{(x \wedge y)} \vee x \wedge y$

C) $x \wedge \overline{(\bar{x} \vee \bar{y})}$

D) $x \wedge y \oplus 1$

99. Bul funksiyalaridan qaysi YOKI-EMAS amallari yordamida berilgan?

A) $\overline{(\bar{y} \vee z)} \vee y \wedge (z \vee \bar{x})$

B) $x \vee \overline{(\bar{y} \vee z)} \vee \bar{y}$

C) $x \wedge y \oplus y$

D) $\overline{(x \vee y)} \vee \bar{x} \wedge \bar{y}$

100. Bul funksiyalaridan qaysi VA-EMAS amallari yordamida berilgan?

A) $y \oplus z \oplus 1$

B) $\overline{(\bar{x} \wedge z)} \wedge \bar{x} \wedge y$

C) $(\overline{\bar{x} \wedge \bar{y}}) \vee x \wedge \bar{y}$

D) $\overline{(\bar{x}_1 \vee x_2)} \wedge (\bar{x}_1 \vee \bar{x}_2)$

101. Birni saqlovchi ikki o‘zgaruvchili funksiyalar soni qancha?

A)8

B)16

C)32

D)4

102. Quyidagi funksiyalarni qaysi biri $x \to y$ ga funksiyaga qo‘shma buladi?

A) $\bar{x} \cdot y$

B) $x \cdot \bar{y}$

C) $x \vee \bar{y}$

D) $\bar{x} \vee y$

103. Quyidagi funksiyalarni Qaysi $\bar{x} \vee \bar{y}$ ga funksiyaga qo‘shma buladi?.

A) $\bar{x} \cdot \bar{y}$

B) $x \cdot y$

C) $x \vee y$

D) $x \vee \bar{y}$

104. Uch o‘zgaruvchili chiziqli funksiyalar soni qancha?.

A) 4

B)16

C)64

D)18

105. To‘rt o‘zgaruvchili chiziqli funksiyalar soni qancha?.

A) 64

B)32

C)22

D)18

106. Qaysi to‘plamda nolni saqlovchi funksiyalar to‘g‘ri ko’rsatilgan?.

A) $x,\ \ y,\ \ x+y,\ \ \bar{x}+1$

B) $x \vee y,\ \ x \to y$

C) $x \cdot y,\ \ x \leftrightarrow y$

D) $\bar{x}+y,\ \ \ \ \bar{y}+x,\ \ \ x+y$

107. Qaysi to‘plamda birni saqlovchi funksiyalar to‘g‘ri ko’rsatilgan?.

A) $x,\ \ y,\ \ \overline{x+y},\ \ \bar{x}+1$

B) $x+y,\ \ x,\ y$

C) $\bar{x}\cdot\bar{y},\ \ x \to y$

D) $x+\bar{y},\ \ x,\ y$

108. Qaysi to‘plamda monoton funksiyalar to‘g‘ri ko’rsatilgan?

A) $x\cdot\ y,\ \ x \vee y,\ \ x \cdot y$

B) $\bar{x}\ ,\ \ x \to y$

C) $x,\ \ \ x \leftrightarrow y$

D) $x+y,\ \ \bar{x}$

109. Qaysi to‘plamda chiziqli funksiyalar to‘g‘ri ko’rsatilgan?.

A) $x+y,\ \overline{x+y},\ \ \overline{(x \leftrightarrow y)},\ \ \bar{x}$

B) $x \cdot y,\ \ \ \ x \vee\ y$

C) $\bar{x}\ ,\ \ x \to y$

D) $x \cdot y, \quad x \leftrightarrow y, \quad x + y$

110. To‘liqmas funksiyalar sistemasini ko’rsating?

A) $\left\{ x + y, \ \bar{x} \right\}$

B) $\{ x \to y, \ 0 \}$

C) $\{ \overline{x \vee y} \}$

D) $\left\{ x \to y, \ \bar{x} \right\}$

111. Qaysi funksiya bilan $\bar{\mathbf{x}}$ funksiya ustma-ust tushadi?.

A) $\{ x + 1 \}$

B) $x \cdot x$

C) x

D) $\{ x \to y \}$

112. Qaysi tenglik o‘rinli?.

A) $x + y = \bar{x} + y + 1$

B) $x + y = \bar{x} \vee \bar{y}$

C) $x + y = \bar{x} + \bar{y} + 1$

D) $x + y = \bar{x} \vee y$

113. Qaysi funksiya bilan $\overline{x \downarrow y}$ funksiya ustma-ust tushadi?

A) $(x+1)(y+1)+1$

B) $x \ y$

C) $(x+1) \cdot y + 1$

D) $x \cdot y + 1$

114. Qaysi funksiya bilan $\mathrm{x} \to \mathrm{y}$ funksiya ustma-ust tushadi?

A) $xy + x + 1$

B) $x \cdot y$

C) $xy + 1$

D) $xy + x$

115. Qaysi funksiya bilan $\mathrm{x} + \mathrm{y}$ funksiya ustma-ust tushadi?

A) $\overline{(x \leftrightarrow y)}$

B) $x \to y$

C) $x + y + 1$

D) $\bar{x} \vee y$

116. Qaysi funksiya bilan $\mathrm{x} \downarrow \mathrm{y}$-Pirs strelkasi funksiya ustma-ust tushadi?

A) $\overline{x \vee y}$

B) $x \to y$

C) $\overline{x \cdot y}$

D) $\overline{x \cdot y}$

117. Qaysi tenglik munosabati o‘rinli?

A) $\bar{x} = x / x$

B) $\bar{x} = x \to y$

C) $\bar{x} = x + x$

D) $\bar{x} = x + y$

118. Quyidagi qaysi tenglik munosabati o‘rinli?

A) $xy = (x/y)\ /\ (x/y)$

B) $x \cdot y = \bar{x} + 1$

C) $x \cdot y = x \to y$

D) $x \cdot y = x \vee y$

119. Quyidagi qaysi tenglik munosabati o‘rinli?.

A) $x \to y = x/(y/y)$

B) $x \to y = x \vee \bar{y}$

C) $x \to y = x \cdot \bar{y}$

D) $x \to y = \bar{x} \cdot y$

120. Quyidagi qaysi tenglik munosabati o‘rinli?

A) $x \vee y = (x+1)\ (y+1) + 1$

B) $x \vee y = \bar{x} \cdot y$

C) $x \vee y = x \to y$

D) $x \vee y = \bar{x} + \bar{y}$

121. Quyidagi qaysi tenglik munosabati o‘rinli?

A) $x \vee y = (x+1)\ (y+1) + 1$

B) $x \vee y = \bar{x} \cdot y$

C) $x \vee y = x \to y$

D) $x \vee y = \bar{x} + \bar{y}$

122. Qaysi funksiya $x \leftrightarrow y$ funksiyaga teng?

A) $\overline{x+y}$

B) $x \vee \overline{y}$

C) $\overline{x \vee y}$

D) $x \vee y$

123. Qaysi Bul funksiyalar sistemasi to‘liq?

A) $\{\cdot, \ -\}$

B) $\{+, \ \to\}$

C) $\{\cdot, \ \leftrightarrow\}$

D) $\{\cdot, \ \vee\}$

124. Qaysi Bul funksiyalar sistemasi to‘liq?

A) $\{+, \ \cdot, \ 1, \to\}$

B) $\{\cdot, \ \to\}$

C) $\{+\}$

D) $\{\leftrightarrow\}$

125. Nol va birni saqlovchi uch o‘zgaruvchili funksiyalar soni qancha?

A)64

B)16

C)128

D)256

126. O‘ziga o‘zi qo‘shma uch o‘zgaruvchili funksiyalar soni qancha?

A)16

B)64

C)36

D)23

127. Uch o‘zgaruvchili Bul funksiyalar soni qancha?

A)256

B)32

C)64

D)128

128. Nolni saqlovchi uch o‘zgaruvchili funksiyalar soni qancha?

A)128

B)32

C)64

D)256

Mulohazalar hisobi

129. Quyidagi formulalarning qaysi biri L nazariyasining teoremasi emas.

A) $\neg(A \to \neg A)$

B) $A \to A$

C) $\neg\neg A \to A$

D) $A \to \neg\neg A$

130. Quyidagi formulalarning qaysi biri L nazariyasining teoremasi emas.

A) $(A \to B) \to (B \to A)$

B) $\neg A \to (A \to B)$

C) $(\neg B \to \neg A) \to (A \to B)$

D) $A \to \neg\neg A$

131. Quyidagi formulalarning qaysi biri L nazariyasining teoremasi emas.

A) $(A \to B) \to (\neg A \to \neg B)$

B) $(A \to (\neg B \to \neg(A \to B)))$

C) $\neg\neg B \to B$

D) $(\neg B \to \neg A) \to (A \to B)$

132. Quyidagilardan qaysi mulohazalar hisobining formulasi bo’ladi?

A) $((p_1 \vee p_2) \vee (p_1 p_2)) \to \overline{p}_3$

B) $(p_1(p_2 \vee p_3)) \to p_3$

C) $(p_1 \to p_2) \to ((p_1 \to \overline{p}_2) \to p_1)$

D) $(p_1 \wedge (\to p_2) \to (p_2 \to \overline{p}_1)$

133. Mulohazalar hisobining ikkinchi guruh aksiomalariga kirmagan formulani toping.

A) $x \wedge y \to x.$

B) $x \wedge y \to y.$

C) $(z \to x) \to ((z \to y) \to (z \wedge x \wedge y))$

D) $(z \to x) \to ((z \to y) \to (z \to x \wedge y))$

134. Mulohazalar hisobining uchinchi guruh aksiomalariga kirmagan formulani toping.

A) $y \to x \vee y$

B) $x \wedge y \to y.$

C) $(x \to z) \to ((y \to z) \to (x \vee y \to z))$

D) $x \to x \vee y$.

135. Mulohazalar hisobining to’rtinchi guruh aksiomalariga kirmagan formulani toping.

A) $\overline{\overline{x}} \to x$.

B) $x \to \overline{\overline{x}}$.

C) $(x \to y) \to (\bar{y} \to \bar{x})$.

D) $x \to x \vee y$.

136. Quyidagi berilgan formulalardan o'rniga qo'yish formulasini toping.

A) $\dfrac{\vdash A}{\vdash \int_x^B (A)}$ C) $\dfrac{\vdash A}{\vdash \int_{x_1, x_2, ..., x_n}^{B_1, B_2, ..., B_n} (A)}$

B) $\dfrac{\vdash A;\ \vdash A \to B}{\vdash B}$ D) $\dfrac{\vdash A_1, \vdash A_2, ..., \vdash A_n, \vdash A_1 \to (A_2 \to (A_3 \to (...(A_n \to L)...)))}{\vdash L}$

137. Quyidagi berilgan formulalardan bir vaqtda o'rniga qo'yish formulasini toping.

A) $\dfrac{\vdash A}{\vdash \int_x^B (A)}$ C) $\dfrac{\vdash A}{\vdash \int_{x_1, x_2, ..., x_n}^{B_1, B_2, ..., B_n} (A)}$

B) $\dfrac{\vdash A;\ \vdash A \to B}{\vdash B}$ D) $\dfrac{\vdash A_1, \vdash A_2, ..., \vdash A_n, \vdash A_1 \to (A_2 \to (A_3 \to (...(A_n \to L)...)))}{\vdash L}$

138. Quyidagi berilgan formulalardan xulosa qoidasi formulasini toping.

A) $\dfrac{\vdash A}{\vdash \int_x^B (A)}$ C) $\dfrac{\vdash A}{\vdash \int_{x_1, x_2, ..., x_n}^{B_1, B_2, ..., B_n} (A)}$

B) $\dfrac{\vdash A;\ \vdash A \to B}{\vdash B}$ D) $\dfrac{\vdash A_1, \vdash A_2, ..., \vdash A_n, \vdash A_1 \to (A_2 \to (A_3 \to (...(A_n \to L)...)))}{\vdash L}$

139. Quyidagi berilgan formulalardan murakkab xulosa qoidasi formulasini toping.

A) $\dfrac{\vdash A}{\vdash \int_{x}^{B}(A)}$ C) $\dfrac{\vdash A}{\vdash \int_{x_1,x_2,...,x_n}^{B_1,B_2,...,B_n}(A)}$

B) $\dfrac{\vdash A;\ \vdash A \to B}{\vdash B}$ D) $\dfrac{\vdash A_1, \vdash A_2, ..., \vdash A_n, \vdash A_1 \to (A_2 \to (A_3 \to (...(A_n \to L)...)))}{\vdash L}$

140. Quyidagi berilgan formulalardan kontropozisiya qoidasi formulasini toping.

A) $\dfrac{\vdash A \to B,\ \vdash B \to C}{\vdash A \to C}.$ C) $\dfrac{\vdash A \to B}{\vdash \overline{B} \to \overline{A}}$

B) $\dfrac{\vdash A \to \overline{\overline{B}}}{\vdash A \to B}$ D) $\dfrac{\vdash \overline{\overline{A}} \to B}{\vdash A \to B}$

JAVOBLAR:

1.To'plamalr nazariyasi

1.1. 1) noto'g'ri 2) to'g'ri 3) to'g'ri 4) noto'g'ri 5) noto'g'ri 6) to'g'ri 7) to'g'ri 8) noto'g'ri 9) noto'g'ri 10) to'g'ri 11) noto'g'ri 12) to'g'ri.

1.2. 1) 4 2) 4 3) 0 4) 0 5) 1 6) 1.

1.3. 1) noto'g'ri 2) to'g'ri 3) noto'g'ri 4) to'g'ri 5) to'g'ri 6) noto'g'ri 7) to'g'ri 8) noto'g'ri 9) to'g'ri 10) to'g'ri.

1.4. 1) to'g'ri 2) noto'g'ri 3) noto'g'ri 4) to'g'ri 5) to'g'ri 6) noto'g'ri 7) noto'g'ri 8) noto'g'ri 9) to'g'ri 10) to'g'ri 11) to'g'ri 12) noto'g'ri 13) noto'g'ri 14) noto'g'ri 15) noto'g'ri 16) to'g'ri

1.5. $A\cap B=\{4,6\}$, $A\cup B=\{2,3,4,5,6,8\}$, $C\cap D=\{4\}$, $B\otimes C=\{2,5,7\}$,
$\overline{A}=\{1,2,7,8\}$, $\overline{B\cap D}=\{1,3,5,6,7,8\}$, $\overline{\overline{A}\cup\overline{B}\cup\overline{C}}=\{4,6\}$,

$(A-B)\cup(C-D)=\{3,5,6,7,8\}$, $2^A\cap 2^B=(\{4\},\{6\},\{4,6\})$,

$2^D-2^B=(\{1\}, \{1,2\}, \{1,4\}, \{1,2,4\})$.

1.7. 1) $M_2\cap M_3$, 2) $M_2\cap M_3\cap M_5$ 4) $(M_2\cap M_5)\setminus M_3$

1.9. Bajariladi: 6), 8), 10), 12), 13), 14), 16), 17).

1.10. 2) yutish qonuni; 4) $A(\overline{A}\cup B)=A\overline{A}\cup AB=AB$;

6) *A-AB=A-B* $|\,A-B=A\overline{B}\,|$- formuladan foydalanamiz, $A-AB=A\overline{AB}=$
$A(\overline{A}\cup\overline{B})=A\overline{A}\cup A\overline{B}=A\overline{B}=A-B.$

8) $A\cup(B-A)=A\cup(B\overline{A})=(A\cup B)(A\cup\overline{A})=A\cup B;$

10) $A\otimes(A\otimes B)=B;$ $|\,A\otimes B=A\overline{B}\cup\overline{A}B\,|$ - formuladan foydalanamiz,
$A\otimes(A\otimes B)=A\otimes(A\overline{B}\cup\overline{A}B)=A(\overline{A\overline{B}\cup\overline{A}B})\cup\overline{A}(A\overline{B}\cup\overline{A}B)==AB\cup\overline{A}B=B$.

12) $A\cup B=(A\otimes B)\cup AB;(A\otimes B)\cup AB=A\overline{B}\cup\overline{A}B\cup AB=$
$=A\cup\overline{A}B=A\cup B;$

14) $\overline{A\otimes B}=\overline{\overline{A}B\cup A\overline{B}}=(A\cup\overline{B})(\overline{A}\cup B)=AB\cup(\overline{A}\overline{B})$;

16) $AB \cup A\overline{B} = A(B \cup \overline{B}) = A$;

18) $A \otimes \overline{B} = A\overline{\overline{B}} \cup \overline{A}\overline{B} = AB \cup \overline{A}\overline{B} = \overline{\overline{A}}B \cup \overline{A}\overline{B} = \overline{A} \otimes B = AB \cup (\overline{A \cup B})$.

1.15. 2) $\overline{A/B} \cap (C \cup D) = \{0,2,5,6,8,9\}$; 4) $(A \cup \overline{B \cup C}) \cap D = \{3,6,9\}$;

6) $(A \cup B \cup C) \cap D = \{3,8,9\}$; 8) $\overline{(\overline{A \cup B} \cup C) \cap D} = \{6,8,9\}$.

1.16. 2) $\overline{A/B} \cap (C \cup D) = \{-4,-3,-2,3,4,5\}$; 4) $(A \cup \overline{B \cup C}) \cap D = \{1,4,5\}$;

6) $(A \cup B \cup C) \cap D = \{-2,1\}$; 8) $\overline{(\overline{A \cup B} \cup C) \cap D} = \{1,4,5\}$.

1.17. 2) $\begin{cases} \overline{A} = B\overline{C}; \\ \overline{C} \subseteq D; \\ AD = \overline{B}CD; \\ B = CD. \end{cases} \Rightarrow \quad \overline{A} = CD\overline{C} = \varnothing$

$\begin{cases} \\ \overline{C} \subseteq D; \\ A = \overline{CD}C = C\overline{D} = CA \Rightarrow C = U \\ B = D \end{cases} \Rightarrow \begin{cases} C = A = U \\ B = D \end{cases}$

1.18. 1) $\begin{cases} X \subseteq Z \subseteq \overline{W}, \\ Y \subseteq W, \\ X \cup Y \subseteq Z \cup W. \end{cases} = \begin{cases} X = Z \\ Y = W. \end{cases}$

2) $\begin{cases} C \otimes D \subseteq A, \\ B \cup D \subseteq A \cup C, \\ A - D \subseteq C - B. \end{cases} = \begin{cases} \overline{A} \subseteq CD, \\ B - C \subseteq \overline{A}, \\ A \subseteq C \cup D. \end{cases}$

1.19. 1) $X = C - B$ 2) $X = (C - A) \cup B$

1.21. Noto'g'ri: 4), 6).

1.22. 1) $\{1, 3, 5\} = \{1, 3, 5, 1\}$; 4) $\{a, b, c\} \neq \{\{a\}, \{b\}, \{c\}\}$

2) $\{11, 13\} \neq \{\{11, 13\}\}$; 5) $\{\{a, b\}, c\} \neq \{a, \{b, c\}\}$

3) $\{a, b, c\} = \{a, b, a, c\}$; 6) $\{x \in R\ 22 \leq x \leq 3\} = \varnothing$.

1.24. 2) $\overline{(A\cup B)\cap(\overline{A}\cup B)\cap(A\cup\overline{B})}=(\overline{A}\cap\overline{B})\cup(A\cap\overline{B})\cup(\overline{A}\cap B)=$

$=\overline{B}\cap(\overline{A}\cup A)\cup(\overline{A}\cap B)=\overline{B}\cup(\overline{A}\cap B)=\overline{B}\cup\overline{A}.$ 4) $A\otimes A\otimes A=\varnothing$

6) $(\overline{A}\cup B\cup\overline{C})\cap(A\cap\overline{B}\cap C)\cap(\overline{A\cup C})=((\overline{A}\cap A\cap\overline{B}\cap C)\cup$

$\cup(B\cap A\cap\overline{B}\cap C)\cup(\overline{C}\cap A\cap\overline{B}\cap C))\cap(\overline{A\cup C})=(\overline{A\cup C})$.

1.26. 6 nafar talabalar kimyo va biologiya to'garaklarga qatnashadilar. 12 nafar talabalar faqat kimyo to'garagiga qatnashadilar.

1.27. a) Ikkala sport turiga qiziqqan talabalar soni kamida 5та. b) Kamida bitta sport turiga qiziqqan o'quvchilar soni 25ta bo'lishi mumkin.

1.29. PMIda 40 talaba bor. Ularning 11tasi faqat Delphini o’rganadi.

1.31. 22 ta **1.32.** 16 kun.

1.33. Uchala tadbirda 46 ta Kostromaliklar bo’lgan.

1.34. 2 ta talaba ortiqcha.

2.16. ha **2.17.** ha **2.18.** ha **2.19.** ha **2.20.** ha.

2.21. τ·σ={<1;2>; <2;2>; <1;1>},σ·τ={<1;2>, <1;3>, <2;2>, <3;2>, <3;3>}.

2.22. Domt= Imt={1,2}, refleksiv, simmetrik, antisim-metrik, tranzitiv.

2.23. Domt={1}, Imt={5}, antirefleksiv, antisimmetrik, tranzitiv.

2.24. Domt=Imt={1,2,3,4,5} simmetrik, tranzitiv.

2.25. Domt=Im={1,2,4,5} antirefleksiv, simmetrik.

2.26. Domt=Imt=N refleksiv.

2.27. Domt={1,4,9,…,n^2,…} Imt=N antirefleksiv.

2.28. Domt=N, Imt=N\{1} antirefleksiv, antisimmetrik, tranzitiv, bog‘li.

2.29. Imt=N\M_{12} Domt=N antirefleksiv, antisimmetrik.

2.30. Domt=Imt=N antirefleksiv, simmetrik tranzitiv.

2.31. Domt=Imt=N refleksiv, simmetrik,tranzitiv.

2.32. Mavjud.Masalan. τ={<1,1>,<2,2>,<4,2>,<8,3>}⊂M_{10}xM_{10}.

2.33. 1, 2^4-1, 2^9-1, 2^n-1.

2.34. Domτ={9,10}, Imt={1,2} antirefleksiv, antisimmetrik, tranzitiv.

2.35. Domτ={1,2,3,4} Imτ={1,4,9} antisimmetrik.

2.36. Domτ=Imτ={2,3,4,6} antirefleksiv, simmetrik.

2.37. Domτ={1,2,3} Imτ=M_{10}\{1} antirefleksiv, antisimmetrik, trnazitiv**.**

2.38. τ^{-1}={<1,1>,<1,2>,<3,2><3,3>}**.** **2.39.** $\sigma^{-1}=\sigma$**.**

2.40. ρ^{-1}={<2,1>,<3,1>,<3,2>}**.**

2.41. $\tau\sigma$ ={<1,1>,<1,3>,<2,3>}; $\sigma\tau$={<1,2>, <2,4>}; τ^2={<1,1>}, σ^2={<1,3>, <2,4>}**.** **2.42.** $\tau\sigma$ ={<1,1>, <1,3>, <2,3>}.

2.43. $\sigma\tau$={<1,2>,<2,4>}. **2.44.** τ^2={<1,1>}.

2.45. σ^2={<1,3>,<2,4>}.

2.Mulohazalar algebrasi

1.1. Bo'ladi:1), 3), 5), 10, 12),13).

1.2.1) yog'on 2) chin. **1.3.** 1) chin 2) yolg'on.

1.4. 1) 33 soni 7 ga qoldiqsiz bo'linmaydi; 2) $255 \geq 258$; 3) ΔABC - to'g'ri burchakli uchburchak emas; 4) Stol – oq emas; 5) Barcha tub sonlar, juft son bo'ladi; 6) ABCD to'rtburchak romb emas.

1.5. Formula bo'la oladi; 3), 6), 7), 12).

1.6. Formulalar soni: 1) 8; 2) 12; 3) 5; 4) 9; 5) 10; 6) 10; 7) 7; 8) 19; 9) 14; 10) 22.

1.7. Qism formulalar soni berilgan formulani hisoblaganda: 1) 11; 2) 8; 3) 9; 4) 7; 5) 8; 6) 8; 7) 11; 8) 11; 9) 7; 10) 9.

1.8. Quyidagi formulalarning chinlik jadvallarini tuzing: 1) $(x\&y) \vee z$;

x	y	z	$x\&y$	$(x\&y) \vee z$;
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

1.9. 1) bajariluvchi; 2) bajariluvchi; 3) bajariluvchi; 4) tavtologiya; 5) aynan yolg'on; 6) bajariluvchi; 7) bajariluvchi; 10) bajariluvchi.

1.10. 1) $\neg(P \to \neg P)$ formula chin bo'lishi uchun $P \to \neg P$ formula yolg'on bo'lishi kerak, implikasiya ta'rifidan, P=1 qiymatni qabul qilishi kerak.

2) implikasiya ta'rifidan, $P \to Q$ chin va $Q \to P$ yolg'on qiymatni qabul qilsa $(P \to Q) \to (Q \to P)$ formula yolg'on qiymatni qabul qiladi. 2-qismdan faqatgina Q chin va P yo'g'on bo'lgandagina berilgan formula yolg'on qiymatni qabul qiladi. Demak, $F(P,Q)=1$ bo'lishi ucun, $F(1,1)=F(1,0)=F(0,0)=1$.

3) $F(1,0,1)=F(1,0,0)= F(0,0,1)$.

4) $F(1,1,1)=F(1,1,0)= F(1,0,0)= F(0,1,1)=F(0,1,0)= F(0,0,1)= F(0,0,0)=1$.

5) $F(1,1,1)=F(1,1,0)= F(0,1,1)=F(0,1,0)= F(0,0,1)= F(0,0,0)=1$.

6) $F(1,1,1)=F(1,1,0)=F(1,0,1)=F(1,0,0)=F(0,1,1)=F(0,1,0)=F(0,0,1)=F(0,0,0)=1$ – tavtologiya.

7) $F(1,1,1)=F(1,1,0)=F(1,0,1)=F(1,0,0)=F(0,1,1)=F(0,1,0)=F(0,0,1)=F(0,0,0)=1$ – tavtologiya.

8) $F(1,1,1)=F(1,1,0)= F(1,0,1)= F(1,0,0)= F(0,1,1) =F(0,1,0) = F(0,0,1)=1$.

9) aynan yolg'on formula;

10) $F(1,1,1)=F(1,1,0)=F(1,0,1)=F(1,0,0)=F(0,1,1)=F(0,1,0)=F(0,0,1)=F(0,0,0)=1$ – tavtologiya.

1.11. 1) $((X \to (Y \wedge Z)) \to (\neg Y \to \neg X)) \to \neg Y$ formula yolg'on bo'lishi uchun implikasiya ta'rifidan $((X \to (Y \wedge Z)) \to (\neg Y \to \neg X))$ chin va $\neg Y$ yolg'on bo'lishi kerak. Bundan Y=1 bo'lishini aniqlaymiz. $((X \to (Y \wedge Z)) \to (\neg Y \to \neg X))$ chin bo'lishini ko'rish uchun teskari isbotdan foydalanamiz. Implikasiya ta'rifidan $(X \to (Y \wedge Z)$ chin va $(\neg Y \to \neg X)$ yolg'on bo'lishi kerak. Y=1 ekanligidan $(\neg Y \to \neg X)$ yolg'on bo'lmasligi aniqlanadi. Demak, $((X \to (Y \wedge Z)) \to (\neg Y \to \neg X))$ formula tavtologiya ekan. $F(1,1,1)=F(1,1,0)= F(0,1,1)=F(0,1,0)=0$. 2) $((X \vee Y) \vee Z) \to ((X \vee Y) \wedge (X \vee Z))$ formula yolg'on bo'lishi uchun implikasiya ta'rifidan $(X\vee Y)\vee Z$ chin va $(X\vee Y)\wedge(X\vee Z)$ yolg'on bo'lishi kerak. $(X\vee Y)\vee Z$ chin bo'lishi uchun $X,Y,Z-$propozisional o'zgaruvchilardan kamida birtasi 1 bo'lishi kerak, ammo 2-shartdan $X\neq 1$.

F(0,0,0) =*F*(0,0,1)=*F*(0,1,0)=*F*(0,1,1)=0.

3) *F*(0,1,1)=0. 5) *F*(1,0)= *F*(0,0)=0. 6) *F*(1,0,1)=*F*(1,0,0)=0.

7) *F*(1,0)= 0. 8) *F*(1,0,1)=0. 9) *F*(1,1,0)=0.

1.13. 1. $B\longrightarrow A=0$; 2. $(\overline{A}\wedge B)\to(\overline{A}\vee B)=1$; 3. $\overline{B}\longrightarrow A=0$;

4. $B\longrightarrow\overline{A}=1$; 5. $(\overline{A}\longrightarrow B)\leftrightarrow A=0$; 6. $\overline{B}\to A=1$;

7. $A=0$; 8. $B=1$; 9. $B\longrightarrow A=0$;

10. $A\to B=0$; 11. $\overline{A}\leftrightarrow\overline{B}=1$; 12. $(A\longrightarrow B)\wedge(\overline{A}\leftrightarrow\overline{B})=1$.

1.14. 1), 2), 3), 4), 5), 7)-ha yetarli; 6)-yetarli emas.

1.15. 1) ha; 2) yo'q; 3) ha; 4) yo'q; 5) yo'q; 6) ha; 7) ha; 8) ha; 9) yo'q; 10) yo'q; 11) yo'q.

1.16. 1), 2), 3)- formula bo'lmaydi; 4), 5), 6)- formula bo'ladi.

1.17. 1) $((A\to B)\to B)$; 2) $(A\vee(B\wedge C))$;

3) $((A\to((B\wedge C)\vee A))\leftrightarrow B)$; 4) $(A\leftrightarrow(B\to(B\vee(C\wedge A))))$;

5) $(((X\vee Y)\to\neg X)\to Z)$; 6) $((\neg X\to(Y\vee\neg X))\to Y)$;

7) $(((X\vee Y)\vee Z)\to(X\vee Z))$; 8) $(((X\to Y)\to Z)\to\neg X)$;

9) $(((X\vee Y)\to Z)\to X)$; 10) $((X\to Y)\to(Y\wedge Z))$;

11) $((\neg X\wedge\neg Y)\to(X\wedge Y))$; 12) $(((\neg X\wedge\neg Y)\vee Z)\to(Z\wedge\neg Y))$.

1.18. 1) $(A\to B)\vee C\wedge D$; 2) $A\wedge B\vee(A\to B)\wedge D$;

3) $A\to B\leftrightarrow A\wedge D$; 4) $(A\vee B\leftrightarrow C\to D)\vee B\wedge C$;

5) $\overline{A\vee B}\wedge C\to A\vee C$; 6) $(((B\leftrightarrow C)\to(A\vee B))\wedge(A\vee D))$;

7) $(A\to(B\to C\wedge D))\vee A$; 8) $((A\leftrightarrow B)\wedge C\wedge D)\leftrightarrow B$.

2.2. 1) $A\leftrightarrow\overline{B}=0$ $(A\leftrightarrow\overline{B}=1)$; 2) $\overline{A}\leftrightarrow B=0$ $(\overline{A}\leftrightarrow B=1)$;

3) $B\to A=1$; 4) $(A\to\overline{B})=1$.

2.3. mavjud emas. **2.4.** 1) ha; 2) yo’q; 3) yo’q; 4) ha. **2.5.** 1) 1; 2) 0; 3) 1.

2.6. $(A\wedge B)\to C$ implikasiyaning qiymatini aniqlash uchun berilganlardan qaysi ortiqcha? 1) hammasi kerak; 2) *A* yoki *B* ortiqcha; 3) *A* va *C* ortiqcha; 4) *A* yoki *B* ortiqcha, C ortiqcha.

2.7. 1) ha; 2) yo’q; 3) ha; 4) yo’q; 5) ha; 6) yo’q.

2.8. 1) *A*=1, *B*=1; 2) *A*=1, *B*=1; 3) *A*=0, *B*=1, *C*=0;

4) a) *A*=1, *B*=1, *C*=0; b) *A*=1, *B*=1, *C*=1; c) *A*=0, *B*=0, *C*=0; d) *A*=1, *B*=0, *C*=0.

5) a) *A*=0, *B*=1; b) *A*=0, *B*=0. 6) a) *A*=1, *B*=1, *C*=0; b) *A*=1, *B*=0, *C*=0.

7) *A*=1, *B*=1, *C*=0. 8) a) *A*=0, *B*=1, *C*=1; b) *A*=0, *B*=0, *C*=1.

9) *A*=1, *B*=1, *C*=1. 10) *A*=0, *B*=1, *C*=0.

11) a) *A*=0, *B*=1, *C*=0; b) *A*=1, *B*=1, *C*=0.

2.12. 1) $A\wedge B\to C\leftrightarrow A\wedge C=1$; 2) $(A\leftrightarrow B)\wedge(A\to C)\to D\to\overline{D}=0$;

3) $(A\vee B)\to A\wedge C=0$.

2.16. 1) ha; 2) ha; 3) ha; 4) yo’q; 5) yo’q; 6) yo’q; 7) ha; 8) ha; 9) yo’q; 10) ha; 11) yo’q; 12) ha.

3.5. 1) $X\vee Y$; 6) $\neg X\vee\neg Y$;

2) $X\vee\neg Z$; 7) $\neg X\vee\neg Y$;

3) $(X\vee Y)\wedge(\neg X\vee\neg Y)$; 8) $\neg X\vee Y$;

4) $(X\vee Y\vee Z)\wedge(\neg X\vee\neg Z)$; 9) X;

5) $X\vee Z$; 10) $\neg X\vee Y\vee\neg Z$.

3.6. 1) $(\neg X\vee(Y\wedge Z))\wedge\neg Z$; 2) $\neg X\vee Y\vee\neg Z$;

3) $U\wedge Z\wedge(\neg Y\vee X)$; 4) $\neg Y\wedge(X\vee\neg Z)$;

5) $(X\vee(\neg Y\wedge X)\vee\neg Z)\wedge(\neg Y\vee\neg Z)$;

6) $\neg X\wedge\neg Y\wedge(\neg Z\vee T)$;

7) $(X\wedge Y\wedge\neg Z)\vee(\neg X\wedge(\neg Y\vee Z))\vee\neg Y$;

8) $X\wedge Y\wedge\neg Z$; 9) $X\vee Y$; 10) $\neg X\wedge\neg Y$.

3.7. 1) $\neg(\neg X \wedge Y \wedge \neg Z)$;

2) $\neg(\neg X \wedge \neg Y)$;

3) $\neg(\neg X \wedge Y \wedge \neg Z)$;

4) $\neg(X \wedge \neg(Y \wedge \neg Z))$;

5) $\neg(\neg X \wedge \neg(Y \wedge \neg Z))$;

6) $\neg(\neg(X \wedge \neg Y) \wedge \neg(Y \wedge Z))$;

7) $\neg(\neg X \wedge \neg Y)$;

8) $\neg(\neg(X \wedge \neg(Y \wedge \neg Z) \wedge \neg(\neg Y \wedge Z))$;

9) $\neg(X \wedge \neg Z) \wedge Y$;

10) $\neg(\neg X \wedge Z)$.

3.8. 1) $X \vee \neg Y \vee Z$; 2) $X \vee Y$;

3) $X \vee \neg Y \vee Z$;

4) $\neg X \vee \neg(\neg Y \vee Z)$;

5) $X \vee \neg(\neg Y \vee Z)$;

6) $\neg(\neg X \vee Y) \vee \neg(\neg Y \vee \neg Z)$;

7) $X \vee Y$;

8) $\neg(\neg X \vee Z) \vee \neg(\neg Y \vee Z) \vee \neg(Y \vee \neg Z)$;

9) $\neg(\neg(\neg X \vee Z) \vee \neg Y)$;

10) $X \vee \neg Z$;

3.10.

1) 1;

2) $P \cup Q$;

3) $P \cap Q$;

4) $\neg P \cap \neg R$

5) $P \cup (Q \cap R)$

6) 1;

7) $P \cap Q \cap \neg R$

8) $\neg P \to Q$;

9) $P \to \neg Q$;

10) $\neg P \cup \neg Q$.

3.12. Bajarilmaydi: 4) , 6) , 8).

3.13. 1) $\neg P$; 2) 1; 3) 1; 4) P; 5) P; 6) 1; 7) 1; 8) P; 9) $\neg P$; 10) P.

3.17. 1) $C \to A$, $(A \wedge B) \to C$;

2) $A \to C$, $B \to C$;

3) $A \to B$, $B \to C$;

4) $P \to Q$, $W \to \neg T$;

5) $(M \vee W) \to ((M \wedge W) \vee S)$, $(P \vee \neg Q \vee R) \to T$;

6) $\neg B \to C$, $(A \wedge B) \to \neg C$;

7) P, Q;

8) $\neg P$, $\neg R$;

9) $A \vee (\neg B \wedge C)$;

10) A, B, C.

3.18.

1) $\neg(A \to B), \neg(C \to (A \wedge B))$;

2) $A \leftrightarrow B$;

3) $P \to Q$;

4) $\neg(A \to B)$, $\neg(A \to C)$;

5) $\neg(A \to B)$, $\neg(C \to B)$;

6) $A \to B$;

7) $A \to (B \vee C)$;

8) $\neg A$, B, C;

9) $A \wedge B \wedge \neg C$;

10) $A \to (B \vee C)$.

4.1. 1) $(\neg X \wedge \neg Y \wedge Z \wedge \neg T) \vee (X \wedge Y \wedge Z \wedge \neg T)$;

2) $(X \wedge Y \wedge Z) \vee \neg Y \vee \neg Z$;

3) $(X \wedge Y \wedge \neg Z) \vee (X \wedge Z) \vee \neg X \vee \neg Y$;

4) $X \vee \neg Z$;

5) $(X \wedge \neg Y) \vee Z$;

6) $\neg X \vee \neg Y \vee Z$;

7) $(\neg X \wedge \neg Y) \vee (\neg X \wedge \neg Z) \vee (X \wedge Z)$;

8) $(X \wedge \neg Y) \vee (\neg X \wedge Y) \vee (X \wedge Z)$;

9) $\neg X \vee \neg Y$;

10) X ;

4.2. 1) $(X \vee \neg Y) \wedge (X \vee \neg Y) \wedge Z \wedge \neg T)$;

2) $X \vee \neg Y \vee \neg Z$;

3) $X \vee \neg X$;

4) $X \vee \neg Z$;

5) $(X \vee Z) \wedge (\neg Y \vee Z)$;

6) $\neg X \vee \neg Y \vee Z$;

7) $(\neg X \vee Z) \wedge (X \vee \neg Y \vee \neg Z)$;

8) $(X \vee Y) \wedge (\neg X \vee \neg Y \vee Z)$;

9) $\neg X \vee \neg Y$;

10) X .

4.3. 1) $(\neg X \wedge \neg Y \wedge Z \wedge \neg T) \vee (X \wedge Y \wedge Z \wedge \neg T)$;

2) $(X \wedge Y \wedge Z) \vee (X \wedge \neg Y \wedge Z) \vee (X \wedge Y \wedge \neg Z) \vee (\neg X \wedge Y \wedge \neg Z) \vee$

$\vee (\neg X \wedge \neg Y \wedge Z) \vee (X \wedge \neg Y \wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge \neg Z)$;

3) $(X \wedge Y \wedge Z) \vee (\neg X \wedge Y \wedge Z) \vee (X \wedge \neg Y \wedge Z) \vee (X \wedge Y \wedge \neg Z) \vee$

$\vee (\neg X \wedge \neg Y \wedge Z) \vee (\neg X \wedge Y \wedge \neg Z) \vee (X \wedge \neg Y \wedge \neg Z) \vee$

$\vee (\neg X \wedge \neg Y \wedge \neg Z)$;

4) $(X \wedge Y \wedge Z) \vee (X \wedge \neg Y \wedge Z) \vee (X \wedge Y \wedge \neg Z) \vee$

$\vee (X \wedge \neg Y \wedge \neg Z) \vee (\neg X \wedge Y \wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge \neg Z)$;

5) $(X \wedge Y \wedge Z) \vee (\neg X \wedge Y \wedge Z) \vee (X \wedge \neg Y \wedge Z) \vee (X \wedge Y \wedge$

$\wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge Z)$;

6) $(\neg X \wedge Y \wedge Z) \vee (X \wedge \neg Y \wedge Z) \vee (\neg X \wedge \neg Y \wedge Z) \vee$

$\vee (\neg X \wedge Y \wedge \neg Z) \vee (X \wedge \neg Y \wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge \neg Z)$;

7) $(X \wedge Y \wedge Z) \vee (X \wedge \neg Y \wedge Z) \vee (\neg X \wedge \neg Y \wedge Z) \vee (\neg X \wedge$

$\wedge Y \wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge \neg Z)$;

8) $(X \wedge Y \wedge Z) \vee (\neg X \wedge Y \wedge Z) \vee (X \wedge \neg Y \wedge Z) \vee (X \wedge \neg Y \wedge$
$\wedge \neg Z) \vee (\neg X \wedge Y \wedge \neg Z)$;

9) $(\neg X \wedge Y) \vee (\neg X \wedge \neg Y) \vee (X \wedge \neg Y)$;

10) $(X \wedge Y \wedge Z) \vee (X \wedge Y \wedge \neg Z) \vee (X \wedge \neg Y \wedge Z) \vee (X \wedge \neg Y \wedge \neg Z)$

4.4. 2) $X \vee \neg Y \vee \neg Z$;

3) Mavjud emas;

4) $(X \vee Y \vee \neg Z) \wedge (X \vee \neg Y \vee \neg Z)$;

5) $(X \vee Y \vee Z) \wedge (X \vee \neg Y \vee Z) \wedge (\neg X \vee \neg Y \vee Z)$;

6) $\neg X \vee \neg Y \vee Z$;

7) $(\neg X \vee Y \vee Z) \wedge (\neg X \vee \neg Y \vee Z) \wedge (X \vee \neg Y \vee \neg Z)$;

8) $(X \vee Y \vee Z) \wedge (X \vee Y \vee \neg Z) \wedge (\neg X \vee \neg Y \vee Z)$;

9) $\neg X \vee \neg Y$;

10) $(X \vee Y \vee Z) \wedge (X \vee Y \vee \neg Z) \wedge (X \vee \neg Y \vee Z) \wedge (X \vee \neg Y \vee \neg Z)$

4.5.

1) $\neg X \wedge \neg Y$;

2) $\neg X \wedge Y$;

3) $X \wedge Y$;

4) $X \wedge \neg Y \wedge \neg Z$;

5) $\neg X \wedge \neg Y \wedge Z$;

6) $X \wedge \neg Y \wedge \neg Z \wedge T$;

7) $\neg X \wedge Y \wedge \neg Z \wedge \neg T$;

8) $\neg X \wedge \neg Y \wedge \neg Z \wedge T$;

9) $X \wedge \neg Y \wedge Z$;

10) $X \wedge Y \wedge Z \wedge \neg T$.

4.6. 1) $(\neg X \wedge \neg Y) \vee (X \wedge Y)$;

2) $X \wedge \neg Y$;

3) $(\neg X \wedge Y \wedge \neg Z) \vee (X \wedge \neg Y \wedge Z) \vee (X \wedge Y \wedge Z)$;

4) $(\neg X \wedge Y \wedge Z) \vee (X \wedge Y \wedge \neg Z)$;

5) $(X \wedge \neg Y \wedge \neg Z) \vee (\neg X \wedge Y \wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge Z)$;

6) $(\neg X \wedge Y \wedge Z) \vee (X \wedge \neg Y \wedge Z) \vee (X \wedge Y \wedge \neg Z) \vee (X \wedge Y \wedge Z)$;

7) $(X \wedge \neg Y \wedge Z) \vee (\neg X \wedge Y \wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge \neg Z)$;

8) $(\neg X \wedge Y) \vee (X \wedge \neg Y) \vee (\neg X \wedge \neg Y)$;

9) $(X \wedge Y \wedge \neg Z \wedge \neg T) \vee (\neg X \wedge \neg Y \wedge Z \wedge T)$;

10) $(\neg X \wedge Y \wedge \neg Z \wedge T) \vee (X \wedge \neg Y \wedge Z \wedge \neg T) \vee (X \wedge \neg Y \wedge \neg Z \wedge \neg T) \vee$
$\vee (X \wedge Y \wedge Z \wedge \neg T)$;

4.7. 1) $X \vee Y$;

2) $\neg X \vee Y$;

3) $\neg X \vee \neg Y$;

4) $\neg X \vee Y \vee \neg Z$;

5) $X \vee Y \vee \neg Z$;

6) $\neg X \vee Y \vee Z \vee \neg T$

7) $X \vee \neg Y \vee Z \vee T$;

8) $\neg X \vee Y \vee \neg Z \vee \neg T$;

9) $X \vee \neg Y \vee \neg Z \vee T$;

10) $\neg X \vee \neg Y \vee \neg Z$;

11) $X \vee \neg Y \vee \neg Z$.

4.8. 1) $(X \vee \neg Y) \wedge (X \vee Y)$;

2) $X \vee \neg Y$;

3) $(X \vee \neg Y \vee \neg Z) \wedge (\neg X \vee \neg Y \vee \neg Z)$;

4) $(\neg X \vee Y \vee Z) \wedge (\neg X \vee Y \vee \neg Z)$;

5) $(X \vee \neg Y \vee \neg Z) \wedge (X \vee Y \vee Z) \wedge (X \vee \neg Y \vee Z)$;

6) $(\neg X \vee \neg Y \vee \neg Z) \wedge (X \vee Y \vee \neg Z) \wedge (\neg X \vee \neg Y \vee Z) \wedge (\neg X \vee Y \vee Z)$;

7) $(\neg X \vee \neg Y \vee Z \vee \neg T) \wedge (X \vee Y \vee \neg Z \vee T) \wedge$

$\wedge(\neg X \vee Y \vee \neg Z \vee T) \wedge (X \vee Y \vee \neg Z \vee \neg T) \wedge (X \vee Y \vee Z \vee T)$;

8) $(X \vee \neg Y) \wedge (\neg X \vee Y) \wedge (\neg X \vee \neg Y)$;

9) $(\neg X \vee Y \vee Z \vee T) \wedge (X \vee \neg Y \vee \neg Z \vee \neg T)$;

10) $(X \vee \neg Y \vee Z \vee \neg T) \wedge (\neg X \vee Y \vee \neg Z \vee T) \wedge$

$\wedge(\neg X \vee Y \vee Z \vee \neg T) \wedge (X \vee \neg Y \vee \neg Z \vee T)$;

11) $(X \vee Y \vee Z) \wedge (X \vee \neg Y \vee Z) \wedge (\neg X \vee \neg Y \vee \neg Z)$.

4.9. 1) $(\neg X \wedge \neg Y) \vee (\neg X \wedge Y) \vee (X \wedge \neg Y)$;

2) $(X \wedge Y \wedge Z) \vee (\neg X \wedge Y \wedge Z) \vee (X \wedge \neg Y \wedge Z) \vee (\neg X \wedge \neg Y \wedge Z)$;

3) $(\neg X \wedge Y \wedge Z) \vee (X \wedge \neg Y \wedge Z) \vee (X \wedge Y \wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge Z) \vee$

$\vee (X \wedge \neg Y \wedge \neg Z)$;

4) $(X \wedge Y \wedge Z) \vee (X \wedge Y \wedge \neg Z) \vee (X \wedge \neg Y \wedge Z) \vee (X \wedge \neg Y \wedge \neg Z) \vee$

$\vee (\neg X \wedge Y \wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge Z) \vee (\neg X \wedge \neg Y \wedge \neg Z)$;

5) $(X \wedge Y \wedge Z \wedge T) \vee (\neg X \wedge Y \wedge Z \wedge T) \vee (X \wedge \neg Y \wedge Z \wedge T) \vee$

$\vee (\neg X \wedge \neg Y \wedge Z \wedge T) \vee (X \wedge \neg Y \wedge \neg Z \wedge T)$;

6) $(\neg X \wedge \neg Y \wedge \neg Z \wedge \neg T) \vee (X \wedge Y \wedge Z \wedge T)$;

7) $F(X,Y,Z) = (\neg X \wedge \neg Y \wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge Z) \vee (\neg X \wedge Y \wedge Z) \vee$
$\vee (X \wedge \neg Y \wedge \neg Z) \vee (X \wedge Y \wedge \neg Z)$.

8) $(X \wedge \neg Y \wedge Z) \vee (X \wedge Y \wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge Z) \vee$

$\vee (\neg X \wedge Y \wedge \neg Z)$;

9) $(\neg X \wedge \neg Y \wedge \neg Z \wedge \neg T) \vee (\neg X \wedge \neg Y \wedge \neg Z \wedge T) \vee (\neg X \wedge \neg Y \wedge Z \wedge$

$\wedge \neg T) \vee (X \wedge Y \wedge Z \wedge \neg T) \vee (X \wedge Y \wedge Z \wedge T)$;

10) $(\neg X \wedge \neg Y \wedge Z) \vee (\neg X \wedge Y \wedge \neg Z) \vee (\neg X \wedge Y \wedge Z) \vee$

$\vee(X \wedge \neg Y \wedge \neg Z) \vee (X \wedge \neg Y \wedge Z) \vee (X \wedge Y \wedge \neg Z);$

11) $(\neg X \wedge \neg Y \wedge \neg Z) \vee (\neg X \wedge Y \wedge \neg Z) \vee (X \wedge Y \wedge \neg Z) \vee$

$\vee (X \wedge Y \wedge Z)$.

4.11. **1.** 1) $A(1,1)= A(1,0)= A(0,1)=1$ 2) $B(0,0)=1$;

3) $C(1,1,1)= C(1,1,0)= C(0,1,1)= C(1,0,1)= C(1,0,0)=1$

4) $D(0,0,0)= D(0,1,0)= D(1,1,0)= D(1,0,1)= D(1,1,1)=1$;

5) $E(1,1)= E(1,0)= E(0,1)= E(0,0)=1$;

4.16. $f_1(x,y,z) = (X \wedge Y \wedge \neg Z) \vee (X \wedge \neg Y \wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge Z);$

$f_2(x,y,z) = (X \wedge Y \wedge \neg Z) \vee (X \wedge \neg Y \wedge \neg Z) \vee (\neg X \wedge Y \wedge \neg Z);$

$f_3(x,y,z) = (X \wedge Y \wedge Z) \vee (X \wedge Y \wedge \neg Z) \vee (X \wedge \neg Y \wedge \neg Z) \vee$

$\vee (\neg X \wedge Y \wedge \neg Z) \vee (\neg X \wedge \neg Y \wedge Z);$

$f_4(x,y,z) = (X \wedge Y \wedge Z) \vee (X \wedge \neg Y \wedge Z) \vee (X \wedge \neg Y \wedge \neg Z) \vee$

$\vee (\neg X \wedge Y \wedge \neg Z);$

$f_5(x,y,z) = (X \wedge \neg Y \wedge \neg Z) \vee (\neg X \wedge Y \wedge Z);$

4.17

x	y	z	3)	4)	5)	6)
1	1	1	1	0	1	1
1	1	0	1	1	1	0
1	0	1	1	0	1	0
1	0	0	1	1	0	1
0	1	1	0	1	0	0
0	1	0	0	0	0	1
0	0	1	1	0	0	1
0	0	0	0	0	1	1

4.23. Aynan chin formulalar- 1),5), 6). Qolgani-aynan yolg’on farmulalar.

4.25. O’zaro teng kuchli formulalar-1), 2), 4), 5), 6), 8), 9). Qolgani- o’zaro teng kuchli formulalar emas.

4.26. 1) $\neg X \vee \neg Y,$ ixtiyoriy tavtologiya;

2) $X \to Y,$ ixtiyoriy tavtologiya;

3) $X \vee Y,$ ixtiyoriy tavtologiya;

4) $\neg X,\ \neg X \vee Y,\ \neg X \vee \neg Y,$ ixtiyoriy tavtologiya;

5) mavjud emas; 6) $X \to Y,$

7) ixtiyoriy formula;

8) $\neg X, Y,\ X \vee Y,\ \neg X \wedge Y,\ X \to Y,\ \neg(X \wedge Y), \neg(X \leftrightarrow Y),$ ixtiyoriy tavtologiya;

9) $\neg Y,\ Y \to X,\ \neg(X \wedge Y),$ ixtiyoriy tavtologiya; 10) ixtiyoriy formula.

4.27. 1) $(X \to Y) \vee Z,$

2) $\neg X \vee Z,\ \neg X \vee Y \vee Z,\ \neg X \vee \neg Y \vee Z,$ ixtiyoriy tavtologiya;

3) $X \to \neg Y,\ Y \to \neg Z,\ Y \to (\neg X \wedge \neg Z), X \to (Y \to \neg Z),\ X \to (Y \to Z),$ $Z \to (Y \to X),\ X \to ((Y \to Z)) \wedge (Z \to (Y \to X)),$ ixtiyoriy tavtologiya;

4) mavjud emas;

5) $(\neg X \wedge \neg Z) \vee Y,\ (X \leftrightarrow Z) \vee Y,\ Z \to Y, Z \to (X \vee Y),\ X \to Y,$ $X \to (Y \vee Z),\ X \to (Z \to Y),$ ixtiyoriy tavtologiya;

6) ixtiyoriy tavtologiya;

7) $(X \vee Y) \wedge \neg Z,$

8) $X \to Y, X \to (Y \vee Z),\ X \to (Z \to Y),$ ixtiyoriy tavtologiya;

9) bunday formula mavjud emas;

10) ixtiyoriy tavtologiya.

3. Bul funksiyalari

1.2. $2^{2^n - 1}$. **1.3.** $2^{2^n - 1}$.

1.5. 1) 11111011; 2) 00100001; 3) 11001111; 4) 00000010; 5) 01111110;

6) 11100001; 7) 00111001; 8) 00010011; 9) 111100111; 10) 11111111.

1.6. Teng kuchli bo'lmagan funksiyalar: 3), 7), 9), 10); Qolgani teng kuchli funksiyalar. **1.8.** Yo'q.

1.13. 1) $x \vee y = (x' \cdot y')'$; 2) $x \to y = (x \cdot y')'$;

3) $x \leftrightarrow y = (x \cdot y')' \cdot (x' \cdot y)'$; 4) $x + y = ((x \cdot y')' \cdot (x' \cdot y)')'$;

5) $x \mid y = (x \cdot y)'$; 6) $x \downarrow y = x' \cdot y'$;

1.16. 1) $x' = x \downarrow x$; 2) $x \vee y = (x \downarrow y) \downarrow (x \downarrow y)$;

3) $x \to y = ((x \downarrow x) \downarrow y) \downarrow ((x \downarrow x) \downarrow y)$;

4) $x \leftrightarrow y = ((x \downarrow x) \downarrow y) \downarrow (x \downarrow (y \downarrow y))$;

5) $x + y = (((x \downarrow x) \downarrow y) \downarrow (x \downarrow (y \downarrow y))) \downarrow (((x \downarrow x) \downarrow y) \downarrow (x \downarrow (y \downarrow y)))$;

6) $x \mid y = ((x \downarrow x) \downarrow (y \downarrow y)) \downarrow ((x \downarrow x) \downarrow y) \downarrow (x \downarrow (y \downarrow y))$;

7) $x \cdot y = (x \downarrow x) \downarrow (y \downarrow y)$.

1.19. 1) x_1 va x_2 -soxta, x_3 -muhim ; 2) hammasi muhim;

3) x_3 -soxta, x_1 va x_2 -muhim; 5) x_1 va x_2 -muhim, x_3 va x_4 -soxta;

6) x_4 -soxta, x_1, x_3 va x_2 -muhim; 7) x_1 -soxta, x_4, x_3 va x_2 -muhim;

8) hammasi soxta; 9) hammasi soxta.

1.21. 1) 0, x; 2) 1, x; 3) 0, $xy, (x \to y)', x, (y \to x)', y, x + y, x \vee y$;

4) $xy, x, y, x \vee y, x \leftrightarrow y, y \to x, x \to y, 1$; 5) x;

6) $xy, x, y, x \vee y$; 7) $0, (x \to y)', (y \to x)', x + y$;

8) $x \leftrightarrow y, y \to x, x \to y, 1$.

1.24. 1) $x \vee y = x' \to y$;

2) $x \cdot y = (x \to y')'$;

3) $x \leftrightarrow y = ((x \to y) \to (y \to x)')'$;

4) $x + y = ((x \to y) \to (y \to x))'$;

5) $x \mid y = x \to y';$

6) $x \downarrow y = (x' \to y)'.$

2.1. n -toq. **2.2.** $2^{2^{n-1}}$.

2.3. $f^* = \overline{x}(y \vee \overline{z}) \vee (x \vee y)(x \vee \overline{z}); \quad g^* = (x\overline{y} \vee z \vee \overline{t})(\overline{x} \vee t)$.

2.6. 1) $x'y'z \vee x'yz' \vee xy'z \vee xyz' \vee xyz;$

2) $x'y'z' \vee x'y'z \vee xyz' \vee xyz;$

3) $x'y'z' \vee xy'z \vee xyz' \vee xyz;$

4) $x'y'z' \vee x'yz \vee xy'z' \vee xy'z \vee xyz;$

5) $x'y'z' \vee x'yz' \vee xy'z' \vee x'yz \vee xyz;$

6) $x'y'z' \vee x'y'z \vee x'yz' \vee x'yz \vee xyz;$

7) $x'y'z' \vee x'yz' \vee x'yz;$

8) $x'y'z' \vee x'yz';$

9) $x'y'z \vee xy'z' \vee xyz;$

10) $x'y'z \vee x'yz' \vee xy'z.$

2.11. O’z-o’ziga ikki taraflama-1), 3), 6), 10);

2) $xyz \vee x'yz \vee xy'z \vee xyz' \vee x'y'z;$

4) $x'y'z' \vee x'yz;$ 5) $x'y'z' \vee x'y'z \vee xy'z' \vee xyz' \vee xy'z \vee xyz;$

7) $x'yz' \vee xyz' \vee xyz;$ 8) $xyz \vee x'yz \vee xy'z \vee xyz' \vee x'y'z;$

9) $x'y'z \vee x'yz' \vee xy'z' \vee xyz'.$

2.12. 1) (00111001); 2) (10100100); 3) (10110010); 4) (11100111);
5) (01001110); 6) (11010100); 7) (00001000); 8) (10101111);
9) (00101011); 10) (01010001).

2.13. O’z-o’ziga ikki taraflama-3), 6), 9).

2.14. Bunday funksiyalar 16 ta: $x;\ x'yz \vee xy'z';\ x'yz' \vee xy'z' \vee xz;\ y;$
$x'y'z \vee xyz \vee xy';\ z;\ x'y'z \vee x'yz' \vee xy'z' \vee xyz;\ x'y'z \vee xyz \vee x'y;\ x';$

$y'z' \vee x'y'z \vee xyz';\ x'y'z' \vee xy'z \vee x'yz \vee xyz'; y';\ z';\ x'z' \vee x'yz \vee xyz';$

$x'y' \vee x'yz \vee xy'z;\ x'y'z \vee xy'z' \vee x'yz'.$

2.17. Masalan: $x'y'zt \vee x'z't' \vee yzt' \vee xy't' \vee xyz't.$

3.1. 1) a) $f(x_1, x_2) = x_1((1 \leftrightarrow x_2) \cdot (1 | x_2)) \vee \bar{x}_1((0 \leftrightarrow x_2) \cdot (0 | x_2));$

b) $f(x_1, x_2) = \overline{x_1} \cdot \overline{x_2};$

4.1. 1) $xy + x + z;$

2) $xyzt + xyz + xyt + xzt + yzt + xz + xt + yz + yt + xy + zt + x + y + z + t;$

3) $x + y + z;$ 4) $xyz + xz + yz + xy + x + y + z;$

5) $xyz + xy + yz + xz;$ 6) $xy\bar{z} + x\bar{y}z + \bar{x}yz + \bar{x}\bar{y}\bar{z}.$

4.6. 1) $xy + xz + y + z;$

2) $yz + x + y + 1;$

3) $xyz + xy + xz + x + y + z + 1;$

4) $yz + x + z + 1;$

5) $xyz + yz + x + z + 1;$

6) $xyz + yz + x + z + 1;$

7) $yz + x + y;$

8) $xyz + xz + 1;$

9) $xyz + xy + yz + x + z + 1;$

10) $xy + yz + x + 1;$

11) $xyz + yz + z + 1.$

4.7. 1) to'g'ri; 2) to'g'ri; 3) noto'g'ri; 4) to'g'ri; 5) to'g'ri; 6) noto'g'ri; 7) to'g'ri; 8) noto'g'ri; 9) noto'g'ri; 10) to'g'ri; 11) to'g'ri.

4.8. 1) 1; 2) 1; 3) 1; 4) 1; 5) 0; 6) 1; 7) 1; 8) 0; 9) 1; 10) 0 ; 11) 1.

4.10. Sakkizta: 0,1, $x, y, x', y', x + y, x \leftrightarrow y.$

4.13. 1) chiziqli emas: $yz + x + y + 1;$

2) chiziqli: $x+y+1$;

3) chiziqli emas: $xy+z+1$;

4) chiziqli emas: $xyz+xy+xz+yz$;

5) chiziqli: $x+1$;

6) chiziqli emas: $xyz+xz+yz+z+1$;

7) chiziqli emas: $xyz+xy+xz$;

8) chiziqli emas: $xyz+xy+xz+yz+x+y+z+1$;

9) chiziqli: $x+y+z$;

10) chiziqli: 1;

11) chiziqli emas: $xyz+xy+xz+y+z+1$.

4.14. Masalan: $f(x,y,z)=xy+yz+y+1$, $g(x,y)=xy$, unda, $h(x,y)=f(x,y,g(x,y))=xy+y(xy)+y+1=y+1$.

4.15. 1) (0, 1, 0), (1, 0, 0)-

2) (0, 1, 1, 0) (0, 1, 0, 1); -

3) (1, 0, 0, 1)< (1, 0, 1, 1);

4) (1, 1, 0)>(1, 0, 0;

5) (0, 1), (1, 0); -

6) (1, 0, 0)<(1, 0, 1);

7) (0, 1, 0, 0)<(0, 1,0,1);

8) (1,0,0,1,0)< (1,0,1,1,0);

9) (0,1,0,0,1) (0,1,0,1,0);-

10) (1, 0, 1)>(1, 0, 0);

4.19. Monoton bul funksiyalari: 1), 3), 6), 10).

4. Mulohazalar hisobi

1.1. Formula bo'ladi: 1), 3), 4), 6), 7); formula bo'lmaydi: 2), 5), 8).

1.3. 1) $\int_{A,B}^{B,C}(L_1)=(B\to C)\to(\overline{C}\vee\overline{B})$;

2) $\int\limits_{A}^{A\to B}(L_2)=(A\to B)\vee((A\to B)\to B);$

3) $\int\limits_{A,C}^{B\to A\wedge B,B}(L_3)=(B\to A\wedge B)\vee(B\to B);$

4) $\int\limits_{A,B}^{A\wedge B,A\vee B}(L_1)=((A\wedge B)\to(A\vee B))\to(\overline{(A\vee B)}\vee\overline{(A\wedge B)})$;

5) $\int\limits_{A,B}^{B,A}(L_2)=B\vee(B\to A);$

6) $\int\limits_{A,B,C}^{A\wedge\overline{A},C,\overline{A}}(L_3)=(A\wedge\overline{A})\vee(C\to\overline{A}).$

1.5. 1) Aksiomma - I_2 ;

2) aksiomma - I_1;

3) aksiomma - I_1;

4) aksiomma - IV_1;

1.12. 1) $W=(F\to(G\to(H\to F)));$

2) $W=(F\to(G\to H))\to((F\to G)\to(F\to H));$

3) $W=\neg G\to\neg G;$

4) $W=(F\to G)\to(H\to(F\to G));$

5) $W=F\to G;$

6) $W=\neg F\to\neg G;$

7) $W=(G\to F)\to(G\to\neg F)\to(G\to F).$

SINOV TESTI JAVOBLARI:

№	0	1	2	3	4	5	6	7	8	9
0		D	A	C	A	C	D	A	C	D
1	B	D	A	C	A	C	D	A	A	D
2	B	B	A	A	A	A	D	A	C	D
3	D	D	A	C	A	B	D	C	C	A
4	B	B	A	B	C	D	D	A	B	C
5	D	C	A	A	A	A	D	B	C	B
6	D	A	A	D	A	A	B	A	B	B
7	C	A	B	D	D	A	D	A	D	D
8	B	D	D	C	B	A	D	A	D	B
9	B	D	A	B	A	C	D	A	D	B
10	B	A	A	A	B	B	A	A	A	A
11	A	A	A	A	A	A	A	A	A	A
12	A	A	A	A	A	A	A	A	A	A
13	A	A	C	C	B	D	A	C	B	D
14	C									

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Mundarija